

Accuracy in variable star work: The three-star single-channel technique

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Abstract

The three-star technique for photoelectric photometers is discussed. The technique is used to study millimag variations of variable stars with periods in the range of 30 minutes to a few days. It is emphasized that for highest precision two comparison stars should be used and observed as often as the variable star. The different types of numbers used to express the precision of measurement are discussed together with possible misinterpretations.

Finally, the successful use of an extinction coefficient derived from differential extinction between the three stars is illustrated with measurements showing an extreme amount of instrumental sensitivity drift.

1. The standard 3-star technique

For over two decades the three-star observing technique used with standard single-channel photoelectric photometers has been the primary method to study short-period variable stars with periods longer than 30 minutes ($f < 50$ cycles d^{-1} or $f < 0.5$ mHz) up to a few days. Although a precision of 2 mmag per single measurement had already been achieved more than 25 years ago (Breger 1966), the improvement since then has been minor. The present limit to the photometric precision is still just slightly under 1 mmag with good photoelectric photometers under excellent weather conditions. In fact, considerable observational care has to be taken in reducing the observational errors sufficiently in order to obtain the 'old' limit near 1 mmag precision per single measurement.

The three-star observing technique consists of observing the variable star, V , together with two comparison stars, $C1$ and $C2$, using the same instrumental equipment. Cycles of $[C1 - V - C2] - [C1 - V - C2] - \text{etc.}$ are chosen. In order to keep scintillation errors to 1 mmag or less, for telescopes in the 0.5 to 1-m class integration times between 30 and 60 s are usually selected. Furthermore, with such long integration times, photon statistics is rarely a problem for relatively bright stars. After considering the additional time required for sky measurements, telescope setting and centering of the star, a complete cycle can be completed in about five minutes. Note that all three stars are observed for identical lengths.

Care must be taken to select the comparison stars in such a manner that the properties of the three stars are as similar to each other as possible. In particular, the stellar brightnesses, positions in the sky and spectral types are important. Since

all three conditions cannot usually be fulfilled, reasonable compromises must be made. Of course, close comparison stars of spectral type K would not be used to study the variation of B stars! To reduce the errors of measurement, only one (or possibly two) filters are usually used.

During the data reduction, the measurements of star V are reduced relative to both comparison stars. Care needs to be taken to choose the best interpolation scheme and not to put undue weight on single measurements. Some numerical experimentation to choose an optimum interpolation scheme is recommended.

The three-star technique relies on the cancellation of most sources of error and avoids the drift problem experienced with multichannel instruments. The price paid for this success lies in the low duty cycle and the restriction to periods of 30 minutes or longer.

An example of the technique is shown in Figure 1, where unpublished measurements of the δ Scuti variable HR 7222 obtained at McDonald Observatory are shown. Two comparison stars, HR 7263 and HR 7286, were used. The measurements were made with the *B* filter, extinction corrections were applied, and the average magnitudes subtracted for clarity. The data were reasonably free of systematic drifts.

