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OBLIQUE WAVE SCATTERING BY A RECTANGULAR SUBMARINE TRENCH

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Abstract

The problem of oblique wave scattering by a rectangular submarine trench is investigated assuming a linearized theory of water waves. Due to the geometrical symmetry of the rectangular trench about the central line $x = 0$, the boundary value problem is split into two separate problems involving the symmetric and antisymmetric potential functions. A multi-term Galerkin approximation involving ultra-spherical Gegenbauer polynomials is employed to solve the first-kind integral equations arising in the mathematical analysis of the problem. The reflection and transmission coefficients are computed numerically for various values of different parameters and different angles of incidence of the wave train. The coefficients are depicted graphically against the wave number for different situations. Some curves for these coefficients available in the literature and obtained by different methods are recovered.

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Keywords and phrases: water wave scattering, rectangular submarine trench, oblique incident wave, multi-term Galerkin approximation, reflection and transmission coefficients.

1. Introduction

The problems of water wave scattering by obstacles situated at the bottom of an ocean form an important area in the linearized theory of water waves for their possible applications in coastal and polar engineering. Oblique water wave scattering by a rectangular submarine trench is one such problem, and it is investigated here by employing a multi-term Galerkin approximation method. In the literature, this method of Galerkin approximation has been widely used to investigate water wave scattering problems involving thin vertical barriers [1, 2, 11] or thick vertical barriers with rectangular cross sections [3–5].

Kreisel [7] first investigated wave propagation over a variable bottom topography for normally incident wave trains by reducing the fluid domain into a rectangular

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strip using an appropriate conformal mapping and then converting the boundary value problem of the potential function to an integral equation, which was solved by iteration. Later, Lassiter [8] studied water wave scattering by a rectangular submarine trench for a normally incident wave train, where the water depths before and after the trench were constant but not necessarily equal. He obtained the reflection and transmission coefficients after formulating the problem in terms of complementary variational integrals of Schwinger's type, using the conditions that the velocity potential and horizontal component of velocity were continuous along the vertical lines before and after the trench. Lee and Ayer [9] used a matching procedure to solve the rectangular trench problem by expressing solutions in two subregions comprising an infinite rectangular region of constant depth and a finite rectangular region of the trench. They obtained the reflection and transmission coefficients numerically and also presented them graphically. A series of laboratory experiments in a wave tank were also performed by Lee and Ayer [9], and the experimental results were compared with the theoretical solutions. All the aforesaid trench problems involved normally incident wave trains. Miles [10] considered the trench problem for normal as well as obliquely incident wave trains. He employed a conformal mapping algorithm to solve the problem for normally incident wave trains, and employed a variational formulation for obliquely incident waves. Also, he gave approximate analytical results for the reflection coefficients for long waves. Shortly afterwards, Kirby and Dalrymple [6] studied the problem of propagation of obliquely incident waves over an asymmetric trench for which the water depths on the sides were unequal but constant. They obtained numerical results based on the numerical solutions of a set of linear integral equations, derived by matching the truncated eigenfunction expansions for each subregion of constant depth along the two vertical boundaries. They compared their numerical results with the data obtained from a small-scale wave tank experiment.

In this paper, we reinvestigate the oblique wave scattering problem involving a rectangular symmetric submarine trench by employing a multi-term Galerkin approximation method. Due to the geometrical symmetry of the rectangular trench about the central line $x = 0$, the problem is split into two separate problems involving the symmetric and antisymmetric potential functions describing the resultant motion in the fluid region, as was done by Kanoria et al. [5], who considered the problem of water wave scattering by a thick vertical barrier of rectangular cross section having four different geometrical shapes. Using eigenfunction expansions of the potential functions along with Havelock's inversion formula [12] followed by a matching procedure, we obtain integral equations for the corresponding unknown horizontal velocity components across the vertical line through the corner point of the trench. By using the multi-term Galerkin approximation method involving ultra-spherical Gegenbauer polynomials as basis functions, the integral equations are solved approximately. Numerical estimates for the reflection and transmission coefficients are obtained for different angles of incidence and different values of various parameters associated with the problem. These coefficients seem to satisfy the energy identity. Some of the curves for these coefficients are compared with those available in the

