F. JOINT DISCUSSION

ON

THE REPORT OF THE WORKING GROUP ON THE IAU SYSTEM OF ASTRONOMICAL CONSTANTS

Thursday 27 August 1964 at $14^h\ 00^m$

ORGANIZING COMMITTEE: D. Brouwer, W. Fricke (Chairman), J. Kovalevsky, A. A. Mikhailov, G. A. Wilkins (Secretary).

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I. REPORT TO THE EXECUTIVE COMMITTEE OF THE WORKING GROUP ON THE SYSTEM OF ASTRONOMICAL CONSTANTS

INTRODUCTION

The principal recommendations of this Report on the system of astronomical constants are in accordance with the resolutions passed at Symposium no. 21 (Paris, May 1963). We first of all give a reference list of the constants of the system and a set of explanatory notes. We have used the term 'primary constant', rather than 'fundamental constant', since the latter has a connotation in astronomical usage that is inappropriate to the manner of selection of the primary constants. In choosing the values for the primary constants we have, perhaps, adopted a conservative view of the likely errors of their determinations, but even so the new system should be of adequate accuracy for astronomical studies for many years. Limits within which the true values are believed to lie are indicated in a later section, in which we also give expressions for differential corrections to the derived constants. Finally we discuss the manner in which this system should be introduced into the national and international ephemerides.

We regret that owing to severe illness Prof. A. Danjon has been unable to share in the preparation of this Report; Dr J. Kovalevsky has, however, been co-opted to the Group in his stead.

NOTES ON THE CONSTANTS

1. The value given for the number of ephemeris seconds in the tropical year at 1900 is taken from the definition of the ephemeris second that was adopted by the Comité International des Poids et Mesures (*Procès-Verbaux des Séances*, deuxième série, **25**, 77, 1957). It is, in fact, derived from the coefficient of T, measured in Julian centuries of 36 525 days, in Newcomb's expression for the geometric mean longitude of the Sun referred to the mean equinox of date. In the list '1900' refers to the fundamental epoch of ephemeris time, namely 1900 January 0 at 12^h E.T., or to 1900·0, as appropriate; the values for constants 20-23 also refer to the fundamental epoch. Throughout the list and this Report the term 'second' must be understood to mean the 'ephemeris second'.

2. The value of the Gaussian gravitational constant (k) is that adopted by the IAU in 1938, and serves to define the astronomical unit of length (A.U.) since the corresponding (astronomical) units of mass and time are already defined. (The unit of mass is that of the Sun and the unit of time is the ephemeris day of 86 400 ephemeris seconds. The units of k are: $(\text{A.U.})^{3/2}$ (ephemeris day)$^{-1}$ (Sun's mass)$^{-1/2}$.) To simplify the later equations an auxiliary constant k', defined as $k/86\,400$, is introduced and a rounded value is given in the list.

3. The value for the measure of the A.U. in metres is a rounded value of recent radar determinations.

4. The value for the velocity of light is that recommended by the International Union of Pure and Applied Physics in September 1963.

5. The term 'equatorial radius for Earth' refers to the equatorial radius of an ellipsoid of revolution that approximates to the geoid. (See also note 16.)

6. The term 'dynamical form-factor for Earth' refers to the coefficient of the second harmonic in the expression for the Earth's gravitational potential as adopted by IAU Commission 7 in 1961. (See also note 16.)

