ANALYTICAL MODELS FOR ROTATING AND FLATTENED PERTURBATIONS IN BULGES

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Abstract. A power series approach for solving the linearized transport equation for perturbations in the central part of flat disks is presented. The application of this method is in principle independent of the mathematical complexity of the unperturbed distribution. As an illustration, this method was used to solve the transport equation in the case of Kalnajs' Omega models.

In this contribution, we want to show that solutions for the linearized Boltzmann equation for two-dimensional disks can be generated by substituting a series expansion form of the perturbed distribution. In combination with Poisson's equation, one can produce self-consistent models for rotating perturbations in the central part of a galaxy, such as bulges. More specifically, the velocity coordinates and the radius are expanded in a power series form, while the angular coordinate and the time appear in a uniformly rotating, harmonic perturbation.

In many strongly flattened rotating galaxies, most stars move on more or less circular orbits, so it is convenient to use these orbits as the zero point of the velocity expansions. The power expansion will then pertain to the description of small deflections from circular orbits. Moreover, since the radius is also expanded in a power series, one can easily see that the resulting distribution will be best suited for the central part of the galaxy. This is in contrast to other perturbation analysis techniques, such as the WKBJ approximation.

The distribution function and the equations of motion are first rewritten using a new coordinate $v'_\theta = v_\theta - r\Omega(r)$ (with $\Omega(r)$ the angular velocity for stars moving on circular orbits). The perturbing potential is has the form

$$V^m(r,\theta,t) = \sum_i a_i^m e^{i(m\theta-\omega t)}. \tag{1}$$

The integer parameter m indicates the order of the perturbing harmonic, while ω stands for the rotation speed. Our central assumption concerns the form of the perturbed distribution

$$df'(r,v_r,v'_\theta,t) = \sum_{i,j,k} p_{i,j,k} r^i v_r^j {v'_\theta}^k e^{i(m\theta-\omega t)}. \tag{2}$$

Since the equations will be linearized by neglecting second order terms in the perturbation, we are allowed to consider every order separately.

Filling in this form in the linearized transport equation generates a set of linear equations by collecting the terms with the same order. The coefficients of the perturbations are easily calculated by solving a subset of these equation recursively. However, the full set of equations is overdetermined. This is acceptable since general solutions of the transport equation can have an (unphysical) $\frac{1}{r}$ singularity in the centre, which can never be generated by a power series. It is possible to show that this limitation on the structure of the solution implies that the full set of equations only have a solution for a restricted set of perturbing potentials, i.e. if $a_i^m = 0$ for all

H. Dejonghe and H. J. Habing (eds.), Galactic Bulges, 381–382.
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https://doi.org/10.1017/S0074180900123691 Published online by Cambridge University Press

$i < |m|$. It is interesting to note that exactly the same restriction on the potential results from solving the Poisson equation for flat disks (Hunter 1963)

As a testcase, we used the previously described method to generate solutions for the linearized transport equation for perturbations in the case of the Omega models (Kalnajs, 1972), for which the full mode analysis has already been done analytically (ibid.). The proposed method should be equivalent to it, since the perturbing potentials are also polynomial. The thus calculated series expansion for the perturbed mass density turns out to be exactly the expansion of the analytic solution. Moreover, we checked that other important aspects of the distribution function are also identical.

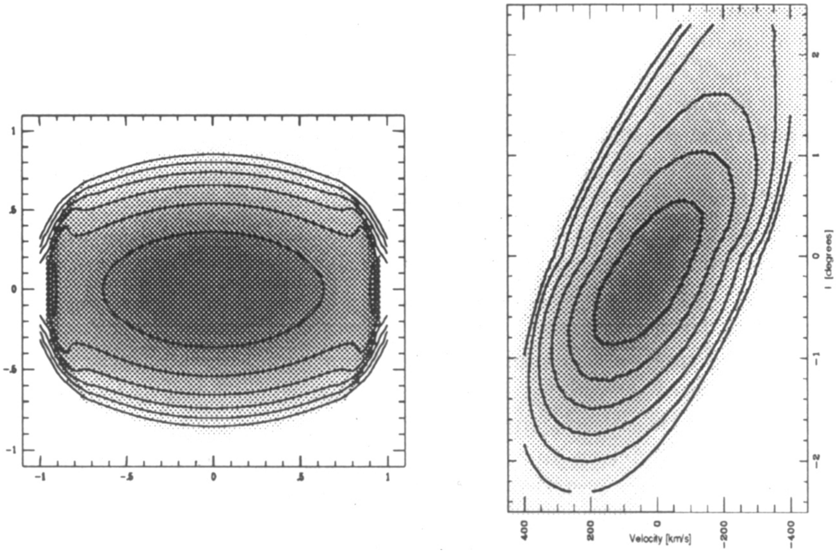

Fig. 1. Left panel: the surface density of a barlike perturbation (m=2). Right panel: the (l, v) diagram based on the barlike perturbation at the Galactic Centre, as seen from the sun. Notice that the rotation speed of the bar is different from the rotation of the bulge.

This series expansion strategy allows in principle to generate solutions for the linearized transport equation for flat perturbations in disks, regardless the mathematical complexity of the underlying unperturbed distribution. Moreover, the algorithm is easy to program and turns out to be very fast.

However, there is no guarantee that these series are convergent to the exact solution, although one can reasonably expect that there should be at least a small region with very small perturbed velocities for which this is the case. The fact that only a series expansion of the solution is generated is, in our opinion, a minor disadvantage, since most analytic solutions are so complex that the only way to handle them is to generate a series expansion anyway.

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https://doi.org/10.1017/S0074180900123691 Published online by Cambridge University Press