

Stellar kinematics and dark matter in dwarf galaxies

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Abstract. In this review I will discuss the current status on determinations of the dark matter content and distribution in Milky Way dwarf spheroidals, for which the available data-sets allow the application of sophisticated mass modeling techniques.

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1. Background

The surroundings of the Milky Way (MW) are populated by a wealth of small passively-evolving satellite galaxies: the “classical” dwarf spheroidal galaxies (dSphs), whose existence has been known since a very long time (e.g. the Sculptor dSph was discovered by Shapley 1938), and the so-called “ultra-faint dwarfs” (UFDs), whose discovery we mainly owe to the SDSS.

If in dynamical equilibrium, “classical” dSphs have the highest mass-to-light ratios known to date, with $M/L \sim 100s M_{\odot}/L_{\odot}$. At face value, UFDs exhibit even larger dynamical mass-to-light ratios, although these values are subject to numerous important caveats. Hence, in terms of their internal dynamics, these systems might well be key in constraining the nature of dark matter (DM).

Aim of several studies in the literature has been to compare the DM content and distribution inferred from the internal kinematics of this population of small galactic systems to the predictions from cosmological theories of structure formation in a DM context. In particular, Λ Cold Dark Matter pure DM N-body simulations show that the density profile of the haloes formed follow very specific functional forms, which are rather steep near the centre (such as the Navarro, Frenk & White profile, NFW, Navarro *et al.* 1995, 1996, and more recently the Einasto form: Navarro *et al.* 2010 and references therein). These high-resolution simulations give robust predictions also for the sub-haloes mass function, which in the case of MW-sized main halos is established down to a simulations’ resolution limit smaller than the mass estimates for the faintest dSphs (see e.g. Springel *et al.* 2008).

However, making a link between a luminous satellite to what should be its corresponding sub-halo in a pure DM N-body simulation is not a straightforward matter. It has in fact become increasingly clear that the inclusion of baryonic effects in such simulations can have a profound impact on the properties of the sub-haloes that will become luminous satellites, such as reducing the central dark matter density, enhancing tidal stripping etc. (e.g. Zolotov *et al.* 2012). On the other hand, this sensitivity to baryonic effects might be useful for constraining model ingredients, such as for example the efficiency of supernovae feedback (e.g. Breddels, Vera-Ciro & Helmi, 2015).

Independently of the comparison with cosmological simulations, determinations of the DM content and distribution of dSphs are central also to interpret results from particle physics experiments of indirect dark matter detection. For example, the γ -ray differential flux from DM decay (annihilation) over a solid-angle can be thought of as the multiplication of two factors, one dependent on the characteristics of the DM particle and the other one being the so-called “astrophysical factor” (Bergström *et al.* 1998). The latter is simply the integral along the line-of-sight (LOS) and over the solid angle of the (squared, in the case of annihilation) DM density. Hence robust determinations of the DM content and distribution of the targeted dSph result in a well constrained astrophysical contribution to the measured signal (see e.g. Bonnivard *et al.* 2015 for the impact of sample sizes, presence of Galactic contaminants in the sample etc. on knowledge of the dSphs’ “astrophysical factor”).

In the following I will focus only on the MW “classical dSphs”[†] as these are the objects for which the largest spectroscopic samples of LOS velocities for individual probable member stars are available (e.g. ~ 2000 and 3000 in Sculptor and Fornax, respectively), which - beside a more robust determination of the “observables” - also allows the application of sophisticated mass modeling techniques. For thorough discussions on the observed internal kinematics of these galaxies, the techniques used to extract information on their gravitational potential, implication of the findings and related uncertainties we refer the reader to recent review articles by Battaglia, Helmi & Breddels (2013), Walker (2013), Strigari (2013). Here I will briefly summarize some of the salient findings in studies of the DM content and distribution of MW “classical” dSphs, with a focus on the assumptions that have been made and their potential impact on the results, closely following the article by Battaglia, Helmi & Breddels (2013).