7. The geocentric gravitational constant (GE) is appropriate for use for geocentric orbits when the units of length and time are the metre and the second; E denotes the mass of the

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REFERENCE LIST OF RECOMMENDED CONSTANTS*

Defining constants: Constantes de définition

1.	Number of ephemeris seconds in 1 tropical year (1900) Nombre de secondes de temps des éphémérides pour l'année tropique (1900)	$s = 31\,556\,925{\cdot}974\,7$
2.	Gaussian gravitational constant, defining the A.U. Constante de la gravitation universelle, définissant l'unité astronomique (U.A.)	$k = 0{\cdot}017\,202\,098\,95$

Primary constants: Constantes primaires

3.	Measure of 1 A.U. in metres Longueur de l'U.A. en mètres	$A = 149\,600 \times 10^6$
4.	Velocity of light in metres per second Vitesse de la lumière, en mètres par seconde	$c = 299\,792{\cdot}5 \times 10^3$
5.	Equatorial radius for Earth in metres Rayon équatorial terrestre, en mètres	$a_e = 6\,378\,160$
6.	Dynamical form-factor for Earth Facteur d'ellipticité géopotentielle	$J_2 = 0{\cdot}001\,082\,7$
7.	Geocentric gravitational constant (units: $m^3\,s^{-2}$) Constante géocentrique de la gravitation	$GE = 398\,603 \times 10^9$
8.	Ratio of the masses of the Moon and Earth Rapport de la masse de la Lune à celle de la Terre	$\mu = 1/81{\cdot}30$
9.	Sidereal mean motion of Moon in radians per second (1900) Moyen mouvement sidéral de la Lune en radians par seconde (1900)	$n^*_{☾} = 2{\cdot}661\,699\,489 \times 10^{-6}$
10.	General precession in longitude per tropical century (1900) Précession générale en longitudes par siècle tropique (1900)	$p = 5025''{\cdot}64$
11.	Obliquity of the ecliptic (1900) Obliquité de l'écliptique (1900)	$\epsilon = 23^\circ\,27'\,08''{\cdot}26$
12.	Constant of nutation (1900) Constante de la nutation (1900)	$N = 9''{\cdot}210$

Auxiliary constants and factors

$k/86400$, for use when the unit of time is 1 second	$k' = 1{\cdot}990\,983\,675 \times 10^{-7}$
Number of seconds of arc in 1 radian	$206\,264{\cdot}806$
Factor for constant of aberration (note 15)	$F_1 = 1{\cdot}000\,142$
Factor for mean distance of Moon (note 20)	$F_2 = 0{\cdot}999\,093\,142$
Factor for parallactic inequality (note 23)	$F_3 = 49\,853''{\cdot}2$

* The above list was confirmed by the Working Group, adopted as definitive at the Joint Discussion, and endorsed by the Twelfth General Assembly (Resolution no. 4, page 95); it now represents the 'IAU System of Astronomical Constants'.

* La liste ci-dessus a été confirmée par le Groupe de Travail, adoptée définitivement à la Discussion Commune, et appuyée par la Douzième Assemblée Générale (Résolution no. 4, page 94); elle représente désormais le 'Système UAI des Constantes Astronomiques'.

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REFERENCE LIST OF RECOMMENDED CONSTANTS*

Derived constants: Constantes secondaires

13.	Solar parallax Parallaxe solaire	$\arcsin (a_e/A) = \pi_\odot = 8''.794\,05$ ($8''.794$)
14.	Light-time for unit distance Temps de lumière relatif à l'U.A.	$A/c = \tau_A = 499^{\mathrm{s}}.012$ $= 1^{\mathrm{s}}/0{\cdot}002\,003\,96$
15.	Constant of aberration Constante de l'aberration	$F_1 k' \tau_A = \kappa = 20''.4958$ ($20''.496$)
16.	Flattening factor for Earth Aplatissement terrestre	$f = 0{\cdot}003\,352\,9$ $= 1/298{\cdot}25$
17.	Heliocentric gravitational constant (units: $m^3\ s^{-2}$) Constante héliocentrique de la gravitation	$A^3 k'^2 = GS = 132\,718 \times 10^{15}$
18.	Ratio of masses of Sun and Earth Rapport de la masse du Soleil à celle de la Terre	$(GS)/(GE) = S/E = 332\,958$
19.	Ratio of masses of Sun and Earth + Moon Rapport de la masse du Soleil à celle du système Terre-Lune	$S/E(1+\mu) = 328\,912$
20.	Perturbed mean distance of Moon, in metres Demi-grand axe perturbé de l'orbite de la Lune, en mètres	$F_2(GE\,(1+\mu)/n_☾^{*2})^{1/3} = a_☾ = 384\,400 \times 10^3$
21.	Constant of sine parallax for Moon Sinus de la parallaxe de la Lune	$a_e/a_☾ = \sin \pi_☾ = 3422''.451$
22.	Constant of lunar inequality Constante de l'inégalité lunaire	$\frac{\mu}{1+\mu}\frac{a_☾}{A} = L = 6''.439\,87$ ($6''.440$)
23.	Constant of parallactic inequality Constante de l'inégalité parallactique	$F_3 \frac{1-\mu}{1+\mu}\frac{a_☾}{A} = P_☾ = 124''.986$

