N-BODY SIMULATIONS

S. J. AARSETH
Institute of Astronomy, Cambridge, U.K.

Abstract. Numerical results on escape and binary evolution in N-body systems are summarized and some results of a new calculation with initial binaries are described.

This brief review of the N-body problem concentrates on the two central aspects, escape and binary evolution. It is particularly important to examine these processes in relation to the fast Monte Carlo and Boltzmann moment methods which are based on simplifying assumptions. One task for N-body calculations is therefore to establish the region of validity of the approximations, since there is no *a priori* guarantee that such results can be applied even to relatively large systems.

Direct integrations of small particle numbers ($N=10$) indicate that multiple encounters contribute significantly to the escape rate, whereas theoretical considerations are based on two-body encounters. However, further calculations show that the latter do increase in relative importance for larger N. These simulations also yield more statistical information about the actual escape mechanisms. Two quantities are particularly useful for the analysis of this data. Let us denote the absolute value of the binding energy per unit mass before and after escape by β and α, respectively, both being expressed in terms of the initial mean kinetic energy, $\frac{1}{2}\overline{mv^2}$. Typically, $\alpha+\beta\simeq 2$ for equal-mass systems with $N=250$, indicating that escape is still due to discrete events. Note that binaries are absent for most of the time. The distribution of escape energies, α, is very wide for unequal masses; in a cluster model with $N=500$ there are 18 escapers out of 46 with $\alpha>1$. Some of these fast particles are produced by 'super-elastic' encounters with energetic binaries, but others are due to close two-body encounters which also dominate when $\alpha<1$. These results have implications for the approximate methods which are based on distant two-body encounters. Thus according to theory, the close encounters (i.e. separations $r\leqslant 2G\bar{m}/\bar{v}^2$) should only contribute in the ratio $1/5\ \ln(0.4\ N)$ with respect to the distant encounters. Because of the numerical results, the neglect of escape by close encounters in large systems ($N\sim 10^5$) may only be justified if the theoretical N-dependence is too weak.

In order to discuss the particle evaporation, we introduce the relative escape rate per crossing time, $L=\Delta N t_{cr}/Nt$. A total of four equal-mass models with $N=250$ have been studied (including one by Dr Wielen), giving $L\simeq 5-11\times 10^{-4}$ for $t/t_{cr}\simeq 30-40$. The introduction of a mass spectrum leads to an increased escape rate, i.e. $L\simeq 4\times 10^{-3}$, whereas $L\simeq 3\times 10^{-3}$ for a similar model with $N=500$ and $t=30t_{cr}$. In the latter case, only about 10% of the escapers have $\beta<1$, although there has been ample time for such orbits to return from the halo for further interactions. Thus it appears that the increased relaxation time of elongated halo orbits is not compensated by the smaller

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binding energy. This feature is also consistent with the absence of preferential escape among all the light particles. Furthermore, well bound particles can only acquire positive energy by large transitions and the presence of such events demonstrates the importance of close encounters.

The N-body calculations show that binaries play a crucial role in small stellar systems. In the first place, a close binary inevitably forms as the end product of the core evolution. Mass segregation favours the combination of heavy particles but, where appropriate, the subsequent evolution proceeds further in the direction of increased mass by capture or exchange. At the same time, the binding energy rapidly exceeds 50%; even for $N = 500$ the corresponding time-scale is only about 12 t_{cr}. This energy sink behaviour can be understood in terms of an asymmetry between incoming and ejected particles since only the latter may exceed the escape velocity. Capture, which is the opposite process to escape, may be viewed as a short-lived event, giving much the same end result. Most of the energy absorbed by the central binary is due to the ejection of strongly bound particles into the halo, whereas escape contributes less to the core evolution. Consequently, the decreased core density leads to a slower evolution rate. At some stage, the core expansion is halted and gives way to a secondary contraction, culminating in a stable hierarchical triple system. Subsequent disruption by external perturbations often produces significant recoil kinetic energy which is transferred to other core members by two-body encounters. The equivalent development is somewhat delayed in equal-mass systems, but the higher number density in the core compensates to some extent for the lack of heavy particles.

It is not yet clear whether binary effects may be neglected in the larger systems simulated by fast methods. Although the presence of only one binary could at most influence the inner core, multiple binary formation during the collapse phase appears likely on theoretical grounds. Another possibility is that binaries are present initially in significant numbers. N-body calculations have recently been made in order to study this effect and some preliminary results are available.