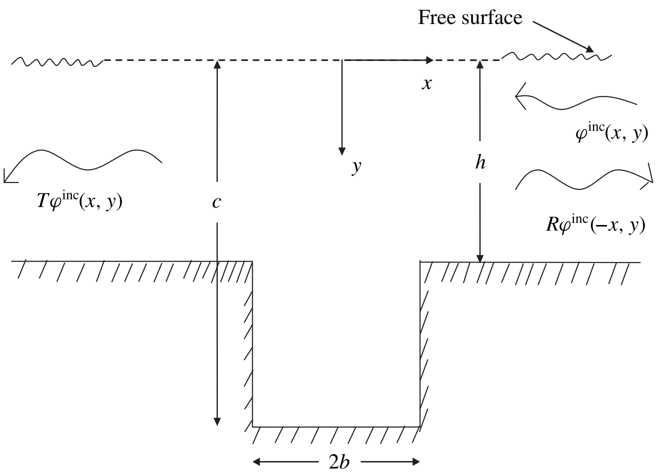

FIGURE 1. Schematic diagram of a submarine trench.

literature obtained using other methods and laboratory experiments, and a very good agreement is achieved. From the numerical results, we observe that the depth and the width of the trench affect the reflection and transmission coefficients significantly. We also notice that for large angles of incidence of the waves, reflection is more while transmission is less, which is plausible.

2. Mathematical formulation

We consider the irrotational motion in water regarded as an incompressible, inviscid and homogeneous fluid, with a free surface over a rectangular submarine trench of width $2b$. The trench is at the bottom of an ocean of uniform finite depth, h, and the depth of the trench from the mean free surface is c (see Figure 1). A rectangular Cartesian coordinate system is chosen, in which the y-axis is taken vertically downwards and the (x, z)-plane corresponds to the undisturbed free surface. Here, we consider the case of a train of surface waves obliquely incident on the right-hand side of the trench with angle of incidence θ. The obliquely incident wave is represented by the velocity potential

$$\Re\{\phi^{\mathrm{inc}}(x, y)e^{i(\nu z-\sigma t)}\},$$

where

$$\phi^{\mathrm{inc}} = \frac{2\cosh k_0(h-y)e^{-i\mu(x-b)}}{\cosh k_0 h},$$

with k_0 being the real positive root of the transcendental equation

$$k\tanh kh = K \quad \text{with } K = \frac{\sigma^2}{g}.$$

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Here, σ is the circular frequency of the incoming wave train, g is the acceleration due to gravity,

$$\mu = k_0 \cos\theta, \quad \nu = k_0 \sin\theta \quad \left(0 < \theta < \frac{\pi}{2}\right)$$

and $e^{i\nu z}$ is the z-dependence of the incident field.

Due to the geometrical symmetry of the rectangular trench, the z-dependent term can be eliminated by assuming the velocity potential to be of the form $\Re\{\phi(x, y)e^{i(\nu z-\sigma t)}\}$. Henceforth, the factor $e^{i(\nu z-\sigma t)}$ will be omitted.

Then $\phi(x, y)$ satisfies the boundary value problem

$$(\nabla^2 - \nu^2)\phi = 0 \quad \text{in the fluid region;} \tag{2.1}$$

$$K\phi + \phi_y = 0 \quad \text{on } y = 0, |x| < \infty; \tag{2.2}$$

$$\phi_x = 0 \quad \text{on } x = \pm b, y \in (h, c)(c > h); \tag{2.3}$$

$$r^{1/3}\nabla\phi \text{ is bounded as} \quad r = \{(x \mp b)^2 + (y - h)^2\}^{1/2} \to 0; \tag{2.4}$$

$$\phi_y = 0 \quad \text{on } y = h, |x| > b; \tag{2.5}$$

$$\phi_y = 0 \quad \text{on } y = c, |x| < b; \tag{2.6}$$

and finally

$$\phi(x, y) \sim \begin{cases} \phi^{\text{inc}}(x, y) + R\phi^{\text{inc}}(-x, y) & \text{as } x \to \infty, \\ T\phi^{\text{inc}}(x, y) & \text{as } x \to -\infty, \end{cases}$$

where R and T are the unknown reflection and transmission coefficients, respectively, to be determined.

