

CORRESPONDENCE.

ON LOGARITHMS CORRECT TO TEN PLACES OF DECIMALS.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—Having had occasion to require the logarithms of certain numbers to 10 places of decimals, my attention was directed to the Tables of Mr. S. Pineto of St. Petersburg, and I have found these so simple and convenient in working, that I feel sure the readers of the *Journal* will be interested in having their method and use explained.

The method referred to depends on the following simple proposition. If A be the number whose logarithm is sought, and M be any other number whatever, then

$$A = AM \times \frac{1}{M}; \text{ and } \text{Log } A = \text{Log } AM + \text{Log } \frac{1}{M}.$$

The tables of Mr. Pineto consist of two parts ;

- 1st. A subsidiary table of the logarithms to 10 places of the reciprocals of certain multipliers, M , which multipliers, when applied to any number whatever, suffice to bring the first seven figures of the product within the limits 1,000,000, and 1,011,000.
- 2nd. A table of the logarithms to 10 places of all numbers between the aforesaid limits, with tables of proportional parts, which enable the logarithms of any number of not more than 11 digits to be found with the greatest ease.

The tables are comprised in a small volume of 56 pages. The following is an example from the introductory chapter illustrating their use.

Let it be required to find to 10 places the logarithm of the number $\pi = 3.14159\ 26536$. From the auxiliary table we find that the requisite multiplier is 32 and that $\text{Log } \frac{1}{32} = 2.49485\ 00216\ 80$

Then			
$\pi =$	3·14159 26536		
	32	Log $\frac{1}{32}$	= 2·49485 00216·80
	$M\pi = 100\cdot53096\ 49152$	Log 100 5309	= 2·00229 95705·75
		Additions by proportional parts	— 2764·80
		{ 64	39·31
		52	·22
		Log π	= 0·49714 98726·88

It is hardly necessary to say that by a converse process the number can be found corresponding to any logarithm to 10 places.

Unfortunately, Mr. Pineto's tables are out of print; but another set of tables by Mons. A. Namur of Thuin-sur-Sambre, Belgium, based on the same principle, can be readily obtained.*

In the latter, the results are tabulated to 12 places of decimals. The numbers to which the logarithms are given lie between the limits 433,300 and 434,300, and Mons. Namur makes use of the features of the logarithmic differences at this part of the table, to find the logarithm of any number between 433,300,000,000 and 434,300,000,000 without the aid of the usual tables of proportional parts. Hence, the very small compass of 11 pages within which his tables are comprised. I must refer the reader for a further explanation to the introduction to the tables, written by Mons. P. Mansion, Professor at the University of Ghent.

It must be added, however, that, in respect of facility in use, Mr. Pineto's tables are superior to those of Mons. Namur. In the latter it is necessary to apply *two* consecutive multipliers in the great majority of cases to the number whose logarithm is sought to bring the first six figures of the product within the limits given by the tables, and the mode of finding the addition to the tabulated logarithm corresponding to the last six figures of the product requires the exercise of great care in fixing the place of the decimal point.

I send for publication in the *Journal* the annexed short table, compiled by an acquaintance of mine on the principle of Pineto's tables, which will enable the logarithms of numbers to be found to eight places of decimals, by means of the extended portion lying between the numbers 100,000 and 108,000 in any good logarithmic tables, such as those of Hutton, Babbage, Callet, Schrön, &c.

The logarithms of $\frac{1}{M}$ have been obtained from the table of logarithms to 20 places given by Hutton.

I am, Sir,

Your obedient servant,

JAS. CHISHOLM.

* The Tables were published in 1877 at Brussels and Paris by the Royal Academy of Belgium, and the price is 1s.

Table for computing to Eight Decimals the Logarithm of any Number by means of the Eight-figure Logarithms of the Numbers from 100,000 to 108,000, given in the Tables of Hutton, Babbage, Schrön and others.