System of planetary masses

		Reciprocal mass		Reciprocal mass
24.	Mercury	6 000 000	Jupiter	1 047·355
	Venus	408 000	Saturn	3 501·6
	Earth + Moon	329 390	Uranus	22 869
	Mars	3 093 500	Neptune	19 314
			Pluto	360 000

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Earth including its atmosphere. Kepler's third law for a body of mass M moving in an unperturbed elliptic orbit around the Earth may be written

$$GE\,(1 + M/E) = n^2 a^3$$

where n is the sidereal mean motion in radians per second and a is the mean distance in metres. The value of GE is based on gravity measurements and observations of satellites.

8. Again the mass of Earth includes the mass of the atmosphere. The reciprocal of 81·30 is 0·012 300 1.

9. The value for the sidereal mean motion of the Moon is consistent with the value of the tropical mean motion used in the improved lunar ephemeris, less the general precession in longitude.

10–12. The values of the principal constants defining the relative positions and motions of the equator and ecliptic are those in current use. Secular terms and derived quantities are already tabulated elsewhere.

13. The rounded value 8″794 for the solar parallax should be used except where extra figures are required to ensure numerical consistency.

14. The value of the light-time for unit distance is numerically equal to the number of light-seconds in 1 A.U. Its reciprocal is equal to the velocity of light in A.U. per second.

15. Apart from the factor F_1 the constant of aberration is equal to the ratio of the speed of a hypothetical planet of negligible mass moving in a circular orbit of unit radius to the velocity of light; it is conventionally expressed in seconds of arc by multiplying by the number of seconds of arc in one radian. The factor F_1 is the ratio of the mean speed of the Earth to the speed of the hypothetical planet and is given by

$$F_1 = \frac{n_\odot}{k'} \frac{a_\odot}{(1 - e^2)^{1/2}}$$

where $n_\odot$ is the sidereal mean motion of the Sun in radians per second, $a_\odot$ is the perturbed mean distance of the Sun in A.U., and e is the mean eccentricity of the Earth's orbit. Newcomb's values for $n_\odot$, $a_\odot$ and e are of ample accuracy for this purpose. The factor F_1 and the constant of aberration take the following values

	F_1	κ''
1800	1·000 142 7	20·495 83
1900	1·000 142 0	20·495 82
2000	1·000 141 3	20·495 81

The rounded value 20″496 should be used except where the extra figures are required to ensure numerical consistency.

16. The condition that the reference ellipsoid of revolution for the Earth shall be an equipotential surface implies that three parameters are sufficient to define its geometrical form and external gravitational field, provided that the angular velocity (ω) of the Earth and the relative mass of the atmosphere (μ_a) are assumed to be known. The variability of the rate of rotation of the Earth can be ignored, and the mass of the atmosphere is only just significant; the required values are:

$$\omega = 0{\cdot}000\ 072\ 921 \text{ radians per second}; \quad \mu_a = 0{\cdot}000\ 001$$