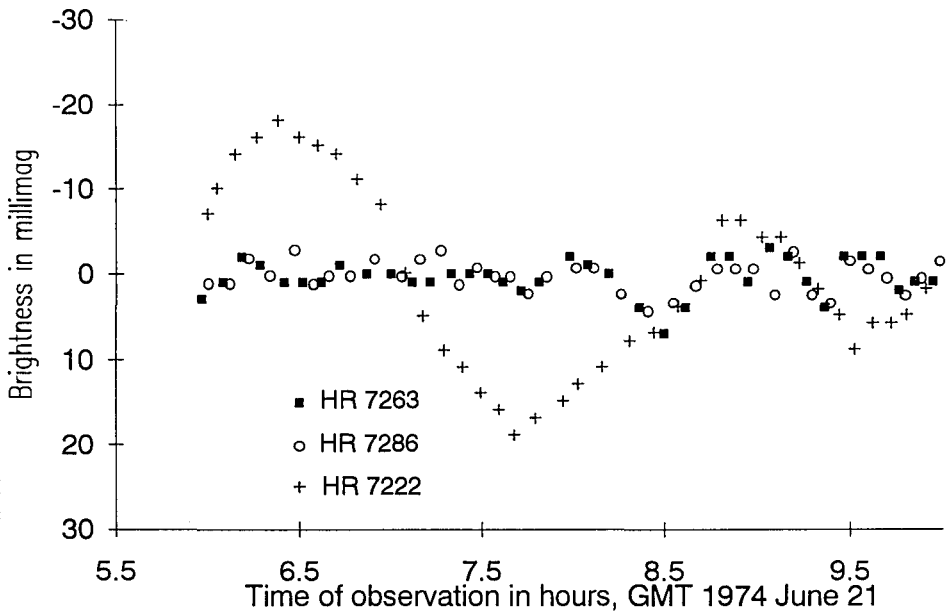


Figure 1 A typical example of the three-star technique. HR 7222 is a δ Scuti variable. The nonrepeatability from cycle to cycle is normal for these stars due to the presence of multiple periods. The comparison stars agree with each other to ± 2 mmag.

2. The variable-comparison-check star technique

A popular variation of the standard three-star technique involves measuring the second comparison star less often, possibly only once every half hour. This star, now called a check star, is therefore used only to provide a rough check of the constancy of the primary comparison star. A motivation for applying this variation lies in the fact that the duty cycle of the variable star measurements can be increased up to 40%, which might be important for very short periods under one hour.

For variable stars with periods of one hour or longer the standard method with two comparison stars is to be preferred. Some of the advantages of the standard method are:

- * The possibility of variability of comparison stars cannot be excluded. The check for constancy is much more reliable when both comparison stars can be compared every five minutes rather than every half hour.

- * If one of the comparison stars turns out to be variable, the use of two comparison stars might permit the observations to be saved. When all three stars are observed equally well and often, inspection of the three power spectra of the reduced magnitude differences $C1 - C2$, $V - C1$, $V - C2$ might reveal which star is constant. Note here that one of the three differences should contain the variability information of both variable stars! The method works best if the measurements are of high quality and the time-scales of variability of the two variable stars are different. The latter is often the case.

- * Inspection of the light curve of the difference ($C1 - C2$) reveals those times at which the precision is low and the data should be discarded. This is especially useful for the beginnings and ends of the nights.

- * The precision of measurement for each night can be determined from a comparison of the two comparison stars. Knowledge of this number is important for the subsequent analysis of the variable star measurements.

- * If all three stars have similar spectral types, it might be possible to derive a differential extinction coefficient from the two comparison stars. If there exists instrumental sensitivity drift, this method to determine extinction becomes extremely useful.

3. What kind of precision are they talking about?

With discussions of millimag or even micromag precision, it is important to define the different types of precision. In the variable-star field, the uncertainties of measurement are usually expressed in one of two ways:

Precision per single measurement. This description of observational quality can be understood as the precision to which the difference between two or more constant stars can be measured, i.e. the repeatability in the instrumental system. (Note here

that the precision of the magnitude difference between two stars is not numerically the same as the precision of the individual measurements.) Since transformation errors can be avoided (no transformations), the measurements can be quite precise.

In fact, a precision of 2 mmag per single measurement had already been obtained more than 25 years ago. Today, the typical precision obtained during multisite campaigns with a variety of telescopes and photometers is about 3 mmag. The best photometry published has uncertainties between 0.5 to 1 mmag. The reason for this relative lack of improvement lies in the observational difficulties in the millimag range. For an excellent discussion of the errors of measurement and their reduction we refer to Young et al. (1991).

Noise in an amplitude, frequency diagram. The relatively new field of asteroseismology requires that for pulsating stars amplitudes of a millimag or smaller be extracted from photometric data. This requires long data strings and stable pulsations. A different description of precision is often used to describe the ability to find such small pulsations, viz. the noise in an amplitude, frequency (Fourier) diagram near the frequency of interest. This noise is the amplitude of a sine curve at a hypothetical frequency forced through data which contain no such variation.

Gilliland and Brown (1992) in their analysis of CCD measurements conclude that the detection of amplitudes as small as 15 μ mag should be possible.