1.1. *Some notes*

Determining the mass content of a system requires observations of the kinematics of suitable tracers. Since dSphs are devoid of a neutral interstellar medium, the only tracers available are stars. The heliocentric distances to MW dSphs have made it unfeasible to obtain accurate proper motions of individual stars in these galaxies with current facilities. Hence to-date the analyses of their internal kinematics are based on their LOS velocity distributions (LOSVD) and their moments.

Since the internal kinematics of dSphs is found to be dominated by random motions, they are commonly treated as non-rotating systems. Another common assumption, less justified from an observational point of view (see Sect. 3), is to regard both the stellar and DM component of dSphs as spherical.

The above, together with the lack of knowledge on the other two components of the LOS velocity vector of the individual stars, causes the so-called “mass-anisotropy” degeneracy, a well-known and cumbersome issue in the dynamical modeling of these objects. It is important to remember that the “mass-anisotropy” degeneracy is a degeneracy between the mass *profile*, the anisotropy *profile* $\beta(r)$ and the tracer density distribution.

Finally, the majority of mass modeling studies of dSphs assumes these galaxies are in dynamical equilibrium, hence that the internal kinematics of the tracer population can be safely used to infer the underlying gravitational potential.

In Sect. 2 I will describe results from mass modeling studies carried out under the above assumptions, and mainly comment on the efforts that have been made to relieve or circumvent the “mass-anisotropy” degeneracy. In Sect. 3 I will comment on the possible effect of relaxing the assumption of sphericity and on the possible impact of tidal effects

[†] Hereafter I will use the terms “classical dSphs” and “dSphs” as interchangeable

onto the internal kinematics of dSphs's. Newtonian dynamics will be assumed throughout this article.

2. Results from dynamical equilibrium modeling analyses

The first attempt in determining the LOS velocity dispersion profile of dSphs was made by Mateo *et al.* (1991) from a sample of ~ 30 stars. Nowadays all the MW classical dSphs have LOS velocity dispersion profiles based on 100s of member stars, where this overwhelming progress has been largely due to wide-area spectrographs with a high multiplex power mounted on 4-8m class telescopes. The LOS velocity dispersion profiles for these galaxies are observed to be approximately flat out to the last measured point, that is often out to and beyond their nominal King tidal radius. This is considered as the best evidence that in dSphs the DM has a different spatial distribution than the stars, so that mass-follows-light models can be excluded in the hypothesis of dynamical equilibrium (see e.g. Evans, An & Walker 2009 for the implications on nonphysical values of the velocity anisotropy in the mass-follows-light case; but see e.g. Lokas 2009 for a different take on how stars should be considered as bound to the dSphs and the effect it has on the shape of the LOS velocity dispersion profile).

It is however still debated whether dSphs inhabit cored or cuspy DM haloes. A Jeans analysis of the LOS velocity dispersion profile of a non-rotating spherical object cannot distinguish between different functional forms of the density profile for extended DM haloes, as exemplified in Fig. 1: there we can see that the LOS velocity dispersion profile of the Sculptor dSph is very well fitted by a cored DM profile traced by stars with an isotropic velocity distribution in the center becoming slightly radial at larger radii; this fit is essentially indistinguishable from the best-fitting cuspy model in the case of the stars having a slightly tangential anisotropy. The kind of conclusions that can be inferred by this type of analysis though is that dSphs display very large dynamical mass-to-light ratios (e.g. Kleyana *et al.* 2001; Koch *et al.* 2007; Walker *et al.* 2007; Battaglia *et al.* 2008)

Much effort has gone in the direction of including higher moments of the LOSVD in the analysis to constrain the DM density profile (e.g. Lokas 2009; Breddels *et al.* 2013; Mamon, Biviano & Boué 2013; Richardson & Fairbairn 2014), in particular the fourth moment because of the information it carries about the velocity anisotropy of the stars. Observationally, the fourth moment of the LOSVD has been derived for most classical MW dSphs and, although the details differ among works, current measurements suggest that the LOSVDs of stars in dSphs are not dramatically different from Gaussians in most cases, so that one can conclude that the velocity ellipsoid is neither strongly radial nor strongly tangential, hence in the regime where both cored and cuspy profile would be consistent with the data.