The expressions for the flattening (f) and the apparent gravity at the equator (g_e) in terms of the primary constants are, to second order:

$$f = \tfrac{3}{2} J_2 + \tfrac{1}{2} m + \tfrac{9}{8} J_2^2 + \tfrac{15}{28} J_2 m - \tfrac{39}{56} m^2$$

$$g_e = (GE/a_e^2)\,(1 - \mu_a + \tfrac{3}{2} J_2 - m + \tfrac{27}{8} J_2^2 - \tfrac{6}{7} J_2 m + \tfrac{47}{56} m^2)$$

where $m = a_e \omega^2 / g_e$ is obtained by successive approximations. The new value of f is given here only for astronomical use (parallax corrections, etc.).

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17. The heliocentric gravitational constant corresponds to GE, but is appropriate for heliocentric orbits when the units are the metre and the second.

18–19. The derived values of the masses of the Earth and of the Earth + Moon differ from those currently in use, but will not supersede them completely until the system of planetary masses is revised as a whole. (See note 24.)

20. The perturbed mean distance of the Moon is the semi-major axis of Hill's variational orbit, and differs from that calculated from Kepler's law by the factor F_2, which depends on the well-determined ratio of the mean motions of the Sun and Moon. (E. W. Brown, *Mem. R. astr. Soc.*, **53**, 89, 1897).

21. The constant of sine parallax for the Moon is conventionally expressed in seconds of arc by multiplying by the number of seconds of arc in one radian. The corresponding value of $\pi_{☾}$ itself is 3422″.608.

22. The constant of the lunar inequality is defined by the expression given and is conventionally expressed in seconds of arc.

23. The constant of the parallactic inequality is defined by the expression given; the coefficient F_3 is consistent with the corresponding quantities in Brown's Tables.

24. The system of planetary masses is that adopted in the current ephemerides and the values given for the reciprocals of the masses include the contributions from atmospheres and satellites. The value for Neptune is that adopted in the numerical integration of the motions of the outer planets; the value used in Newcomb's theories of the inner planets is 19 700. In planetary theory the adopted ratio of the mass of the Earth to the mass of the Moon is 81·45 (compared with 81·53 in the lunar theory), and the ratio of the mass of the Sun to the mass of the Earth alone is 333 432. This system of masses should be revised within the next few years when improved values for the inner planets are available from determinations based on space-probes.

CORRECTION FACTORS AND LIMITS

To first order, relative errors of the derived constants are given by:

$$\frac{\Delta\pi_{\odot}}{\pi_{\odot}} = \frac{\Delta a_e}{a_e} - \frac{\Delta A}{A} \qquad \frac{\Delta\tau_A}{\tau_A} = \frac{\Delta A}{A} - \frac{\Delta c}{c}$$

$$\frac{\Delta\kappa}{\kappa} = \frac{\Delta A}{A} - \frac{\Delta c}{c} = \frac{\Delta\tau_A}{\tau_A} \qquad \frac{\Delta f}{f} = \frac{\Delta J_2}{J_2}$$

$$\frac{\Delta(GS)}{GS} = \frac{3\Delta A}{A} \qquad \frac{\Delta(S/E)}{S/E} = -\frac{\Delta(GE)}{GE} + \frac{3\Delta A}{A}$$

$$\frac{\Delta\{S/E(1+\mu)\}}{S/E(1+\mu)} = \frac{3\Delta A}{A} - \frac{\Delta(GE)}{GE} - \frac{\Delta\mu}{1+\mu}$$

$$\frac{\Delta a_{☾}}{a_{☾}} = \frac{1}{3}\frac{\Delta(GE)}{GE} - \frac{2}{3}\frac{\Delta n^*_{☾}}{n^*_{☾}} + \frac{1}{3}\frac{\Delta\mu}{1+\mu} \qquad \frac{\Delta\sin\pi_{☾}}{\sin\pi_{☾}} = \frac{\Delta a_e}{a_e} - \frac{\Delta a_{☾}}{a_{☾}}$$

$$\frac{\Delta L}{L} = \frac{\Delta\mu}{\mu(1+\mu)} + \frac{\Delta a_{☾}}{a_{☾}} - \frac{\Delta A}{A} \qquad \frac{\Delta P_{☾}}{P_{☾}} = -\frac{2\Delta\mu}{1-\mu^2} + \frac{\Delta a_{☾}}{a_{☾}} - \frac{\Delta A}{A}$$

