STELLAR SEISMOLOGY

WERNER DÄPPEN
Space Science Department of ESA
ESTEC
2200 AG Noordwijk
The Netherlands

ABSTRACT. *Stellar* acoustic oscillation frequencies will likely be accurately observed in the near future, in analogy to the well-known *solar* five-minute oscillation frequencies. Of course, we will never expect the wealth of solar data, which is a result of spatial resolution. We will therefore not be able to solve the inverse problem, that is to probe physical quantities as functions of depth, and the low number of anticipated observed frequencies will make an unambiguous mode identification difficult. Despite this restriction to the forward problem, however, observed stellar oscillation frequencies will become valuable constraints for the determination of stellar parameters. One should not forget that the present knowledge of stellar ages and compositions relies on the calibration of theoretical models (matching effective temperature and luminosity). Additional observational constraints will improve these calibrations, even if the theoretical models themselves are not questioned. We hope, however, that the observation of stellar oscillation frequencies will also lead to improvements in the *physics* of stellar models, in analogy to the solar case. Again, of course, stellar seismologists will be less ambitious than helioseismologists, since there are more open parameters in stellar models. However, stellar observations will allow tests of models with different age and composition.

1. Introduction

Helioseismology has been extremely successful in probing the solar interior. The precisely observed p-mode oscillation frequencies act as a window, enabling us to peep into the Sun (see *e.g.* Christensen–Dalsgaard, these proceedings; for reviews see *e.g.* Christensen–Dalsgaard and Berthomieu 1990, Deubner and Gough 1984). The high spatial resolution of the solar disk has allowed to observe, and unambiguously identify, p modes in a range of angular degree l from zero (radial modes) to a few thousand. This in turn has lead to direct inversion of the oscillation frequencies to obtain physical quantitities (such as sound speed) as a function of depth – without recourse to models.

It is clear that little of this beautiful picture will remain in the stellar case. The lack of spatial resolution will restrict the observable modes to those of low l (not greater than 3 or 4), with higher-degree modes disappearing in the integrated light. It is worth mentioning that clever techniques exist, which are based on line profile variations (*e.g.* Smith 1985;

G. Berthomieu and M. Cribier (eds.), Inside the Sun, 357–370.

further references are found in Baade 1988a,b). They promise observation of some nonradial modes of somewhat higher angular degrees (l up to 8 or perhaps 16). But even these few extra modes can never bring the wealth of solar seismological data (but never say never!).

As a consequence there will be no direct inversions of stellar oscillations frequencies. Nevertheless, despite this lack of analogy, stellar seismology does have exciting prospects. The reason is that stars can offer something that the Sun can't: their large number. The Sun can be subjected to helioseismological tests as thoroughly as wanted, but as far as they are to test the theory of stellar evolution, they do it merely for the case of one mass, age, and chemical composition. In stars, however, we see these parameters differing from one object to another. Without seismology, the number of observable stellar quantities that can serve as tests of the theory of stellar evolution is not that large (one knows effective temperatures quite well, luminosities for nearby stars with known parallaxe, and mass in the case of close binary system). Though the theory of stellar evolution has helped enormously to decipher the phenomenology of these observations, there are still sizeable uncertainties in the physics of the stellar models used. These uncertainties matter: our cosmic distance-scale, as well as the age determination of globular clusters, relies on good models of variable stars (Cepheids and RR Lyrae; for a recent review see van den Berg 1989).

Despite the considerable precision that solar models have attained one should not forget the underlying standard assumptions of stellar evolution, which are, *e.g.* , an initially chemically homogeneous gas cloud, nuclear reaction rates that are determined or extrapolated from laboratory values, opacities from elaborate theoretical calculations, plus a crude mixing-length formalism for convective heat transport. But the fact that solar standard models work quite well is no guarantee to use the same assumptions for stars with different parameters. Helioseismology does not equally strongly test all these assumptions. Indeed, solar physicists are happy about those uncertainties in the physics of the models that are not so important; these uncertainties do not obstruct the way towards constructing a 'good' solar model. As an example I mention opacity in the bulk of the solar convection zone. Since there the temperature gradient is to a very good approximation given by the thermodynamic adiabatic gradient, opacity doesn't play a role. This is good for solar modellers, but bad for those who test stellar opacitites. For certain stars the same opacities that have little importance for the Sun do matter. So, for instance, in the case of stars that are slightly more massive than the Sun, where the convection zone is much shallower, and where thus opacity at temperatures and densitites corresponding to the solar convection zone significantly influences the structure of the star.

