

## Cross-fringe versus single-fringe probabilities

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High-resolution TEM is well-suited to characterizing nanocrystals, where lattice fringes serve as a source of structural information [1, 2]. Based on 2D lattice fringe images taken at different specimen orientations, 3D lattice parameters can be determined [3-6]. Recent work has shown that lattice fringe-visibility maps, a thin-specimen extension of bend-contour and channeling-pattern maps, can assist crystallographic study in direct space much as do Kikuchi maps in reciprocal space [7]. A nanocrystal can be tilted while the condition for visualizing a set of lattice fringes is maintained so as to "acquire" new lattice fringe normals (co-vectors), and thus continually refine a basis triplet containing information on both the nanocrystal's lattice and its orientation. Local specimen thickness measurements are another promising possibility.

Probabilities inferred from fringe-visibility maps further allow one to quantify the abundance of fringes in collections of randomly-oriented nanoparticles. In the thin-specimen limit, a fringe-visibility band has a band-width proportional to  $d/t$ , rather than the  $\lambda/d$  proportionality expected for large  $t$ , where  $d$  is the lattice spacing,  $t$  is specimen thickness, and  $\lambda$  is electron wavelength. This follows from the expression for bandwidth half-angle at arbitrary thickness:

$$\alpha_{\max} = \sin^{-1} \left[ \frac{df}{t} + \frac{\lambda}{2d} \left( 1 - \left( \frac{df}{t} \right)^2 \right) \right], \quad (1)$$

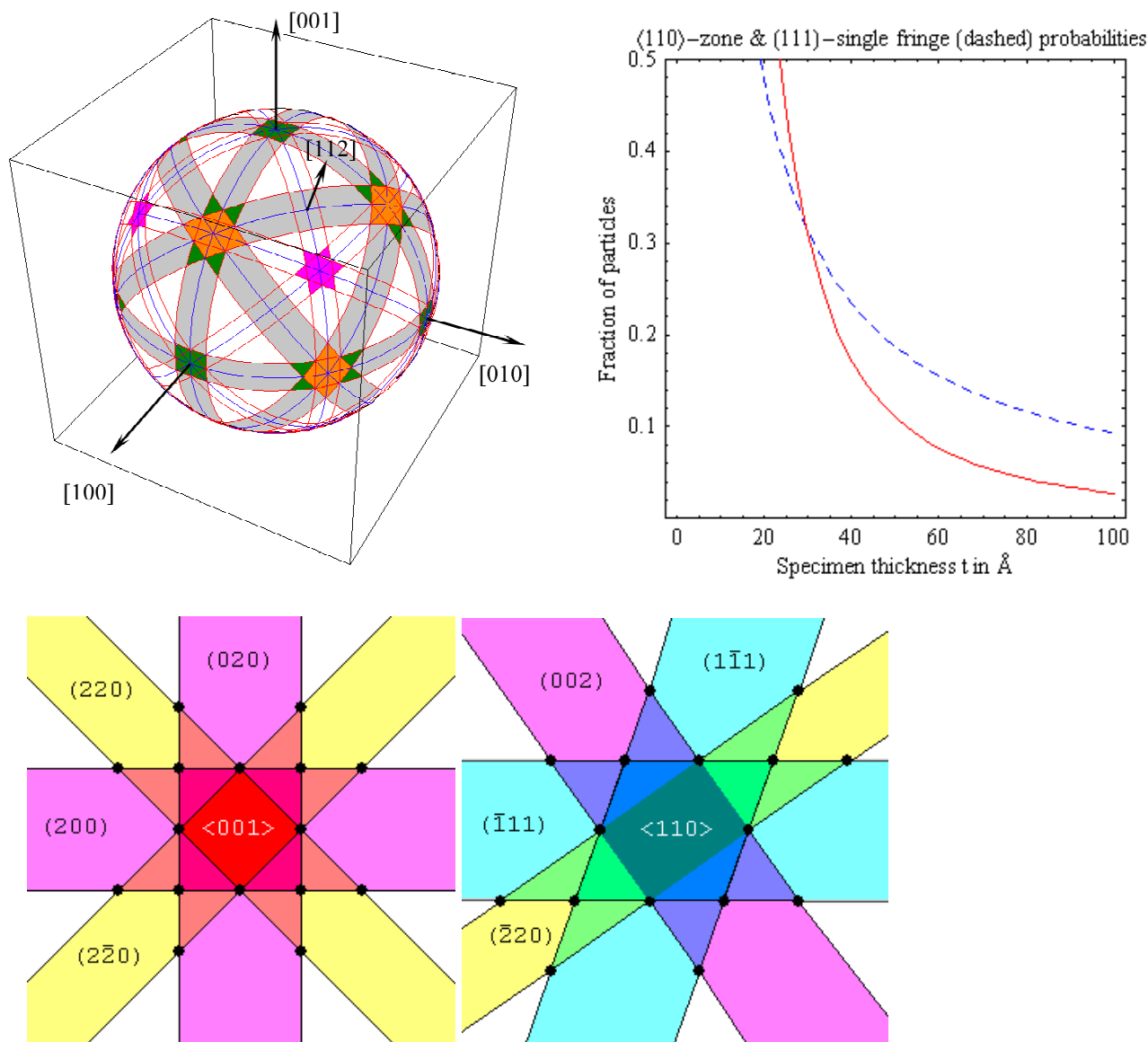
where  $f$  is a "visibility factor" on the order of 1 that empirically accounts for signal-to-noise in the method used to detect fringes [7]. The first term in (1) dominates for  $t < 2 d^2 f/\lambda$  and therefore in typical TEMs for inter-atom spacings in particles 10 nm in size and smaller.