3. Method of solution

Due to the geometrical symmetry of the rectangular trench about $x = 0$, $\phi(x, y)$ can be split into symmetric and antisymmetric parts, $\phi^s(x, y)$ and $\phi^a(x, y)$, respectively, so that

$$\phi(x, y) = \phi^s(x, y) + \phi^a(x, y),$$

where

$$\phi^s(-x, y) = \phi^s(x, y) \quad \text{and} \quad \phi^a(-x, y) = -\phi^a(x, y).$$

Therefore, we consider only the region $x \geq 0$. Now $\phi^s(x, y)$ and $\phi^a(x, y)$ satisfy the equations (2.1)–(2.6) together with

$$\phi^s{}_x(0, y) = 0 \quad \text{and} \quad \phi^a(0, y) = 0.$$

Let the behaviour of $\phi^{s,a}(x, y)$ for large x be represented by

$$\phi^{s,a}(x, y) \sim \frac{\cosh k_0(h - y)}{\cosh k_0 h}\{e^{-ik_0(x-b)} + R^{s,a}e^{ik_0(x-b)}\} \quad \text{as } x \to \infty,$$

where R^s and R^a are unknown constants. These constants are related to R and T by

$$R, T = \tfrac{1}{2}(R^s \pm R^a)e^{-2ik_0 b}. \tag{3.1}$$

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Now the eigenfunction expansions of $\phi^{s,a}(x,y)$ satisfying the equations (2.1)–(2.3), (2.5) and (2.6) for $x > 0$ (in different regions) are given as follows.

Region I ($x > b,\ 0 < y < h$):

$$\phi^{s,a}(x,y) = \frac{\cosh k_0(h-y)}{\cosh k_0 h}\{e^{-i\mu(x-b)} + R^{s,a}e^{i\mu(x-b)}\} + \sum_{n=1}^{\infty} A_n^{s,a}\cos k_n(h-y)e^{-s_n(x-b)}, \tag{3.2}$$

where k_n $(n = 1, 2, \ldots)$ are the real positive roots of the equations

$$k\tan kh + K = 0$$

and

$$s_n = (k_n^2 + \nu^2)^{1/2}.$$

Region II ($0 < x < b,\ 0 < y < c$):

$$\begin{pmatrix}\phi^s(x,y)\\ \phi^a(x,y)\end{pmatrix} = \begin{pmatrix}C_0^s\cos(\alpha_0^2-\nu^2)^{1/2}x\\ C_0^a\sin(\alpha_0^2-\nu^2)^{1/2}x\end{pmatrix}\frac{\cosh\alpha_0(c-y)}{\cosh\alpha_0 c} + \sum_{n=1}^{\infty}\begin{pmatrix}C_n^s\cosh t_n x\\ C_n^a\sinh t_n x\end{pmatrix}\cos\alpha_n(c-y), \tag{3.3}$$

where $\pm\alpha_0, \pm i\alpha_n$ $(n = 1, 2, \ldots)$ are the roots of the transcendental equation

$$\alpha\tanh\alpha c = K$$

and

$$t_n = (\alpha_n^2 + \nu^2)^{1/2} \quad (n = 1, 2, \ldots).$$

Now we have the matching conditions

$$\phi_x^{s,a}(b+0,y) = \phi_x^{s,a}(b-0,y), \quad 0 < y < h.$$

Let us define

$$\phi_x^{s,a}(b+0,y) = f^{s,a}(y), \quad 0 < y < h, \tag{3.4}$$

$$\phi_x^{s,a}(b-0,y) = g^{s,a}(y), \quad 0 < y < c, \tag{3.5}$$

so that

$$g^{s,a}(y) = \begin{cases} f^{s,a}(y), & 0 < y < h,\\ 0, & h < y < c.\end{cases} \tag{3.6}$$