An enthusiastic and possibly misleading comparison of the precision obtained with different techniques is given by Kjeldsen and Frandsen (1992, hereafter called KF). In this very informative paper they compare high-precision CCD photometry with standard photoelectric techniques, including the three-star technique described in this paper. In their Table 1 the noise at 3 mHz is compared. They list ~ 1 mmag/min for CCD techniques, ~ 2 mmag/min for single-channel work with no comparison stars and ~ 8 mmag/min for single-channel work with two comparison stars.

Are measurements with comparison stars really so much worse than those without comparison stars? After noticing that the evaluation was made at periods of 5 mins (3 mHz), we can explain most of these numbers without having to examine the potential of CCD techniques or comparing the observational quality of the different papers used in the comparison. Since one of the papers cited contains data familiar to the author (HR 2724, see next section), we can use these data to examine the analysis of KF. The SAAO observations of HR 2724 used in the table are ~ 200 hours of single-channel measurements of a cycle of up to 4 stars measured in 4 colors each. (Some averaging was undertaken before calculating an amplitude-frequency diagram.) We note:

* The observations of HR 2724, as listed, are spaced 20 mins apart. Hypothetical amplitudes of sine curves at periods ~ 5 mins (3 mHz) are not meaningful. The original publication, naturally, does not present a power spectrum in this range of periods. This and other data obtained with the three-star technique *should not be used for frequencies beyond the Nyquist frequency*, as was done in the table of KF.

* The duty cycle of the data $< 10\%$. This can partially explain the fact that in the table of KF observations made without comparison stars appear to be more accurate. Nevertheless, in the range of periods intended for the three-star technique (and discussed in this paper), comparison stars must be observed.

We conclude that the seemingly poor performance of the three-star method as listed by KF is caused by using its data for purposes for which they had not been intended and should never be used. At periods of one hour or longer the game is completely different than at 5 mins. Any measurements made without comparison stars would probably show additional errors due transparency variation and equipment drift. (Of course, here the situation is reversed: these measurements without comparison stars are not intended to be used to search for such long periods.) Furthermore, a realistic analysis of the noise needs to include the effects of aliasing (caused by day-time observing gaps) of the multiple periods.

Finally, it needs to be emphasized again that the 'best' observing method strongly depends on the length of the periods one wants to determine.

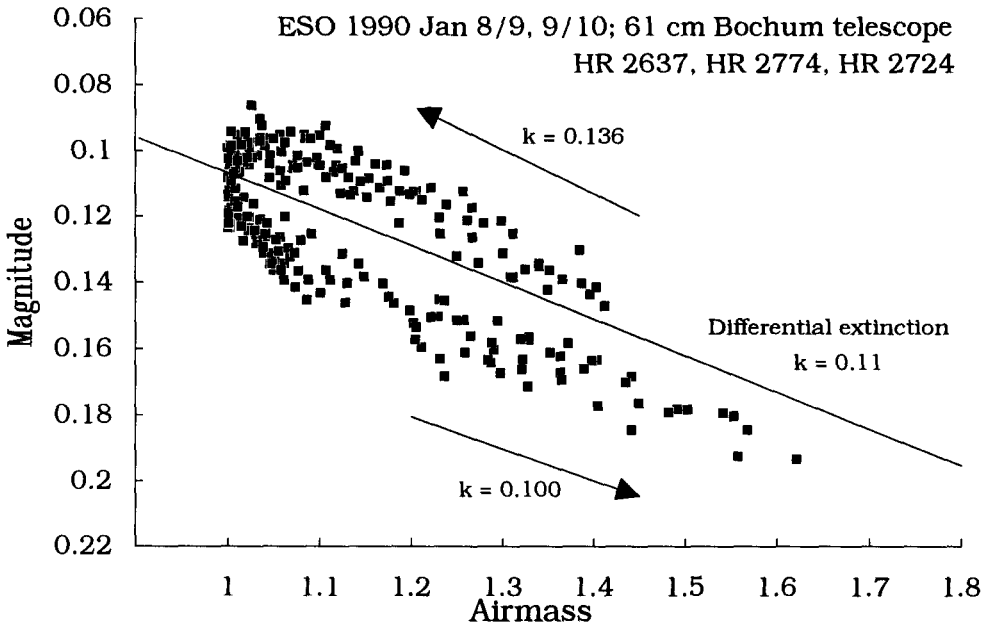


Figure 2 Bouguer plot of measurements (squares) of three stars obtained during two nights during which the instrumentation with instrumental sensitivity drift. The feature at small air masses is typical of drift. A solution for the extinction coefficient from differential extinction shows $k = 0.11$, in agreement with values obtained on subsequent nights with less or no instrumental sensitivity drift.