Fig. 1 shows the {200}, {111} and {220} fringe-visibility bands of a spherical f.c.c. nanocrystal. The probability of the (hkl) lattice plane to show is therefore that fraction of the solid angle subtended by the corresponding visibility band, i.e.  $p_{(hkl)} = \sin[\alpha_{\max}] \cong d_{hkl}f/t$  in the thin-specimen limit. Band intersections correspond to regions of visible cross-fringes. Calculation of the area of an intersection between visibility bands [7] indicates that flat-polygon intersection areas are an excellent small-angle approximation, in some cases with errors on the order of  $\alpha_{\max}^6$ . With this approximation, the probability of cross-fringes from lattice planes 1 and 2, whose fringe-visibility bands have half-widths  $\alpha_1$  and  $\alpha_2$  and intersect at an angle of  $\phi$ , is  $p_{1 \times 2} = 2\alpha_1\alpha_2/(\pi \sin\phi)$ .

Figure 2 illustrates the fraction of randomly-oriented fcc particles showing only un-crossed (111) fringes, and that showing  $\langle 110 \rangle$  zone cross-fringes. It is obvious that cross-fringe grains become more abundant than single-fringe grains as the grain diameter  $t$  decreases below 3 nm. This is because the zone area increases as  $(df/t)^2$  while the single-fringe region increases as  $(df/t)$  and decrease in length at the expense of the zones. This model suggests, moreover, that the crossover size is quite sensitive to the visibility factor  $f$  for a given microscope/specimen combination. In such a small size range, though, the broadening of reciprocal spots warrants caution, as deceptive lattice fringes that are "Moires" instead of direct representations of the lattice planes may be formed, as a result of the low-pass filtering of projected potentials by the microscope [2]. Figures 3 and 4 show how zone axis areas (overlaps of bands) change as smaller spacings become reliably visible.

References:

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**Figure 1** (upper left): The  $\{200\}$ ,  $\{111\}$  (both shaded) and  $\{220\}$  (not shaded) fringe-visibility bands of a spherical f.c.c. nanocrystal.

**Figure 2** (upper right): The reversal in relative abundance for cross-fringe versus single-fringe particles as a function of particle diameter.

**Figure 3** (lower left): Polygonal cross-fringe zones for fcc  $\langle 001 \rangle$  in the thin-specimen/small-angle approximations, built up as first  $\{200\}$  and then  $\{220\}$  fringes become visible.

**Figure 4** (lower right). Polygonal cross-fringe zones for fcc  $\langle 110 \rangle$  in the thin-specimen/small-angle approximations.