Due to the edge condition (2.4),

$$f^{s,a}(y) = O(|y-h|^{-1/3}) \quad \text{as } y \to h.$$

Substituting the expansions of $\phi^{s,a}(y)$ from (3.2) in (3.4),

$$i\mu\frac{\cosh k_0(h-y)}{\cosh k_0 h}\{R^{s,a} - 1\} - \sum_{n=1}^{\infty} A_n^{s,a}s_n\cos k_n(h-y) = f^{s,a}(y), \quad 0 < y < h. \tag{3.7}$$

Use of Havelock's inversion formula [12] in (3.7) yields

$$1 - R^{s,a} = \frac{4ik_0}{\delta_0 \mu} \cosh k_0 h \int_0^h f^{s,a}(y) \cosh k_0(h-y)\, dy, \tag{3.8}$$

with $\delta_0 = 2k_0h + \sinh 2k_0h$ and

$$A_n^{s,a} = -\frac{4k_n}{\delta_n s_n} \int_0^h f^{s,a}(y) \cos k_n(h-y)\, dy,$$

with

$$\delta_n = 2k_nh + \sin 2k_nh \quad (n = 1, 2, \dots).$$

Now, using (3.3) and (3.5),

$$\begin{aligned} &C_0^{s,a}(\alpha_0^2 - \nu^2)^{1/2}(-\sin(\alpha_0^2 - \nu^2)^{1/2}b, \cos(\alpha_0^2 - \nu^2)^{1/2}b) \frac{\cosh \alpha_0(c-y)}{\cosh \alpha_0 c} \\ &+ \sum_{n=1}^{\infty} (\sinh t_n b, \cosh t_n b) C_n^{s,a} t_n \cos \alpha_n(c-y) = g^{s,a}(y), \quad 0 < y < c. \end{aligned}$$

Applying Havelock's inversion formula and using (3.6),

$$\begin{aligned} C_0^{s,a} = &\frac{4\alpha_0 \cos h\alpha_0 c}{\gamma_0(\alpha_0^2 - \nu^2)^{1/2}(-\sin(\alpha_0^2 - \nu^2)^{1/2}b, \cos(\alpha_0^2 - \nu^2)^{1/2}b)} \\ &\times \int_0^h f^{s,a}(y) \cosh \alpha_0(c-y)\, dy, \end{aligned}$$

with

$$\gamma_0 = 2\alpha_0 c + \sinh 2\alpha_0 c$$

and

$$C_n^{s,a} = \frac{4\alpha_n}{\gamma_n t_n(\sinh t_n b, \cosh t_n b)} \int_0^h f^{s,a}(y) \cos \alpha_n(c-y)\, dy,$$

with

$$\gamma_n = 2\alpha_n c + \sin 2\alpha_n c.$$

Now matching $\phi^{s,a}(x, y)$ across the line $x = b$ yields

$$\begin{aligned} &\frac{\cosh k_0(h-y)}{\cosh k_0 h}\{1 + R^{s,a}\} + \sum_0^{\infty} A_n^{s,a} \cos k_n(h-y) \\ &= C_0^{s,a}(\cos(\alpha_0^2 - \nu^2)^{1/2}b, \sin(\alpha_0^2 - \nu^2)^{1/2}b) \frac{\cosh \alpha_0(c-y)}{\cosh \alpha_0 c} \\ &\quad + \sum_{n=1}^{\infty} (\cosh t_n b,\ \sinh t_n b) C_n^{s,a} \cos \alpha_n(c-y), \quad 0 < y < h. \end{aligned}$$

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This ultimately produces the first-kind integral equations

$$\int_0^h F^{s,a}(u)\mathcal{M}^{s,a}(y,u)\,du = \frac{\cosh k_0(h-y)}{\cosh k_0 h}, \quad 0<y<h, \tag{3.9}$$

where

$$F^{s,a}(y) = \frac{4k_0\cosh^2 k_0 h}{\mu\delta_0(1+R^{s,a})} f^{s,a}(y), \quad 0<y<h,$$

and

$$\begin{aligned}\mathcal{M}^{s,a}(y,u) = \frac{\delta_0\mu}{k_0\cosh^2 k_0 h}\sum_{n=1}^{\infty}\bigg[&\frac{k_n\cos k_n(h-y)\cos k_n(h-u)}{\delta_n s_n}\\ &+(\coth t_n b, \tanh t_n b)\frac{\alpha_n\cos\alpha_n(c-y)\cos\alpha_n(c-u)}{\gamma_n t_n}\\ &+\Big\{(-\cot(\alpha_0^2-\nu^2)^{1/2}b, \tan(\alpha_0^2-\nu^2)^{1/2}b)\cosh\alpha_0(c-y)\\ &\times\cosh\alpha_0(c-u)\frac{\alpha_0}{\gamma_0(\alpha_0^2-\nu^2)^{1/2}}\Big\}\bigg], \quad 0<y,\ u<h.\end{aligned}$$