An alternative way for relieving the “mass-anisotropy” degeneracy was proposed by Battaglia *et al.* (2008), where the idea was to use the multiple “chemo-dynamical” stellar components observed in dSphs (e.g. Tolstoy *et al.* 2004; Battaglia *et al.* 2006) as independent tracers of the same gravitational potential. They carried out a spherical Jeans analysis of the Sculptor dSph, modeling it as a metal-rich centrally concentrated component, and a metal-poor hot and extended one, both embedded in a dark matter halo. This analysis shows that models in which the velocity anisotropy is constant as a function of radius for both the metal-rich and a metal-poor component can be excluded (see Battaglia 2007). While the metal-poor component is better fit with a nearly flat anisotropy profile, the metal-rich one, because of its rapidly falling velocity dispersion profile, requires a radially anisotropic ellipsoid. In this case, the cored models provided better fits but that NFW models could not be ruled out.

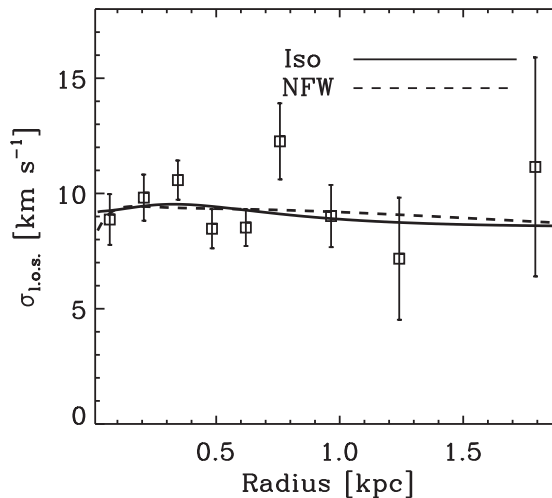


Figure 1. Example of the “mass-anisotropy” degeneracy. The squares with error-bars show the observed Sculptor dSph LOS velocity dispersion profile overlaid to the best-fitting NFW model with constant β (dashed line; $c=35$, virial mass = $6.1 \times 10^8 M_{\odot}$ and $\beta = -0.5$) and pseudo-isothermal sphere with an Osipkov-Merrit anisotropy profile (solid line; core radius = 0.5kpc, $M(<1.8\text{kpc}) = 3.2 \times 10^8 M_{\odot}$ and anisotropy radius = 0.4kpc). Both models share the same χ^2 value. Data and analysis by Battaglia *et al.* (2008).

Another very interesting result of the recent literature is the finding that for dispersion-supported systems there exists one radius where the enclosed mass inferred from the LOS velocity dispersion is largely insensitive to the velocity anisotropy (see Strigari *et al.* 2007, Walker *et al.* 2009, Wolf *et al.* 2010 for an analytic proof using the spherical Jeans equation) for a system with a flat projected velocity dispersion profile. The enclosed mass at the radius can be expressed in a very simple way, just in terms of the (luminosity-weighted) LOS velocity dispersion and this characteristic radius. Walker & Peñarrubia (2011) have analyzed a set of dSphs and applied this type of mass estimator to the dSph’s metal-rich and metal-poor component independently. This yields the enclosed mass at two different radii, and hence to a measurement of the slope. Application of this method to the Fornax and Sculptor dSph rules out NFW-like profiles at high significance levels, $\gtrsim 96\%$ and $\gtrsim 99\%$, respectively.