In the spirit of this interdisciplinary conference, I will stress the common mathematical language with other physical disciplines. Review articles that are more astrophysically slanted are found elsewhere (*e.g.* Christensen–Dalsgaard 1984, or Däppen *et al.* 1988). One of the goals of this article is to show that computing stellar oscillation frequencies is a relatively easy part of stellar seismology, while the hard work is being done in the computation of equilibrium models of evolved stars, where the nonlinear partial differential equations of stellar evolution must be solved. Furthermore, to interrogate the physics of stellar interiors by oscillation frequencies, one usually has to compare models that are the same up to the physical quantity to be tested (notice that we are restricted to the forward problem). Again, computing the series of similar evolutionary sequences is time-consuming. In contrast, the physical description of the pulsation itself remains unchanged, as long as one is content with the adiabatic approximation, which is sufficient for interpreting oscillation frequencies. Of course the situation becomes more complicated if other pulsational features are examined, such as exitation, life-time, or amplitudes.

2. Modelling Stellar Oscillations

In this section, I present three different approaches. Since I do not consider nonlinearities here, the problem of stellar oscillations is equivalent to finding all normal modes. The first approach will be an outline of the complete numerical solution. The similarity with other eigenvalue problems in mathematical physics is stressed (*e.g.* vibrating strings, bound states in quantum mechanics). The second approach will be based on a simplified wave equation and on propagation diagrams that reflect local conditions in a star. This approach is best suited for a qualitative discussion of the frequency spectrum of modes. Further characteristics of the modes, like their type (p mode or g mode, see below) or penetration depth are also directly visible in propagation diagrams. Finally, I will present yet an other approach to represent oscillation frequencies, namely asymptotic theory for high-order modes. Its advantage is a relatively easy, quantitative link between the observed frequency spectrum and the mass and age of the star.

Before discussing these three different approaches, I begin with some very basic facts from stellar evolution, because it is clear that one needs some ideas about the equilibrium solution before one can study the deviations from it.

2.1. SOME ELEMENTARY PREREQUISITES FROM STELLAR EVOLUTION

The basic framework to model stellar oscillations is given by the hydrodynamic equations of motion for the stellar fluid moving around its equilibrium. This equilibrium solution is assumed as given; it results from a calculation of stellar structure and evolution. A still excellent introduction to the basic principles of stellar evolution is the book by Schwarzschild (1958) (though the book can't of course cover all the fascinating progress that has been made possible since, especially thanks to a greatly increased computing power). For our purposes we need to know that stars in the equilibrium are self-gravitating gas balls, with a pressure gradient balancing the local gravitational force, and with a temperature gradient associated to the heat flux going from the nuclear-burning central regions to the (much cooler) outside. These two gradients are at the base of the nonlinear stellar structure equations.

As long as the star still lives from its hydrogen supply in the center, things happen very slowly (the Sun, *e.g.* , has been around for about 5 10^9 years, with now about half of its hydrogen fuel used). More massive stars have a much shorter life, because their energy output (*i.e.* their luminosity) increases with at least the third power of the mass of the star; therefore the hydrogen-burning phase of a 100 solar-mass star is only of the order of

10^6 years. Less massive stars make much more economical use of their resources; their life time easily exceeds the currently assumed age of the universe.

During this hydrogen-burning phase stars are on the so-called main sequence, which is a phenomenological name, originating in the fact that if stellar luminosity is plotted against surface (effective) temperature (in the so-called Hertzsprung–Russel diagram), then most stars lie on a line, *i.e.* the main sequence. It was an important success of the theory of stellar evolution to identify this observational fact with hydrogen burning. Another important result of the theory of stellar evolution is that during the hydrogen-burning phase a star moves the main sequence slightly upwards (the Sun is generally thought to have been about 25% less luminous at the begin of its main-sequence life). Stars thus climbing up the main sequence look practically the same as more massive but younger ones, if the only discriminating observables are temperature and luminosity. It is precisely one of the goals of stellar seismology to lift this degeneracy by providing additional observables (see section 3).