Note that $\mathcal{M}^{s,a}(y,u)$ $(0<y,u<h)$ are real and symmetric in y and u.

Now, if we define

$$C^{s,a} = -i\frac{1-R^{s,a}}{1+R^{s,a}}, \tag{3.10}$$

then, by equations (3.8) and (3.9),

$$C^{s,a} = \int_0^h F^{s,a}(y)\frac{\cosh k_0(h-y)}{\cosh k_0 h}\,dy,$$

where

$$F^{s,a}(y) = \frac{4\cosh^2 k_0 h}{\delta_0(1+R^{s,a})} f^{s,a}(y) \tag{3.11}$$

and $F^{s,a}(y)$ and $C^{s,a}$ are real quantities. Thus, if the integral equations (3.9) are solved, these solutions can be used to evaluate $C^{s,a}$ from the relations (3.11), and these produce the actual reflection and transmission coefficients

$$|R| = \frac{|1+C^sC^a|}{\triangle} \quad \text{and} \quad |T| = \frac{|C^s-C^a|}{\triangle},$$

with

$$\triangle = \{1+(C^s)^2+(C^a)^2+(C^sC^a)^2\}^{1/2},$$

which are obtained from the equations (3.10) and (3.1).

Next we consider the Galerkin approximation method to solve the integral equations (3.9). The unknown functions $F^{s,a}(y)$ are approximated as

$$F^{s,a}(y) \approx \mathcal{F}^{s,a}(y), \quad 0<y<h,$$

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where $\mathcal{F}^{s,a}(y)$ have the multi-term Galerkin expansions in terms of suitable basis functions given by

$$\mathcal{F}^{s,a}(y) = \sum_{n=0}^{N} a_n^{s,a} f_n^{s,a}(y), \quad 0 < y < h, \tag{3.12}$$

where $a_n^{s,a}$ are unknown constants. Following similar arguments given by Kanoria et al. [5], the basis functions are

$$f_m^{s,a}(y) = -\frac{d}{dy}\left[e^{-ky} \int_y^h \widehat{f_n}(t)\, dt\right], \quad 0 < y < h, \tag{3.13}$$

where $\widehat{f_n}(y)$ is chosen in terms of the ultra-spherical Gegenbauer polynomials of order $1/6$ as

$$\widehat{f_n}(y) = \frac{2^{7/6}\Gamma(1/6)(2n)!}{\pi\Gamma(2n+1/3)h^{1/3}(h^2-y^2)^{1/3}} C_{2n}^{1/6}\left(\frac{y}{h}\right), \quad 0 < y < h. \tag{3.14}$$

Now, using (3.14) and (3.13) and substituting these into (3.12), we get the approximate forms of $\mathcal{F}^{s,a}(y)$. Using these approximate forms in (3.9), multiplying both sides by $f_n^{s,a}(y)$ and integrating over the interval $(0, h)$, we obtain the linear systems

$$\sum_{n=0}^{N} a_n^{s,a} K_{mn}^{s,a} = d_m^{s,a}, \quad m = 0, 1, \ldots, N, \tag{3.15}$$

where

$$K_{mn}^{s,a} = \int_0^h \int_0^h \mathcal{M}^{s,a}(y,u) f_n^{s,a}(u) f_m^{s,a}(y)\, du\, dy, \quad m, n = 0, 1, \ldots, N, \tag{3.16}$$

and

$$d_m^{s,a} = \int_0^h \frac{\cosh k_0(h-y)}{\cosh k_0 h} f_m^{s,a}(y)\, dy, \quad m = 0, 1, \ldots, N. \tag{3.17}$$