The large number of LOS velocities available for some of the dSphs has also allowed the application of more sophisticated and non-parametric mass modeling techniques, such as e.g. Schwarzschild modeling, which is widely used in integrated light spectroscopic studies of more distant galaxies. So far this has been done treating the stellar component of dSphs as a single component. Some of the advantages of the Schwarzschild modeling are that the distribution function is guaranteed to be positive - unlike for the Jeans modeling - and that the velocity anisotropy is an outcome of the analysis. Breddels *et al.* (2013), Breddels & Helmi (2013), Jardel *et al.* (2013) have carried out Schwarzschild modeling of MW dSphs in the spherical case, while Jardel & Gebhardt (2012) have considered a non-spherical light distribution. Even though neither this method can make a distinction between cored and cuspy profiles for the individual dSphs, a striking result from the work of Breddels & Helmi 2013 is that the mass distribution of each of the analyzed dSphs is the same for all the DM models used (NFW, Einasto, cored models) from about the half-mass radius to the last measured kinematic data point. This means that the mass distribution is robustly determined over a large range of radii, and that a slope of the density profile can be determined at some intermediate point.

All in all, there is still debate as to whether the spherical dynamical modeling of dSphs favors cored or cuspy DM density distributions. In general, using two (or multiple) components disfavor NFW/cuspy profiles for dSphs, at least for Fnx and Scl, as also shown by Amorisco & Evans (2012), Agnello & Evans (2012) - but see Strigari, Frenk & White (2014). Efforts are under way to use multiple stellar populations also in Schwarzschild modeling (G. van de Ven talk, EWASS 2015). At the same time it would also be desirable to understand the extent down to which these systems's properties are better described using a few independent components, rather than to assume that the properties of the stars change gradually throughout the system, and understand how this might affect the modeling.

3. The possible impact of assumptions

3.1. Sphericity

The stellar component of dSphs appears flattened on the sky, with a mean projected ellipticity ~ 0.3 and values ranging from ~ 0.1 to ~ 0.6 . For an axisymmetric system this means that the ratio between the shortest and longest *projected* axis is between 0.4 & 0.9. Since we lack information on the inclination of the dSphs's stellar component with respect to the plane of the sky, we can only say that, unless we are seeing the dSph edge-on, most likely its 3D stellar density distribution is flatter than what we see in projection. It is then clear that the density distribution of the dSphs stellar component might deviate considerably from the assumption of sphericity. As for the DM halo, the expectations differ according to the characteristics of the DM particle. In Λ Cold Dark Matter (pure N-body + semi-analytics) simulations it appears that those sub-haloes that are expected to become luminous satellites tend to be closer to spherical with respect to field sub-haloes as a consequence of tidal stripping (e.g. Barber *et al.* 2015).

The effect of moving away from the assumption of sphericity has started to be investigated by several groups (e.g. Jardel & Gebhardt 2012, Hayashi & Chiba 2012, Kowalczyk *et al.* 2013, Laporte *et al.* 2013, Bonnivard *et al.* 2015). Hayashi & Chiba (2012) explore the variation of the LOS velocity dispersion profile along the projected major and minor axes for different flattenings and density distributions of the DM and stellar components. The application of these axisymmetric Jeans models to the MW dSphs results into significantly flattened DM haloes for several systems, and in values of the mass within a spheroid of major-axis length of 300pc that can differ up to an order of magnitude with respect to the values from a spherical analysis, depending on the flattening retrieved for the system. It should be noticed that also this analysis might suffer from the assumptions made on the shape of the velocity ellipsoid, specifically that the velocity dispersion in the radial R and vertical z direction are identical, as there is a degeneracy between the flattening of the velocity ellipsoid and the flattening of the halo. However it is clearly worth to understand how relaxing the assumption of sphericity may affect the results from different methods.

3.2. Dynamical equilibrium

The dSphs are small galaxies orbiting a much larger system, hence one of the questions related to their dynamical modeling is to what extent they can be considered in dynamical equilibrium.