4. Some comments on extinction

The use of close comparison stars eliminates uncertainties in the extinction coefficients, at least to the first order. However, the remaining small errors are important since they are systematic. Different recommendations in the literature on how to correct for extinction include the use of

- * mean seasonal extinction coefficients
- * nightly extinction coefficients
- * extinction coefficients changing throughout the night.

Which of these reduction schemes should be adopted? We cannot give a universally valid recommendation. Since the present technique results in a large number of measurements obtained at different air masses, a solution might be to determine the extinction differentially between the stars, i.e. to examine the relation between $\Delta(\text{mag})$ and $\Delta(\text{air mass})$ between the stars. This assumes, of course, that the stars are situated several degrees apart and have similar spectral-energy distributions.

Figure 2 gives an example of extreme instrumental sensitivity drift. One of the three stars shows millimag variations, which can be disregarded in this context. The measurements were made by H.-G. Grothues (see Breger, Balona and Grothues 1991). This example shows clearly that solving for time-variable extinction coefficients would not be suitable for these data.

References:

- Breger, M. 1966, ApJ 146, 958
 Breger, M., Balona, L. A. and Grothues, H.-G. 1991, A&A 243, 160
 Gilliland, R. L. and Brown, T. M. 1992, PASP 104, 582
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 Young, A. T., Genet, R. M., Boyd, L. J., Borucki, W. J., Lockwood, G. W., Henry, G. W., Hall, D. S., Pyper Smith, D., Baliunas, S. J., Donahue, R., Epan, D. H. 1991, PASP 103, 221

Discussion

C.L. Sterken *I recall that on one occasion we saw that the non-differential signal was more precise than the differential result: we were observing Spica (α Vir) during a fortnight of continuous, excellent, weather conditions at La Silla, with the Danish 50 cm telescope in 1985.*

Breger: If your measuring errors are purely random in nature, the difference between two stars will be $\sqrt{2}$ times the individual errors. Such excellent observing conditions, with excellent equipment, I have not seen personally so far, but one should always check that reductions of the variable relative to the comparison stars does not increase the scatter.

R.M. Genet: *We have made observations with a precision of 0.0005 magnitude. These observations matched predictions by Andy Young of observational error versus air mass. Thus reducing errors further requires a larger telescope or longer integration times.*

Breger: The present limit with standard photoelectric photometers seems to be ± 0.0005 mag, which has been obtained by several astronomers. But can we measure more precisely, or are all stars variable at that level (with periods > 24 hours)?

R.R. Shobbrook: *Scintillation requires some 50s - 100s integration times on telescopes of $D \sim 0.5$ m. One can improve precision slightly by fitting a polynomial (in time, during the night) to the magnitudes of the comparison stars. The first term is the gain drift and the other terms account for the occasional $\sim 1\%$ transparency variations.*

Breger: In practice, we also have found it useful to fit polynomials through the data or to draw a curve by eye. In this respect it is important for the fitted curve not to follow every little wiggle.

C.L. Sterken: *Differential Bouguer method: Observers of δ Scuti stars, and other variables with periods of the order of hours, frequently monitor such stars throughout a complete night. If, as you suggest, both comparison stars are equally frequently observed, and if the stars are at an appropriate declination, the determination of extinction coefficient is often done using the original Bouguer method, viz., from a plot of observed magnitude versus air mass X . Sometimes, however, the so-called "differential Bouguer method" is used, viz a plot of differential magnitude (of the comparison stars) versus associated differential air mass. Provided that the ΔX -range remains substantial, that method should yield the same result as the classical Bouguer method when atmospheric conditions are good, but the method may break down when this is not the case. The classical method is more likely to show that conditions are not optimal.*