2.2. OUTLINE OF NUMERICAL COMPUTATIONS OF STELLAR OSCILLATIONS

The purpose of this subsection is to convince the reader that calculations of stellar oscillations are a relatively simple affair, if reasonable assumptions and simplifications are made. As briefly mentioned before, it is the problem of finding the stellar equilibrium solution that is the difficult part.

Assuming the existence of a stable and constant equilibrium configuration, to discuss the pulsational motion of a star we start out from the hydrodynamic equations for compressible fluids

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\nabla\mathbf{v} = -\frac{1}{\rho}\nabla p + \nabla\psi \ . \tag{1}$$

Here, $\mathbf{v}$ is the (Eulerian) velocity field, p and ρ pressure and density, respectively, and ψ is the (self-) gravitational potential. For simplicity, we have disregarded viscosity. To this equation, one must add the usual equation of continuity and also an energy equation, which - in the simplest case - is replaced by a condition of adiabaticity, normally expressed in the form of constant co-moving specific entropy (per mass). Under the assumption of adiabaticity, stellar pulsation is frictionless and energy conserving. It is clear that this assumption precludes a discussion of mode excitation (or damping), necessary to understand why we see this pulsation mode but not the other. Nevertheless, this hypothesis of adiabaticity still leads to many important results by telling, *e.g.* , what the linear eigenfrequencies are. Virtually the whole success of helioseismology has been so far in the framework of adiabatic pulsations; only very recently have serious attempts been made to go beyond and to address questions like mode excitation, damping, and amplitudes (for a review see Cox *et al.* 1990).

In the following I discuss a few of the key steps used in manipulating Equation (1). I will be very brief, but details can be found, *e.g.* , in the book by Unno *et al.* (1979). We restrict ourselves to a simple, but still relevant case, in which the equilibrium configuration is assumed to be at rest (*i.e.* no rotation) and spherically symmetric. The gravitational potential is assumed to be static, *i.e.* its distortions due to the stellar pulsation itself is neglected (this is the so-called Cowling approximation, see Unno *et al.* 1979). Restricting even more to *small* perturbations, we introduce *linearization* of Eq. (1).

The static equilibrium and the linearized equations allow the separation of the time dependence in the form of $\exp(i\omega t)$. The spherically symmetric equilibrium configuration allows expressing the field variables (displacement vector, pressure, and density) as a series of spherical harmonics Y_l^m of angular degree l and azimuthal order m, so that each term in the series is itself a solution of the equations. The adiabatic assumption is used to link pressure and density fluctuation with the help of a thermodynamical quantity (the adiabatic exponent), which is a given quantity of the equilibrium model. One arrives therefore at a fourth-order system for the (independent) three displacement-vector components and the pressure fluctuation. The Cowling approximation allows yet another simplification; with it, the tangential part of the displacement field becomes proportional to the tangential component of the gradient of the (Eulerian) pressure fluctuation, and thus the only independent fields remaining are the radial component of the displacement vector and the pressure fluctuation. Their amplitudes are governed by the following (schematic) system of ordinary differential equations

$$
\begin{aligned}
\frac{dy_1}{dr} &= f_{11}(r)y_1 + f_{12}(r;l;\omega^2)y_2 \\
\frac{dy_2}{dr} &= f_{21}(r;\omega^2)y_1 + f_{22}(r)y_2 \\
y_1 &\equiv \frac{\xi_r}{r} \\
y_2 &\equiv \frac{p'}{gr\rho} \; .
\end{aligned}
\tag{2}
$$

Here, the functions $f_{ij}(r;\omega^2)$ are expressions involving quantities of the equilibrium model (like pressure, density, local gravity, sound-speed, etc.). As is standard practice, the labels l and ω are dropped in the y_i's. The prime $'$ denotes the Eulerian (first-order) displacement from equilibrium, (and not a derivative as everywhere else in the paper).