Clear signs of the tidal disturbance exerted by the MW and M31 over some of their small companions are observed in the form of tidal tails or isophote twists around surviving dSphs (e.g. NGC 205, NGC 147, Carina, Hercules: Choi *et al.* 2002; Crnojević *et al.* 2014; Battaglia *et al.* 2012; McMonigal *et al.* 2014; Roderick *et al.* 2015), and stellar streams of

heavily or entirely disrupted objects (e.g. the spectacular cases of the Sagittarius stream around the MW, Ibata *et al.* 1994; the Giant Stream around M31, McConnachie *et al.* 2010). Other systems show outer breaks in their stellar number density profiles and/or increasing LOS velocity dispersion profiles (e.g. Leo I, Sohn *et al.* 2007, Mateo *et al.* 2008), features that might be caused by tidal disturbance, although this need not be the only explanation.

N-body simulations of dwarf-like systems orbiting a MW-like potential agree in that the central velocity dispersion (or the dispersion at the half-light radius) is a good indicator of the present maximum circular velocity and bound mass, as long as the objects retain a bound core (e.g. Muñoz *et al.*, 2008; Peñarrubia *et al.*, 2008b; Klimentowski *et al.*, 2009; Kazantzidis *et al.*, 2011). However, at large radii, the presence of unidentified unbound stars can significantly inflate the measured LOS velocity dispersion (e.g. Read *et al.* 2006, Muñoz *et al.* 2008, Klimentowski *et al.* 2009).

The impact of tidal disturbance is expected - and it has been shown - to depend on several (mainly unknown) factors, such as the degree of embedding of the stellar component in the DM halo, the DM density profile, the orbit of the object around the host galaxy. However, even if an object has suffered a large degree of mass loss, this does not necessarily imply lack of current dynamical equilibrium. Tidally perturbed stars progressively become unbound and are eventually dispersed, with the object eventually settling in a new equilibrium configuration (Peñarrubia *et al.*, 2009). Also, the degree of contamination of kinematic data-sets from stars originating in tidal tails can vary along the orbit, as a consequence of the varying orientation of the inner regions of the tidal tails with respect to the LOS (e.g. Klimentowski *et al.* 2009). Given the difficulty in unambiguously identifying unbound stars lingering close to the main body of the MW dSphs, it is quite possible that, depending on the specific orbital history and current location along the orbit, the observed kinematics of stars is a faithful tracer of their underlying potential in some of the MW dSphs and not in others.

N-body simulations tailored to reproduce the observed structural and internal kinematic properties of *specific* MW dSphs appear particularly well suited to explore this issue and so far confirm the expectation that not all dSphs are in the same situation. For example, it has been shown that, for *observationally motivated orbits*, the observed Carina's structural and internal kinematic properties are well matched also by modeling this galaxy as a tidally disturbed system initially embedded in "mass-follows-light" DM halo, rather than in an extended halo (Muñoz *et al.* 2008), while on the other hand this is not the case for the Fornax dSph, for which the effects produced by the Galactic tidal field are relatively small, and the final system is close to equilibrium in its own gravitational potential and appears to require an extended DM halo (Battaglia, Sollima & Nipoti 2015).

It is remarkable that also the internal kinematics of tidal streams can potentially be used to infer information on the DM density distribution of the tidally disturbed dwarf galaxy (Errani, Peñarrubia, Tormen 2015).

4. A few closing remarks

Much progress has been made in the study of the internal kinematics of dSphs, with impressive spectroscopic data-sets being already in place. The near future will see other major advances: the Gaia mission is likely to provide more accurate systemic proper motions for the MW dSphs and hence a better understanding of their orbital history around the MW; several fiber spectrographs with much larger field-of-view, multiplex power and simultaneous wavelength coverage at intermediate spectral resolution than

the existing ones are planned to be mounted on 4m and 8m class telescopes in the next years (e.g. WHT/WEAVE, VISTA/4MOST, VLT/MOONS, Subaru/PFS). This appears as the right time for modelers to explore in depth what the need for future data-sets is, so that the full information on discrete LOS velocities can be exploited at the same time allowing to take into account other observed properties such as non-spherical density distributions and “multiple stellar components”.

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