The true values of the primary constants are believed to lie between the following limits:

A :	149 597 to 149 601 × 10^6 m	μ^{-1}:	81·29 to 81·31
c :	299 792 to 299 793 × 10^3 m s^{-1}	$n^*_{☾}$:	correct to number of places given
a_e :	6 378 080 to 6 378 240 m	p :	5026″.40 to 5026″.90
J_2 :	0·001 082 4 to 0·001 082 9	ϵ :	23° 27′ 08″.16 to . . . 08″.36
GE:	398 600 to 398 606 × 10^9 m^3 s^{-2}	N :	9″.200 to 9″.210

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Correspondingly the limits for the derived constants are:

$\pi_\odot$	:	$8''.793\,88$ to $8''.794\,34$	f^{-1}	:	298·33 to 298·20
τ_A	:	$499^{s}.001$ to $499^{s}.016$	$a_☾$	:	384 399 to 384 401 $\times 10^{3}$ m
κ	:	$20''.4954$ to $20''.4960$	$\sin\pi_☾$	:	$3422''.397$ to $3422''.502$
GS	:	132 710 to 132 721 $\times 10^{15}$ $m^3 s^{-2}$	L	:	$6''.4390$ to $6''.4408$
S/E	:	332 935 to 332 968	$P_☾$	:	$124''.984$ to $124''.989$
$S/E(1+\mu)$	:	328 890 to 328 922			

IMPLEMENTATION OF THE REPORT

We regard it as essential that the new system should be introduced into the national and international ephemerides as soon as possible after its adoption. Accordingly we are requesting the directors of the principal ephemeris offices to study the consequences of the introduction of the new system so that a firm timetable can be drawn up at the meetings of Commission 4 in Hamburg. We provisionally suggest that the new system be introduced into the almanacs for the year 1968; for the Sun, Moon and planets we suggest that differential corrections to the ephemerides based on the current system be tabulated until such time as new or revised theories have been completed.

We intend to meet again at the beginning of the 1964 Assembly so that we may consider any fresh observational evidence or theoretical arguments that may have been brought to our attention since our meeting in January 1964. We will then confirm or amend as necessary the list of constants given above, and this final list will be our recommendation for the 'IAU System of Astronomical Constants'. We therefore request that the Executive Committee submit the following draft resolution for consideration by Commissions 4, 7, 8, 19, 20 and 31, with a view to its adoption by the General Assembly:

The International Astronomical Union endorses the final list of constants prepared by the Working Group on the System of Astronomical Constants and recommends that it be used in the national and international astronomical ephemerides at the earliest practicable date.

(L'Union Astronomique Internationale donne son accord à l'ensemble des constantes préparé par le Groupe de Travail sur le système des constantes fondamentales et recommande qu'il soit utilisé dans les éphémérides astronomiques nationales et internationales dès que ce sera possible.)

Acknowledgements

The Working Group wishes to acknowledge the assistance given to the Group by those persons who have responded to our request in a circular letter dated 30 August 1963 for comments and results relevant to the system of astronomical constants proposed by the Paris Symposium.

The Group is also grateful to the Astronomer Royal for his hospitality and for providing facilities for the meeting of the Group at the Royal Greenwich Observatory, Herstmonceux Castle, on 8–10 January 1964.

28 February 1964

W. FRICKE (*Chairman*), D. BROUWER, J. KOVALEVSKY, A. A. MIKHAILOV G. A. WILKINS, (*Secretary*)

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