Adding boundary conditions to equation (2) leads to an eigenvalue problem. The one at the *centre* is the usual regularity condition due to the singular nature of Eq. (2). From the specific nature of the coefficients one knows (see Unno 1979) that Eq. (2) has a regular-singular point at $r = 0$, and so there is a regular and a singular solution. Picking the regular one gives the boundary condition (this is explicitly done by a standard power-series development). The *outer* boundary condition is not so simple. In principle, one would have to put a good stellar atmosphere (for which there are elaborate models) at the outer end, and impose smooth matching as the outer boundary condition. Until now, nobody has succeeded in doing that, and simpler approaches must be chosen. Often an isothermal atmosphere is assumed, and the outer boundary condition is determined by a discussion of propagation and reflection of sound waves in a stratified atmosphere analogous to Lamb (1932) (we come to that in 2.3.). Here, we can afford something even simpler, namely a mechanical boundary condition of the form $\delta p = 0$ where δ stands for the Lagrangian displacement. Such a boundary condition is the three-dimensional analogon of a frictionless lid. In terms of our variables y_1 and y_2, the condition $\delta p = 0$ is given by $y_1 + y_2 = 0$.

I hope that I have convinced everyone that computing stellar oscillation frequencies is the easy part of seismology, if one can borrow a good equilibrium model from a friend. Finding the eigenvalues of Eq. (2) is really not much harder than those of a vibrating string, whose equation is $d^2y/dx^2 + \omega^2 y = 0$ with boundary conditions $y = 0$ at two different x values. The only complication in Eq. (2) comes from the nonconstant coefficents, but numerically it is still an easy task. The hard part is therefore finding the equilibrium solution which delivers the coefficients for Eq. (2). In studies of the parameter dependence of oscillation frequencies one generates series of similar evolutionary models, with slightly different stellar parameters (age, mass, composition, or quantities from basic physics like the opacity). This can be time consuming, especially if rather evolved stars are considered, where many time steps are required for the solution.

2.3. QUALITATIVE DISCUSSION USING PROPAGATION PROPERTIES

We adopt the asymptotic discussion of Deubner and Gough (1984), which itself is similar to the treatment of acoustic waves by Lamb (1932) [see also Christensen-Dalsgaard (1986)]. For wavelengths much shorter than the solar radius, normal oscillation modes can be quite accurately discussed using the simplified wave equation

$$\Psi'' + K^2(r)\Psi = 0 \, . \tag{3}$$

Here, $\Psi = \sqrt{\rho}c^2 \mathrm{div}(\delta \mathbf{R})$, where ρ and c are density and sound speed of the equilibrium configuration, and $\delta\mathbf{R}$ is the fluid displacement vector. The local wave number is given by

$$K^2(r) = \frac{\omega^2 - \omega_c^2}{c^2} + \frac{l(l+1)}{r^2}\left(\frac{N^2}{\omega^2} - 1\right) \, , \tag{4}$$

with the acoustic cut-off frequency defined by

$$\omega_c^2 = \frac{c^2}{4H^2}(1 - 2\frac{dH}{dr}) \, , \tag{5}$$

and the Brunt-Väisälä frequency N by

$$N^2 = g(\frac{1}{H} - \frac{g}{c^2}) \, , \tag{6}$$

where H is the density scale height and g the local gravity. From the form of Eq. (3) (to which upper and lower boundary conditions must be added), one immediately realizes that in propagation zones necessarily $K^2 > 0$.

Our present qualitative discussion of the influence of mass and evolution on oscillation frequencies aims at showing the maximum of effects with a minimum of curves. Here, we

https://doi.org/10.1017/S0252921100068068 Published online by Cambridge University Press

restrict ourselves to the role of N^2 and ω_c^2 in K. For finer details we refer the reader to Deubner and Gough (1984), Christensen-Dalsgaard (1984) or Gough (1985).

With the convenient definition of the Lamb frequency

$$S_l^2 = \frac{l(l+1)}{r^2} c^2 , \tag{7}$$

we obtain the simplified necessary conditions for propagation of an acoustic wave, $\omega > \omega_c$ and $\omega > S_l$. Additionally, in order to have a trapped standing wave, it is also necessary that in some surface layer ω_c becomes greater than ω. This happens indeed; the height of this (outer) ω_c mountain is the greater, the cooler the local temperature at the edge of the star is [this is seen from Eq. (5)]. 'Mathematical' stars with zero temperature at the outer boundary have an infinitely high ω_c mountain; they can therefore trap modes of arbitrary high frequency. Real stars have an 'inversion' temperature (just above the photosphere); further up temperature begins to rise again. The maximum p-mode frequency 'measures' this inversion temperature.