The integrals (3.16) and (3.17) can be evaluated explicitly as in Kanoria et al. [5] by using the different properties and standard results on the Gegenbauer polynomials. Thus,

$$\begin{aligned} K_{mn}^{s,a} = \frac{\delta_0 \mu}{k_0 \cosh^2 k_0 h}\Bigg[4(-1)^{m+n} \sum_{r=1}^{\infty} \Bigg\{ & \frac{k_r^{2/3} \cos^2 k_r h}{\delta_r s_r h^{1/3}} J_{2m+1/6}(k_r h) J_{2n+1/6}(k_r h) \\ & + \frac{\alpha_r^{2/3}(\coth t_r b, \tanh t_r b)}{\gamma_r t_r h^{1/3}} \cos^2 \alpha_r c J_{2m+1/6}(\alpha_r h) J_{2n+1/6}(\alpha_r h) \\ & + \frac{(-\cot(\alpha_0^2 - \nu^2)^{1/2} b, \tan(\alpha_0^2 - \nu^2)^{1/2} b)}{\gamma_0 (\alpha_0^2 - \nu^2)^{1/2} h^{1/3}} \cosh^2 \alpha_0 c\, I_{2m+1/6}(\alpha_0 h) I_{2n+1/6}(\alpha_0 h) \Bigg\}\Bigg] \end{aligned}$$

for $m, n = 0, 1, \ldots, N$ and

$$d_m^{s,a} = \frac{I_{2m+1/6}(k_0 h)}{(k_0 h)^{1/6}}, \quad m = 0, 1, \ldots, N.$$

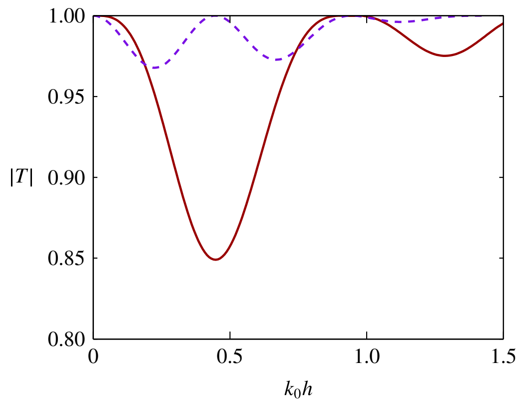

FIGURE 2. Plot of $|T|$ against K_0h with fixed values of $c/h = 2, b/h = 5$ and different values of $\theta = 0°$ (– – –) and 45° (–).

The constants $a_n^{s,a}$ $(n = 0, 1, \ldots, N)$ are now obtained by solving the linear systems (3.15) and, from (3.11),

$$C^{s,a} = \sum_{n=0}^{N} a_n^{s,a} d_n^{s,a}.$$

Then $|R|$ and $|T|$ are evaluated from (3.10).

4. Numerical results

To solve the linear system (3.15) and to obtain numerical results correct up to six decimal places, a suitable choice of N is 40. The reflection and transmission coefficients, $|R|$ and $|T|$, are evaluated numerically for different values of Kh. These coefficients are also depicted graphically for various values of the parameters b/h and c/h, and for different incident angles in a number of figures. To verify the results obtained here, we have compared our results with those given by Kirby and Dalrymple [6], obtained by employing a different method. Kirby and Dalrymple [6] plotted $|T|$ against the nondimensional wave number k_1h_1 ($\equiv k_0h$ here) in their Figure 4(a) and (b) for various values of h_2/h_1 ($\equiv c/h$ here) and a fixed value of the ratio of the trench length and water depth in the left-hand side of the trench ($\equiv 2b/h$ here) and different values of the incident angle θ. In Figure 4(a) of Kirby and Dalrymple [6], $|T|$ was plotted against k_1h_1 ($\equiv k_0h$ here) for $2b/h = 10$, $c/h = 2$ with $\theta = 0°$ and 45°. In this paper, $|T|$ is plotted against k_0h for the same values of $b/h, c/h$ and θ in Figure 2. Also, the curves in Figure 4(b) of Kirby and Dalrymple [6] depicting $|T|$ against k_1h_1 for $2b/h = 10$, $c/h = 3$, $\theta = 0°$ and 45° almost coincide with the corresponding curves in Figure 3 here, which are plots of $|T|$ against k_0h for $b/h = 5, c/h = 3, \theta = 0°$ and 45°. Also, $\theta = 0°$ corresponds to the case of normal incidence and Figure 4 plots $|R|$ against $k_0h/2\pi$ with $c/h = 2, b/h = 2.5$ and $\theta = 0°$. This curve is compared with Figure 2 of Lee and Ayer [9], in which $|R|$ was plotted against h/λ ($\equiv k_0h/2\pi$ here) with the same

