Abstract

In this dissertation, I focus on a program in the philosophy of mathematics known as neo-logicism that is a direct descendant of Frege's logicist project. That program seeks to reduce mathematical theories to logic and definitions in order to put those theories on stable epistemic and logical footing. The definitions that are of greatest importance are abstraction principles, biconditionals associating identity statements for abstract objects on one side, with equivalence classes on the other. Abstraction principles are important because they provide connections between logic on the one hand, and mathematics and its ontology on the other.

Throughout this work, I advocate that the epistemic goals of neo-logicism be taken into account when we're looking to solve problems that are of central importance to its success. Additionally, each chapter either discusses or advocates for a methodological shift, or sets up and implements a novel methodological position I believe to be broadly beneficial to the neo-logicist project.

Chapter 2 traces thinking about the status of higher-order logic through the mid-twentieth century, setting the stage for issues dealt with in later chapters. Chapter 3 asks neo-logicists to look beyond set theory and consider other foundational theories, or something entirely new, when looking for reductions of foundational mathematical theories. Chapter 4 is an extended argument involving nonstandard analysis showing that Hume's Principle ought not be considered analytic in Frege's sense of the term.[1]

Chapters 5 and 6 move away form the (somewhat) historical work in the first three chapters, and set up new strategies for solving central neo-logicist problems by integrating formal and epistemic considerations. Chapter 5 introduces the notion of a canonical equivalence relation via a discussion of content carving, the latter notion being a particular way of understanding the relationship between equivalence relations and abstracts. Finally, Chapter 6 makes use of canonical equivalence relations to introduce a new direction in the search for solutions to the Bad Company objection.

As whole, the project can be seen as providing, as the title suggests, new directions that ought to be considered by those wishing to vindicate neo-logicism.

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William D. Simmons, ***Completeness of Finite-Rank Differential Varieties***, University of Illinois at Chicago, USA, 2013. Supervised by David Marker. MSC: 03C60, 12H05, 12H20. Keywords: model theory of fields, differentially closed fields, complete variety, quantifier elimination, differential algebra.

Abstract

The fundamental theorem of elimination theory states that projective varieties over an algebraically closed field K are *complete*: If V is such a variety and W is an arbitrary variety over K, then the projection map $\pi : V \times W \to W$ takes Zariski-closed sets to Zariski-closed sets. This property is tightly linked to projectiveness, as shown by the example of the affine hyperbola $xy - 1 = 0$. The images of the projections to either axis lack 0 but contain every other point of the affine line. We must "close up" the variety with a point at infinity to ensure a closed projection.

The situation is much more complicated in differential algebraic geometry. A *differential ring* is a commutative ring R with 1 and a finite set of maps Δ such that for each $\delta \in \Delta$ (*derivations* on R) and $x, y \in R$, $\delta(x + y) = \delta(x) + \delta(y)$ and $\delta(xy) = \delta(x)y + x\delta(y)$. We consider differential fields of characteristic zero with a single derivation δ. Over such a field,

[1] A version of Chapter 4 has been published as E. Darnell and A. Thomas-Bolduc, "Is Hume's Principle Analytic?" *Synthese*, 2018. DOI: 10.1007/s11229-018-01988-8.

a *differential polynomial equation* defines a closed set in the Kolchin topology, the differential analogue of the Zariski topology. In particular, we focus on differentially closed fields (close analogues of algebraically closed fields that enjoy many good model-theoretic properties, including quantifier elimination).

Our question, first looked at in essentially this form by E. R. Kolchin (1974) and later by W. Y. Pong (2000), is this: If V is a projective δ-variety over a differentially closed field K of characteristic zero and W is an δ-variety over K, then does the projection map $\pi : V \times W \to W$ take Kolchin-closed sets to Kolchin-closed sets?

The answer is "not necessarily," but Pong showed one reason for the counterexamples: δ-completeness requires a variety to have *finite rank* (in one of several equivalent senses). In this thesis, we take the basic model-theoretic and algebraic setup used by Pong and further develop it into several strategies for attacking the δ-completeness problem. Our work includes new examples of complete δ-varieties. Importantly, we also give the first example of an incomplete finite-rank projective δ-variety; hence, we now know that this class is not empty.

Our methods are as follows:

1. Modify Pong's "valuative criterion" for δ-completeness and produce alternative versions in terms of the Kolchin closure of the image of the projection as well as differential elimination ideals. We use these results to give multiple explicit elimination algorithms proving completeness of several new varieties.
2. Reduce from the differential setting to the algebraic by showing how the modified valuative criteria transfer the problem to a sequence of complex algebraic varieties. This enables one to use tools from analysis or standard (nondifferential) commutative algebra on the δ-completeness problem.
3. More speculatively, we isolate two conjectural properties (interesting in their own right as questions of algebraic geometry) of these complex varieties, both depending on the notion of generically perturbing the coefficients of the associated systems of equations. We explain how asymptotic properties of the complex varieties might imply δ-completeness of the original variety, given the above conjectures.

The thesis concludes with an evaluation of these methods and their prospects for classifying complete δ-varieties. We also discuss applications of δ-completeness. The appendices contain a new case of the differential algebraic phenomenon of nontrivial projective δ-varieties contained entirely in a single affine chart, as well as algorithmic elimination proofs of δ-completeness.

[1] E. R. Kolchin, *Differential equations in a projective space and linear dependence over a projective variety*, ***Contributions to Analysis*: *A Collection of Papers Dedicated to Lipman Bers*** (L. V. Ahlfors, I. Kra, B. Maskit, and L. Nirenberg, editors), Academic Press, New York, 1974, 195–214.

[2] W. Y. Pong, *Complete sets in differentially closed fields*. ***Journal of Algebra***, vol. 224 (2000), pp. 454–466.

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Filippo Cavallari, ***Regular Tree Languages in the First Two Levels of the Borel Hierarchy***, University of Turin, Italy, and University of Lausanne, Switzerland, 2018. Supervised by Luca Motto Ros (Turin) and Jacques Duparc (Lausanne). MSC: 03D05, 03E15, 03E75. Keywords: Descriptive Set Theory, Automata Theory, regular languages, Wadge hierarchy, Rabin-Mostowski index.