In propagation zones (if a constant adiabatic exponent of 5/3 is assumed), a further simplification follows from the fact that $\omega > g/c$ implies $\omega > \omega_c$. And finally, we choose the approximation of identifying (the absolute value of) g/c with N, which certainly gives the correct order of magnitude in radiative zones (but would be entirely wrong in convection zones, where $N \approx 0$). The advantage of this choice is that the same curves will give information about g modes, too.

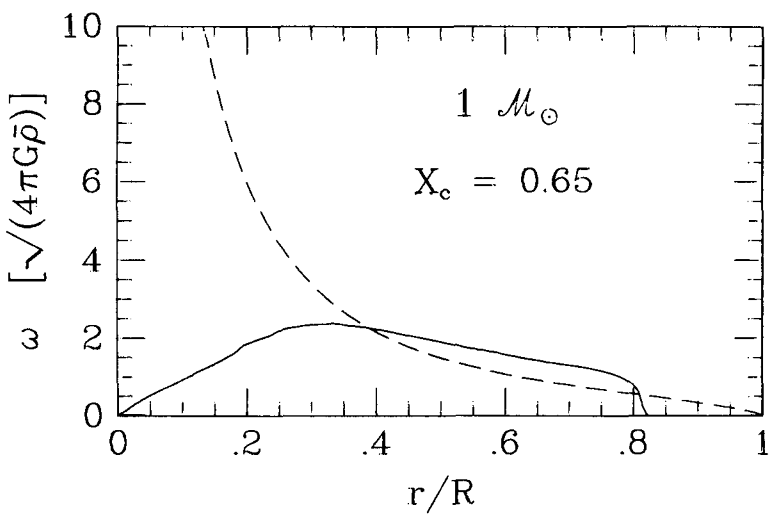

Figure 1. Critical frequencies as functions of the fractional radius r/R. The solid line denotes the Brunt-Väisälä frequency N, the dashed line the Lamb frequency S_l for $l = 1$. The model parameters are: hydrogen abundance $X = 0.70$, heavy-element abundance

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$Z = 0.01$, and the mixing-length parameter $l/H_p = 1.5$. Stellar age is indicated by the central hydrogen abundance X_c.

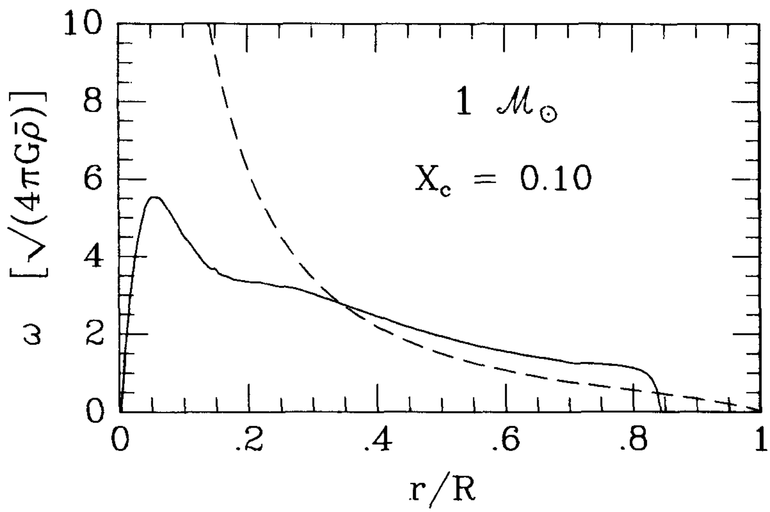

Figure 2. Same as Figure 1, but for a more evolved model.

Let us now consider S_l (we choose the representative case $l = 1$) and N in a model of a 1 $M_\odot$ star (Figure 1). Due to the rather deep convection zone, N cannot represent the increase of ω_c close to the surface, and so the diagram does not show the upper turning point that is caused by the large spatial inhomogeneity near the surface. In Figure 1, S_1 defines the penetration depth of the $l = 1$ modes; for $l > 1$ the corresponding curves would be shifted to the right.

Figure 1 also shows how to distinguish p modes from g modes. The distinction is only asymptotic. Modes with $\omega \to \infty$ would become true p modes (but they cease to be trapped above a certain frequency, see above). Modes with $\omega \to 0$ become true g modes [they have a large K^2 due to N/ω, see Eq. (4)]. Outside the asymptotic regime one still speaks of p modes and g modes, but they are not 'pure', though most of them are of a dominant type. Only in highly evolved stars do genuine dual-status modes appear (see section 3.).

