the total moment about OX is

$$\frac{h}{2}+\frac{h}{2(1+h)^2}+\frac{h}{2(1+2h)^2}+\dots+\frac{h}{2(1+\overline{n-1}h)^2}.$$

This *exceeds* the moment of the area under the curve, so that if we take the distances from OX too small, namely, $\frac{1}{2}\cdot\frac{1}{1+h}$, $\frac{1}{2}\cdot\frac{1}{1+2h}$, etc., the result will be a better approximation for the curve. In this case the sum is

$$\frac{h}{2(1+h)}+\frac{h}{2(1+h)(1+2h)}+\frac{h}{2(1+2h)(1+3h)}+\dots$$

or $\frac{h}{2}$ times the proposed series. Hence the series stated means an approximate value of $\frac{2}{h}$ times the moment about OX of the area under the graph of $\frac{1}{x}$ and between the ordinates at $x=1$ and $x=1+nh$.

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The equation of the polar of a point with respect to a circle.—The equation of the polar of a point (x', y') with respect to a circle $x^2+y^2=r^2$ can be found very easily by the use of

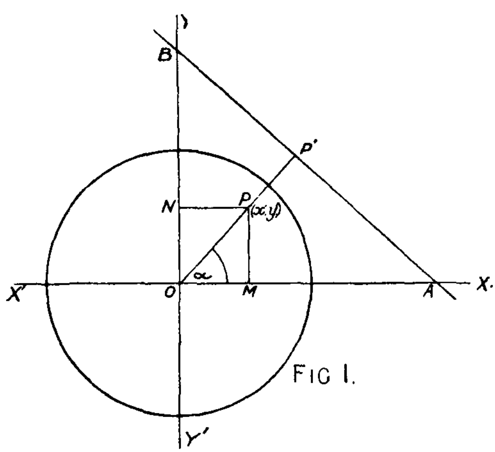

FIG 1.

either the perpendicular or the intercept form of the equation of a straight line.

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Thus if the equation of AB (Fig. 1), the polar of P, is $p = x\cos\alpha + y\sin\alpha$, $p = \text{OP}'$, $\cos\alpha = \dfrac{x'}{\text{OP}}$, $\sin\alpha = \dfrac{y'}{\text{OP}}$. Hence the equation becomes on substitution of these values $xx' + yy' = \text{OP} . \text{OP}'$, that is $xx' + yy' = r^2$, since P and P′ are inverse points with respect to the circle.

Again, if the equation of AB is $\dfrac{x}{a} + \dfrac{y}{b} = 1$, then $a = \text{OA}$ and $b = \text{OB}$. But from the cyclic quadrilaterals PMAP′, PNBP′ (Fig. 1), $\text{OM} . \text{OA} - \text{ON} . \text{OB} = \text{OP} . \text{OP}' = r^2$.

Therefore $a = \dfrac{r^2}{x'}$ and $b = \dfrac{r^2}{y'}$, and the equation becomes

$$xx' + yy' = r^2.$$

A simple geometrical method of finding the equation of the polar of a point with respect to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ did not occur to me until my colleague, Dr M‘Whan, showed me the following very ingenious device for obtaining the equation of

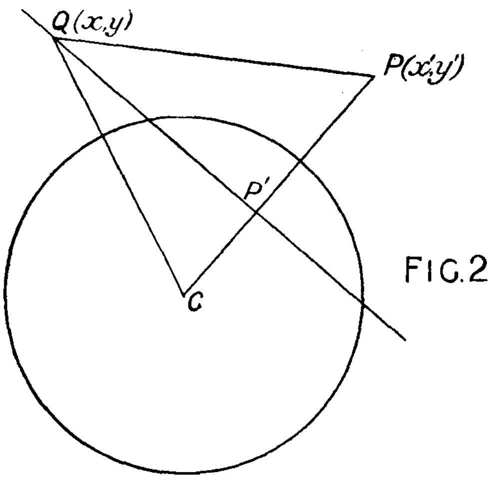

FIG. 2

the tangent at a point on the circle:—If $\text{P}(x', y')$ is on the circle and Q is any point on the tangent at P, the power of Q with respect to the circle is equal to QP^2, and hence the equation of the tangent is

$$x^2 + y^2 + 2gx + 2fy + c = (x - x')^2 + (y - y')^2 \quad \ldots\ldots\ldots\ldots (1)$$

It was thus suggested that the standard equation of the tangent or polar, viz.,

$$xx' + yy' + g(x + x') + f(y + y') + c = 0$$

could be written

$$(x - x')^2 + (y - y')^2 = x^2 + y^2 + 2gx + 2fy + c$$
$$+ x'^2 + y'^2 + 2gx' + 2fy' + c,$$

which reduces to (1) if (x', y') is on the circle. This form of the equation shows that the polar of a point P, (x', y') with respect to a circle is the locus of a point Q, (x, y), which moves so that PQ^2 is equal to the sum of the powers of P and Q with respect to the circle. This well-known property of the polar can be very easily proved, for, from the triangle PCQ (Fig. 2), we have

$$\begin{aligned} PQ^2 &= CP^2 + CQ^2 - 2CP \,.\, CP' \\ &= CP^2 + CQ^2 - 2r^2 \\ &= (CP^2 - r^2) + (CQ^2 - r^2) \\ &= \text{sum of the powers of P and Q.} \end{aligned}$$

Thus, reversing the above steps, we have an easy geometrical way of obtaining the equation of the polar.

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Two Theorems on Determinants, and their Application to the Proof of the Volume-Formula for Tetrahedra

—*Theorem I.* If to each of the elements of any row of a determinant there be added the same fraction of the difference between the corresponding elements in two other rows, the value of the determinant is unaltered. This is a case of a well known elementary theorem.

Theorem II. If to each of the elements of two rows of a determinant there be added the same fraction of the difference between the corresponding elements in these rows, the value of the determinant is unaltered.

For

$$\begin{vmatrix} a_1 + \lambda(a_2 - a_1), & b_1 + \lambda(b_2 - b_1), & c_1 + \lambda(c_2 - c_1) \\ a_2 + \lambda(a_2 - a_1), & b_2 + \lambda(b_2 - b_1), & c_2 + \lambda(c_2 - c_1) \\ a_3, & b_3, & c_3 \end{vmatrix} = \begin{vmatrix} a_1 + \lambda(a_2 - a_1), & \ldots\ldots, & \ldots\ldots \\ a_2 - a_1, & b_2 - b_1, & c_1 - c_1 \\ a_3, & b_3, & c_3 \end{vmatrix}$$

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