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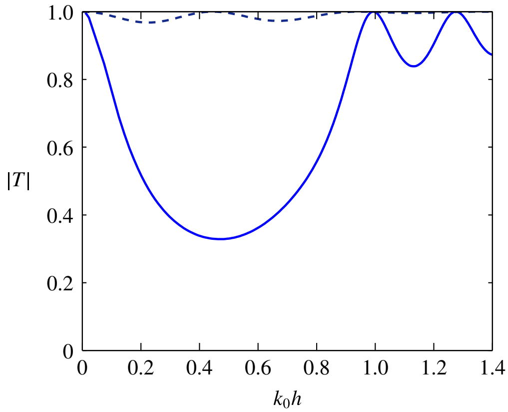

FIGURE 3. Plot of $|T|$ against K_0h with fixed values of $c/h = 3, b/h = 5$ and different values of $\theta = 0°$ (− − −) and 45° (–).

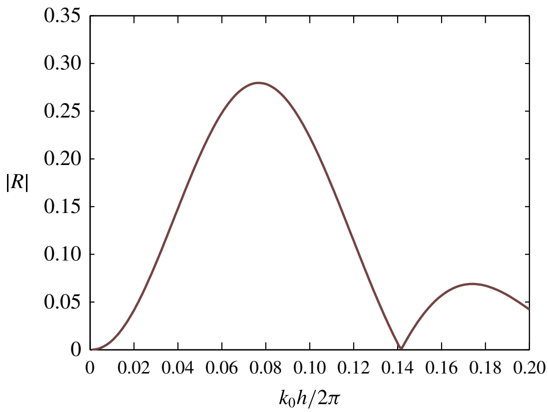

FIGURE 4. Plot of $|R|$ against $k_0h/2\pi$; $c/h = 2$, $b/h = 2.5$ and $\theta = 0°$.

values of c/h and b/h for the case of normal incidence. Here, it is observed that Lee and Ayer's result (see [9, Figure 2]) is recovered from our present obliquely incident result (Figure 4 here) by taking the incident angle θ to be 0°.

Some more figures of $|R|$ and $|T|$ are drawn against the nondimensional wave number Kh in Figures 5(a) and (b), 6 and 7. In Figure 5(a) and (b), $|R|$ and $|T|$ are depicted, respectively, with fixed values of $c/h = 2$, $b/h = 3$ and different values of the incident angle θ, such as $\theta = 30°$, 45° and 60°. In these figures, we observe that the amplitude of $|R|$ gradually increases and the number of zeros of $|R|$ gradually decreases with increasing values of the incident angle. The value of $|T|$ gradually decreases and also the number of zeros of $|T|$ becomes less as θ increases. Thus, more energy is reflected and less is transmitted as the incident angle increases.

In Figure 6, $|R|$ and $|T|$ are plotted with fixed values of $c/h = 2$ and $\theta = 45°$ for different values of b/h, such as 2, 3 and 5. From these figures, it is clear that as the