2.4. ASYMPTOTIC THEORY OF OSCILLATION FREQUENCIES

The simplest theoretical analysis of oscillation frequencies is asymptotic theory (Tassoul 1980), which - to second order - yields the following expression for the frequencies $\nu_{n,l}$

$$\nu_{n,l} = (n + l/2 + \sigma)\nu_0 + \epsilon_{n,l} \; . \tag{8}$$

Here, σ is a constant of order unity, ν_0 will be defined below (Eq.10), and $\epsilon_{n,l}$ is small compared with $\nu_{n,l}$. (This expression is found by manipulating Eq. (2), exploiting the simplifications due to the high-order modes with short wave-length). Given a set of observed oscillation frequencies, the constants appearing in Equation (8) can be determined. As shown below, they are related to integral quantities of the star.

At this point it is useful to introduce two definitions pertaining to the structure in the periodogram of high order pulsators.

(i) Large frequency-separation:

$$D_{n,l} \equiv \nu_{n+1,l} - \nu_{n,l} \; . \tag{9}$$

To first order asymptotic theory it is well known that

$$D_{n,l}{}^{-1} \approx \nu_0{}^{-1} \equiv 2 \int_0^R \frac{1}{c} dr \; , \tag{10}$$

with c denoting local sound speed and R the radius of the star. In simplified stellar models (polytropes) it is easy to show that

$$\nu_0 \propto \sqrt{\frac{g}{R}} = \sqrt{\frac{GM}{R^3}} \; . \tag{11}$$

(ii) Small frequency-separation:

$$d_{n,l} \equiv (\nu_{n,l+1} - \nu_{n,l}) - \frac{1}{2}(\nu_{n+1,l} - \nu_{n,l}) \; . \tag{12}$$

The small separation serves to cancel the first-order term of Eq. (8), and thus reveals second-order details. The ratio between small and large separation pertains to the central regions of the star, and is, to second-order asymptotic theory, given by (Tassoul 1980)

$$\frac{d_{nl}}{D_{nl}} \approx \left(\frac{l+1}{2\pi^2 \nu_{nl}}\right) \int_0^R \frac{dc}{dr} \frac{1}{r} dr \; . \tag{13}$$

This equation is somewhat simplified; for a more thorough discussion see Gabriel (1990). Since sound speed increases steeply from the surface to the centre of a star, $D_{n,l}$ probes more the surface regions and $d_{n,l}$ more the central regions.

3. Determination of Stellar Age and Mass from Seismology

3.1. QUALITATIVE DISCUSSION

The two Figures 1-2 show the principal effects of evolution and mass on oscillation frequencies, thus demonstrating the power of the qualitative tools of section 2.3. The first effect of evolution is an increase of the inner 'N-mountain', caused by the growing spatial chemical inhomogeneity in the central regions. The inner mountain is capable eventually to prevent radial modes from penetrating to the center. Note that this might sound paradoxical, because Eq. (4) tells us that N does not influence modes with $l = 0$. But recall that in our simplified figures we also use N to represent the (inner) ω_c mountain, which is of the same order as N.

In evolved stars, modes with $l \geq 1$ can acquire 'dual status', i.e. they become gravity modes in the core and remain acoustic modes in the envelope. Thus they penetrate deeper into the interior, while simultaneously their frequency spectrum becomes denser. The main effect of mass (in the range around and slightly above 1 $M_\odot$) on the frequency spectrum is related to the disappearance of the convective envelope and the forming of a convective core. This has a major influence on g-mode propagation.

3.2. QUANTITATIVE DISCUSSION

The small separation (Eq. 12) carries an important signature of stellar age, because as hydrogen is converted into helium in the stellar core, the mean molecular weight increases, which causes a decrease of sound speed and its gradient, thus reducing the small separation. An excellent diagnostic diagram that extracts the information contained in the small and large separation has been invented by Christensen-Dalsgaard (1984) (for a more detailed calculation see Christensen-Dalsgaard 1988). In this diagram [hereinafter JCD diagram (for Jørgen Christensen–Dalsgaard)], contours of constant stellar mass and age are plotted against the theoretically computed large and small separations. Since the mass and age contours are rather perpendicular than parallel to each other, they reveal the considerable diagnostic potential of these diagrams.