N	M	$\text{Log } \frac{1}{M}$	Log
108	96	$\bar{2}\cdot 01772,87670$	·987
112	9	$\bar{1}\cdot 04575,74906$	·955
120	88	$\bar{2}\cdot 05551,73279$	·945
121	84	$\bar{2}\cdot 07572,07139$	·936
125	8	$\bar{1}\cdot 09691,00130$	·904
134	77	$\bar{2}\cdot 11350,92748$	·887
140	72	$\bar{2}\cdot 14266,75036$	·877
143	7	$\bar{1}\cdot 15490,19600$	·846
153	66	$\bar{2}\cdot 18045,60645$	·820
162	63	$\bar{2}\cdot 20065,94505$	·817
164	61	$\bar{2}\cdot 21467,01650$	·810
167	6	$\bar{1}\cdot 22184,87496$	·779
179	58	$\bar{2}\cdot 23657,20064$	·772
182	55	$\bar{2}\cdot 25963,73105$	·741
195	54	$\bar{2}\cdot 26760,62402$	·739
197	51	$\bar{2}\cdot 29242,98239$	·731
200	5	$\bar{1}\cdot 30102,99957$	·699
216	48	$\bar{2}\cdot 31875,87626$	·686
224	45	$\bar{2}\cdot 34678,74862$	·676
228	44	$\bar{2}\cdot 35654,73235$	·645
244	41	$\bar{2}\cdot 38721,61433$	·634
250	4	$\bar{1}\cdot 39794,00087$	·603
269	39	$\bar{2}\cdot 40893,53930$	·592
276	38	$\bar{2}\cdot 42021,64034$	·588
277	36	$\bar{2}\cdot 44369,74992$	·557
300	34	$\bar{2}\cdot 46852,10830$	·550
303	33	$\bar{2}\cdot 48148,60601$	·524
323	31	$\bar{2}\cdot 50863,83062$	·510
334	3	$\bar{1}\cdot 52287,87453$	·478
359	29	$\bar{2}\cdot 53760,20021$	·464
371	27	$\bar{2}\cdot 56863,62358$	·431
400	25	$\bar{2}\cdot 60205,99913$	·413
430	24	$\bar{2}\cdot 61978,87583$	·376
455	22	$\bar{2}\cdot 65757,73192$	·355
477	21	$\bar{2}\cdot 67778,07053$	·334
500	2	$\bar{1}\cdot 69897,00043$	·302
539	19	$\bar{2}\cdot 72124,63990$	·288
556	18	$\bar{2}\cdot 74472,74949$	·256
599	17	$\bar{2}\cdot 76955,10786$	·237
625	16	$\bar{2}\cdot 79588,00173$	·209
667	15	$\bar{2}\cdot 82390,87409$	·179
715	14	$\bar{2}\cdot 85387,19643$	·147
771	13	$\bar{2}\cdot 88605,66477$	·114
829	121	$\bar{3}\cdot 91721,46297$	·112
834	12	$\bar{2}\cdot 92081,87540$	·080
900	112	$\bar{3}\cdot 95078,19773$	·074
910	11	$\bar{2}\cdot 95860,73148$	·042
981	102	$\bar{3}\cdot 99139,98282$	·037
991	101	$\bar{3}\cdot 99567,86262$	·033

DIRECTIONS FOR USE OF THE TABLE.

The column M contains multipliers, one of which, properly selected, will always bring the natural number to be dealt with within the limits 10,000,000 and 10,800,000. The column $\text{Log } \frac{1}{M}$ contains the log., to 10 places, of the reciprocals of those multipliers.

To find the log. to eight places of any number over 10,800,000. Take the multiplier corresponding to the first three figures of the number in the column N., or to the next lower if those figures are not found there. Multiply by it the number to be dealt with. Find the log. of the result from one of the tables referred to. Add to that log. the $\text{Log } \frac{1}{M}$ from this table. The sum, properly reduced to eight places, gives the log. required.

To find the natural number to eight places of any log. over .033 . . . Add thereto the $\text{Log } \frac{1}{M}$ corresponding to the first three places of the log. in the above table, or, if not found there, to the next lower places there found. The resulting log. will be between .0000000 and .0330000. Find its natural number, which is then to be multiplied by the corresponding M. The product is the natural number sought.

It will make the result in all cases rather more accurate if 25 in the ninth and tenth places be added to the tabular log. where the last figure has not been increased, and subtracted where it has been increased in those tables which give information on that point.

PREMIUMS FOR CONTINGENT ASSURANCES.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—I have read with interest Mr. Chatham's letter which appears in the last number of the *Journal* (see *J.I.A.*, xxv, 439). For some time I have thought it would be desirable to tabulate corrections, such as those which he gives on pp 441 and 442, by means of which the premium for assuring x against y and for t years longer might be easily deduced from the premium for assuring x against y .

It appears to me, however, not unlikely that the H^M Table, which by some of the best authorities is held inapplicable to the calculation of premiums generally, may, on examination, prove specially inappropriate for quoting rates for contingent assurances.