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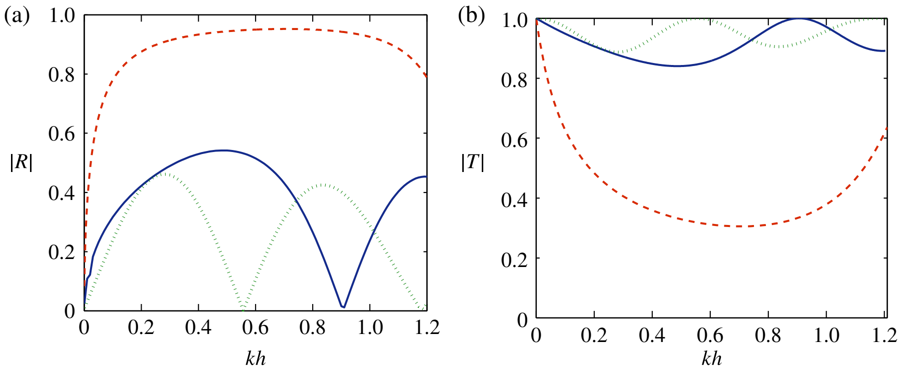

FIGURE 5. Plots of (a) $|R|$ and (b) $|T|$ against Kh with fixed values of $b/h = 3, c/h = 2$ and different values of $\theta = 30°$ (– · – ·), 45° (· · ·) and 60° (–).

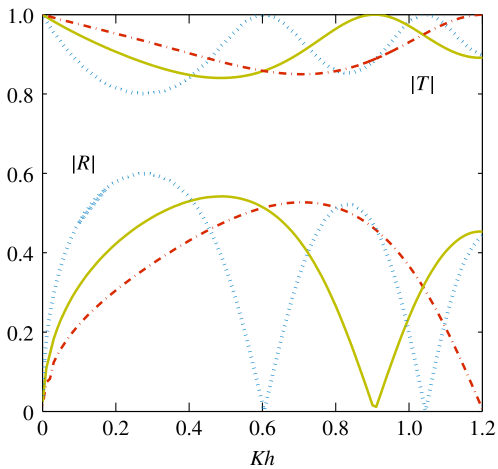

FIGURE 6. Plots of $|R|$ and $|T|$ against Kh with fixed values of $\theta = 45°$, $c/h = 2$ and different values of $b/h = 2$ (– · – ·), 3 (–) and 5 (· · ·).

trench width becomes large, more energy is reflected and less energy is transmitted. Figure 7 depicts $|R|$ and $|T|$ against Kh. Here, the values of $b/h\,(= 4)$, $\theta\,(= 45°)$ remain fixed, and $c/h\,(= 1, 2)$ varies. From this figure, it is observed that as c/h increases, $|R|$ gradually decreases and $|T|$ gradually increases. Also observe that the energy identity relation $|R|^2 + |T|^2 = 1$ is satisfied numerically. This is used as a partial check for the correctness of the numerical results.

5. Conclusion

A multi-term Galerkin approximation method involving ultra-spherical Gegenbauer polynomials of order $1/6$ was employed here to reinvestigate the oblique wave

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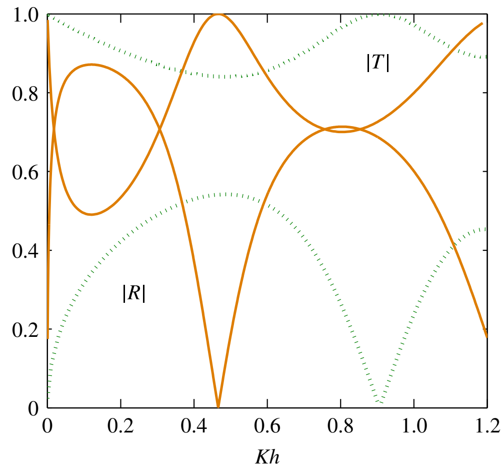

FIGURE 7. Plots of $|R|$ and $|T|$ against Kh with fixed values of $\theta = 45°$, $b/h = 4$ and different values of $c/h = 1$ $(\cdots)$ and 2 (–).

scattering problem involving a symmetric, rectangular submarine trench. Very accurate numerical estimates for the reflection and transmission coefficients for different values of wave number and other parameters involved in the physical problem have been obtained. The results for normal incidence are recovered from the present solution by putting $\theta = 0$. The numerical results are illustrated in a number of figures, some of which match quite well with those available in the literature, drawn by using different mathematical techniques. The incident angle as well as the trench length affect the reflection and transmission coefficients significantly. If the water depths before and after the trench are different, that is, for the case of an asymmetric submarine trench, we expect that the same method can be employed with appropriate modifications.

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