Going a step further, Gough (1987) discussed the accuracy of seismological mass and age determination, using JCD diagrams and stellar models computed by Ulrich (1986, 1988). Gough's discussion was purely formal: taking the theoretical model for granted, he computed the uncertainty in the mass and age determination, assuming given errors for the observed stellar parameters (effective temperature, luminosity, heavy-element abundance, large and small frequency separation). Gough's (1987) result is that mass and age determination are so sensitive to the heavy-element abundance that they cannot be carried out in this way, unless other stellar parameters are known by independent means. If, for instance, in the case of a binary system we can determine mass, or if we can estimate it from surface gravity, then the large separation can reveal the evolutionary information contained in the deviation from the simple relation (11) (otherwise the large separation mainly fixes M/R^3). Thus a more accurate age determination could become possible (see Gough 1987).

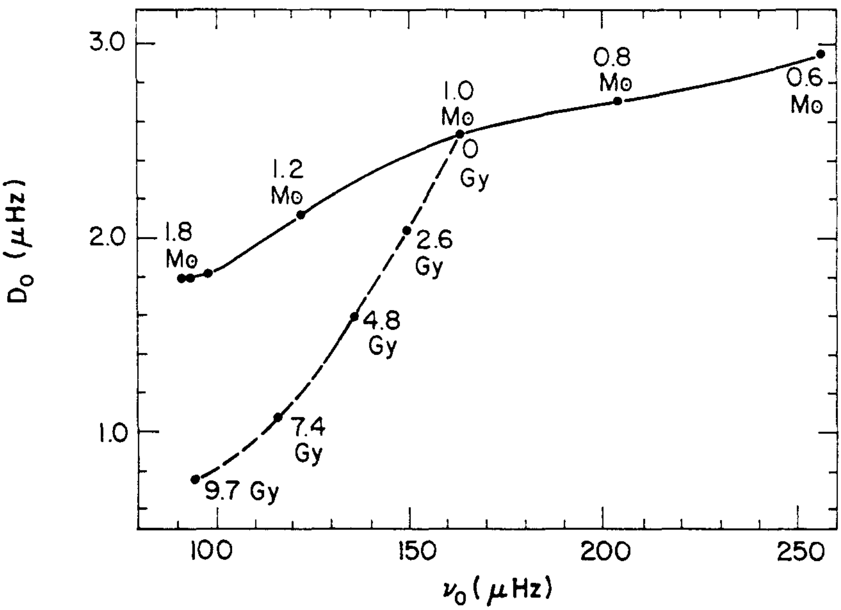

Figure 3. The location of a ZAMS (zero-age main sequence) (*solid line*) and of a 1 $M_\odot$ evolutionary sequence (*dashed line*) in a (D_0, ν_0) JCD diagram. Here, ν_0 is as in Eq. (8), and D_0 is a suitably defined average over small separations $d_{n,l}$ (for details see Christensen-Dalsgaard 1984, from which this figure is taken).

4. The Observational Situation: Procyon, α Centauri, and ϵ Eridani

Gelly et al. (1986) have reported p modes in Procyon and α Centauri. Only the large separation has been observed. Noyes et al. (1984) have reported three individual p-mode frequencies and the large separation for ϵ Eridani. To illustrate the potential, and the difficulties, of such observations for testing stellar structure and evolution, consider the recent controversial theoretical articles dealing with Noyes *et al.* 's (1984) observations. While Guenther and Demarque (1986) have concluded that a model of a very old star (12 Gyr) fits the data best (though they are aware of indications of stellar activity that speak against such a high age), Soderblom and Däppen (1989) conclude that a model of a very young star (1Gyr) is equally well suited, provided that one accepts the unusually small value of the mixing-length parameter of 0.45. The discrepancy of the two interpretations is well in line with the aforementioned error analysis by Gough (1987). However, an erroneous frequency determination could also have been the source of these difficulties. More and better resolved frequencies will be needed.