For simplicity we adopt an equal-mass system with 100 particles of mass m_1 and replace 20 of these particles by 10 close binaries, each with a binding energy $E_b = 5\, m_1 \bar{v}_1^2$. This choice of energy is based on theoretical considerations of maximum efficiency discussed elsewhere in this volume. In order to set up a self-consistent dynamical structure, the cluster is exposed to violent relaxation in the form of a moderate initial contraction. The next phase is characterized by mass segregation; hence a modest initial binary population may eventually dominate the central region if their combined mass exceeds the average mass of the single particles. At this stage ($t \simeq 5\, t_{cr}$), the number of halo-type orbits is very similar in the comparison system where 10 single particles of mass $m_2 = 2\, m_1$ replace the binaries. Although one of the close binaries is destroyed in a two-binary collision, their total internal energy is slightly increased. Further favourable interactions occur during the subsequent evolution, until finally at $t \simeq 24\, t_{cr}$ about 75% of the total energy is contained in bound pairs compared to 50% initially. Hence treating the binaries as single particles implies that the cluster members are less bound by a factor of 2 and the dynamical time-scale is increased by $2\sqrt{2}$. The binaries are also less bound to the cluster centre by a similar factor, with the most energetic binary ($E_b \simeq 20\, \bar{m}_1 \bar{v}_1^2$) in an elongated halo orbit.

Of the nine surviving binaries, three remained unperturbed, even preserving their eccentricity to two decimal places, and two more retained their identity with somewhat increased binding energy. A further three suffered exchange of one companion and one binary actually lost both of its original members. Significantly, the binaries with new companions are also the most energetic. The modest escape rate, i.e. $L \simeq 1 \times 10^{-3}$, reflects the core expansion effect. In the comparison system one particle escapes during ten crossing times, also giving $L \simeq 1 \times 10^{-3}$. The heavy particles are now much more strongly bound to the centre, with two of them combined into a close binary of energy $E_b \simeq 4\, m_1 \bar{v}_1^2$. Considering the future evolution of the first system, it is likely that the binaries will eventually be destroyed by further collisions or be ejected altogether, in both cases leaving behind a more loosely bound cluster. Finally, we note that the number of centrally concentrated binaries was only three or four on average. This small population illustrates the effectiveness of superelastic encounters between the binaries, a process which acts in the opposite sense to the mass segregation.

A more extensive survey of the N-body problem will be published jointly with Dr M. Lecar in *Annual Review of Astronomy and Astrophysics*, Volume 13.

Some of the ideas discussed above were formulated at the 1974 Theoretical Astrophysics Workshop held at the Aspen Center for Physics.

DISCUSSION

King: The disagreement of the number of close to distant encounters is because you have applied the theory where it was not meant to apply. The theoretical escape formula depends on assuming that $\langle v_e^2 \rangle / \langle v^2 \rangle = 4$, which is far from true in the region from which escape takes place in your systems, and the resulting escape rate is very sensitive to this number. The formula is more applicable if there is a tidal limit, and I think that you would find that escape by diffusion was relatively much more important there. Correspondingly, the existence in your systems of Hénon's paradox (that stars close to escape have vanishingly small diffusion) is also due to the lack of a tidal limit. In fact, Hénon's paradox is asymptotically true for real systems, as the tidal limit becomes infinitely large; but for a typical rich cluster, with a typical tidal limit, escape should be predominantly by diffusion.

Aarseth: The above discussion is only concerned with isolated systems, as are the Monte-Carlo calculations. Furthermore, I am primarily questioning the escape mechanism rather than the escape rate and the former depends only weakly on the velocity ratio.

Spitzer: I would like to point out that the difference between close encounters and distant encounters becomes rather small if N is small, and even for N equal to 250, it may be difficult to tell these two apart. For example, I compute that for a cluster with $N = 250$ the mean change of energy for a star of zero energy making one traverse through the core and experiencing many so-called 'distant encounters' is about $\frac{3}{5}$ of the mean kinetic energy for all cluster stars; this 'step size' for energy changes is evidently, much larger than one would commonly associate with a diffusion picture.

Miller: In your discussion for $\Delta = (\frac{1}{2}v_\infty^2 - E')/\frac{1}{2}\overline{mv^2}$, the energy change leading to escape, what happens with encounters that lead to large energy changes but do not lead to escapes? Have they been as carefully studied? Are there collisions, starting from lower E' which have just as large a Δ but do not lead to escape? What is known of the distribution of Δ, as it depends on E' (but independent of whether process leads to escape)?

Aarseth: There is no detailed analysis of energy changes which do not lead to escape. It is quite conceivable that there are such large energy changes but the number of small changes will in any case be much greater.

Severne: Can you indicate what value the ratio of close to distant encounters should attain to give a value $\bar{\Delta} \simeq 2$?

Aarseth: The ratio should probably be somewhat greater than one.