CORRIGENDA TO THE PAPER ON THE STABILITY OF CRYSTAL LATTICES

IX. COVARIANT THEORY OF LATTICE DEFORMATIONS AND THE STABILITY OF SOME HEXAGONAL LATTICES*

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There are several mistakes and misprints in the above paper.

(1) In introducing Voigt's notation for the strain components the well-known factor 2 in the normal components has been omitted, and (3·11) should read

$$\left.\begin{matrix} e_{11}, & e_{22}, & e_{33}, & e_{23}, & e_{31}, & e_{12}, & e_1(k), & e_2(k), & e_3(k) \\ 2x_1, & 2x_2, & 2x_3, & x_4, & x_5, & x_6, & y_1(k), & y_2(k), & y_3(k). \end{matrix}\right\} \qquad (3{\cdot}11)$$

The following formulae in which this notation is used are correct as they stand.

(2) In § 4, Central Forces, some terms of the second order in the expression of Φ are omitted. One has in (4·1) to replace r^2 not by $r^2+2\rho$, where ρ are the terms of the 1st order in $e_{\alpha\beta}$ and $e_\alpha(k)$, but by

$$r^2+2\rho+\sigma,$$

where σ represents the second order terms, namely,

$$\sigma = 2\delta g_{\mu\nu}\,\delta\lambda^\mu\xi^\nu + \delta\lambda_\mu\,\delta\lambda^\mu.$$

This can be written in the S-coordinate system in the symmetric form

$$\sigma = e_{\alpha\beta}\,e_\gamma(x^\alpha\delta_{\beta\gamma}+x^\beta\delta_{\alpha\gamma})+e_\alpha\,e_\beta\,\delta_{\alpha\beta}.$$

Now the second of the formulae (4·2) has to be altered and reads

$$\Phi_2 = \tfrac{1}{4}N\sum_l{}'\sum_{kk'}\left\{D\phi\begin{pmatrix} l \\ kk'\end{pmatrix}\sigma\begin{pmatrix} l \\ kk'\end{pmatrix}+D^2\phi\begin{pmatrix} l \\ kk'\end{pmatrix}\rho^2\begin{pmatrix} l \\ kk'\end{pmatrix}\right\}. \qquad (4{\cdot}2)$$

In consequence of this two of the coefficients (4·4) have to be modified:

$$\left.\begin{aligned} \begin{bmatrix}\rho,\gamma \\ k\end{bmatrix} &= 2\begin{bmatrix}\alpha\beta,\gamma \\ k\end{bmatrix} = N\sum_l{}'\sum_{k'}\{(x^\alpha\delta_{\beta\gamma}+x^\beta\delta_{\alpha\gamma})\,D\phi+x^\alpha x^\beta x^\gamma D^2\phi\}, \\ \begin{bmatrix}\alpha\beta \\ kk'\end{bmatrix} &= -N\sum_l{}'\{\delta_{\alpha\beta}D\phi+x^\alpha x^\beta D^2\phi\} \quad (k\neq k'), \end{aligned}\right\} \qquad (4{\cdot}4)$$

where the arguments $\begin{pmatrix} l \\ kk'\end{pmatrix}$ in the x^α, $D\phi$ and $D^2\phi$ are omitted. The subsequent symmetry relations are correct; but two of the expressions (4·11) have to be changed:

$$\left.\begin{aligned} \begin{bmatrix}\rho,\gamma \\ k\end{bmatrix} &= N\sum_p\sum_{k'}\left\{\left[\nu_p\begin{pmatrix}\alpha \\ kk'\end{pmatrix}\delta_{\beta\gamma}+\nu_p\begin{pmatrix}\beta \\ kk'\end{pmatrix}\delta_{\alpha\gamma}\right]B_p(kk')+\nu_p\begin{pmatrix}\alpha\beta\gamma \\ kk'\end{pmatrix}C_p(kk')\right\}, \\ \begin{bmatrix}\alpha\beta \\ kk'\end{bmatrix} &= -N\sum_p\left\{\nu_p(kk')\,\delta_{\alpha\beta}\,B_p(kk')+\nu_p\begin{pmatrix}\alpha\beta \\ kk'\end{pmatrix}C_p(kk')\right\}; \quad (k\neq k'). \end{aligned}\right\} \qquad (4{\cdot}11)$$

* *Proc. Cambridge Phil. Soc.* 38 (1942), 82.

It may be remarked that the corresponding formulae of my article in the *Mathematical Encyclopaedia* (where another notation is used) are correct.

(3) There is a misprint in Table 3, p. 93. A few figures are interchanged and a bracket one line too high. The beginning of the 1st and 4th column ought to read as follows:

$\frac{r}{a}$	$\frac{x^1}{a}$	$\frac{x^2}{a}$	$\frac{x^3}{a}$
1	± 1	0	0
	$\pm \frac{1}{2}$	$\pm \frac{1}{2}\sqrt{3}$	0
γ	0	0	$\pm \gamma$
$\sqrt{3}$	$\pm \frac{3}{2}$	$\pm \frac{1}{2}\sqrt{3}$	0
	0	$\pm \sqrt{3}$	0

Table 3.

All operations performed with these figures are correct. The quantities $\nu\binom{\alpha}{kk'}$ should also be contained in this table; but this does not matter as they all vanish in virtue of the central symmetry of the lattice.

(4) Owing to the latter circumstance all the expressions (7·2), (7·3), (7·4), (7·5) are correct; only the expressions (7·6) have to be modified:

$$\left.\begin{aligned} V &= \begin{bmatrix} 11 \\ 11 \end{bmatrix} = Na^2(6B_3' + 6B_6' + 12B_9' + \ldots + 1C_3' + 4C_6' + 14C_9' + \ldots), \\ W &= \begin{bmatrix} 33 \\ 11 \end{bmatrix} = Na^2(6B_3' + 6B_6' + 12B_9' + \ldots + \gamma^2\{\tfrac{3}{2}C_3' + \tfrac{3}{2}C_6' + 3C_9' + \ldots\}). \end{aligned}\right\} \quad (7\cdot 6)$$

All numerical conclusions about the ratio of the elastic constants of the close packed hexagonal lattice are unchanged. For as only first neighbours are taken into account one has according to (7·8) $B_3' = 0$; hence the lowest additional terms in (7·6) vanish.

Although the errors corrected here do not modify the actual special results of the paper I thought it advisable to give the correct general formulae for the case of application to other examples.

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