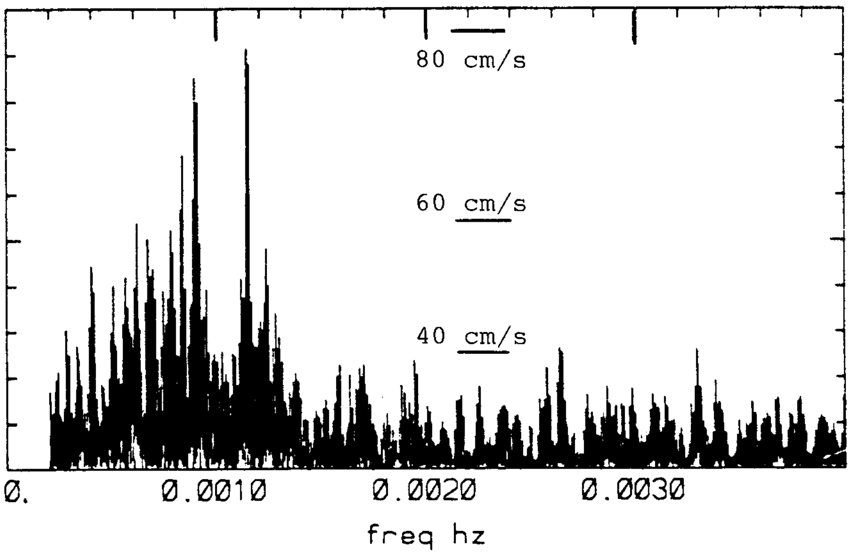

Figure 4. Power spectrum of Procyon, obtained during 6 nights of observation. Power is in arbitrary units, the corresponding velocities are indicated in the figure.

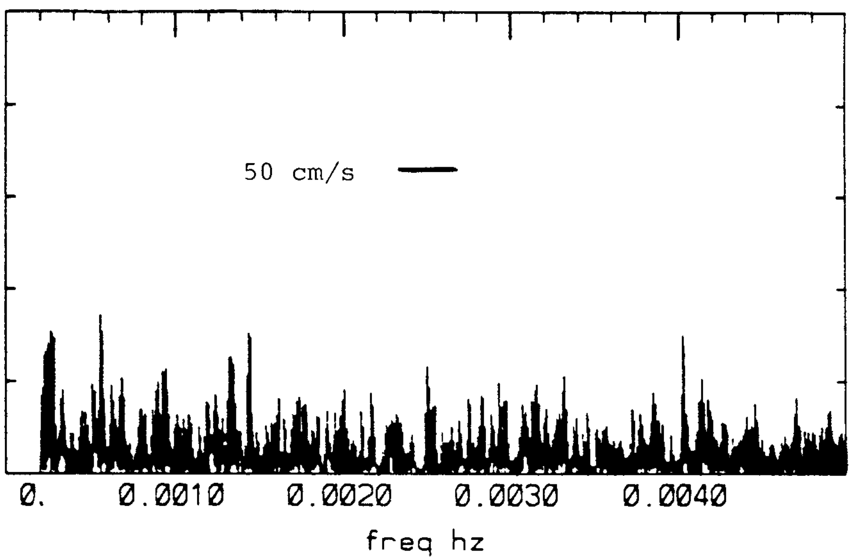

Figure 5. Same as Figure 4, but for the Sun (5 days of observation).

We end this section with a recent very promising result. Brown, Gilliland, Noyes, and Ramsey (private communication) have carried out Doppler velocity measurements of Procyon and obtained a clear signal around and below 1 mHz (Figures 4 and 5).

5. Prospects

Despite little analogy with the solar case, and despite the fact that not much has been achieved yet, stellar seismology has excellent prospects. Together with new observational techniques (like high S/N spectroscopy, astrometrical space missions), which will allow better determinations of stellar parameters (age, mass, chemical composition), the expected seismological information will put additional constraints on theoretical models of stellar evolution. The observational data will enable us to develop better physical models for the theory of stellar evolution (equation of state, convection, opacity, nuclear reactions, etc.). Since many of the diagnostically powerful small frequency-separations lie around 10 μHz, which is close to the diurnal frequency (11.6 μHz), it will greatly help to go to space or to coordinate several ground-based telescopes.

ACKNOWLEDGMENTS: I am grateful to Tim Brown, Ron Gilliland, Robert Noyes, and Lawrence Ramsey for their permission to show the results on Procyon.

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