38

Hybrid correlators

38.1 Light hybrid correlators

We shall be concerned with the two-point correlator (standard notations):

$$\begin{aligned}\Pi^{\mu\nu}_{V/A}(q^2) &\equiv i\int d^4x\ e^{iqx}\ \langle 0|\mathcal{T}\mathcal{O}^{\mu}_{V/A}(x)\left(\mathcal{O}^{\nu}_{V/A}(0)\right)^{\dagger}|0\rangle \\ &= -(g^{\mu\nu}q^2-q^{\mu}q^{\nu})\Pi^{(1)}_{V/A}(q^2)+q^{\mu}q^{\nu}\Pi^{(0)}_{V/A}(q^2),\end{aligned} \tag{38.1}$$

built from the hadronic local currents $\mathcal{O}^{V/A}_{\mu}(x)$:

$$\mathcal{O}^{\mu}_{V}(x)\equiv: g\bar{\psi}_i\lambda_a\gamma_{\nu}\psi_j G^{\mu\nu}_a:\,,\qquad \mathcal{O}^{\mu}_{A}(x)\equiv: g\bar{\psi}_i\lambda_a\gamma_{\nu}\gamma_5\psi_j G^{\mu\nu}_a: \tag{38.2}$$

which select the specific quantum numbers of the hybrid mesons; A and V refer respectively to the vector and axial-vector currents. The invariant $\Pi^{(1)}$ and $\Pi^{(0)}$ refer to the spin one and zero mesons. The correlator is represented in Fig. 38.1.

The perturbative QCD expressions of the invariants are:

$$\begin{aligned}\frac{1}{\pi}\mathrm{Im}\Pi^{(1)}_{V/A}(t)_{\rm pert} &= \frac{\alpha_s}{60\pi^3}t^2\left\{1+\frac{\alpha_s}{\pi}\left[\frac{121}{16}-\frac{257}{360}n_f+\left(\frac{35}{36}-\frac{n_f}{6}\right)\log\frac{\nu^2}{t}\right]\right\}\\ \frac{1}{\pi}\mathrm{Im}\Pi^{(0)}_{V/A}(t)_{\rm pert} &= \frac{\alpha_s}{120\pi^3}t^2\left\{1+\frac{\alpha_s}{\pi}\left[\frac{1997}{432}-\frac{167}{360}n_f+\left(\frac{35}{36}-\frac{n_f}{6}\right)\log\frac{\nu^2}{t}\right]\right\}.\end{aligned} \tag{38.3}$$

The anomalous dimension of the current can be easily deduced to be:

$$\gamma_H=\beta_1+\frac{32}{9}\,, \tag{38.4}$$

where $\beta_1=-1/2(11-2n_f/3)$ is the first coefficient of the beta function. The short-distance tachyonic gluon mass effect is given by the diagram in Fig. 38.2 and reads [462]:

$$\begin{aligned}\frac{1}{\pi}\mathrm{Im}\Pi^{(1)}_{V/A}(t)_{\lambda} &= -\frac{\alpha_s}{60\pi^3}\frac{35}{4}\lambda^2 t\\ \frac{1}{\pi}\mathrm{Im}\Pi^{(0)}_{V/A}(t)_{\lambda} &= \frac{\alpha_s}{120\pi^3}\frac{15}{2}\lambda^2 t\,.\end{aligned} \tag{38.5}$$

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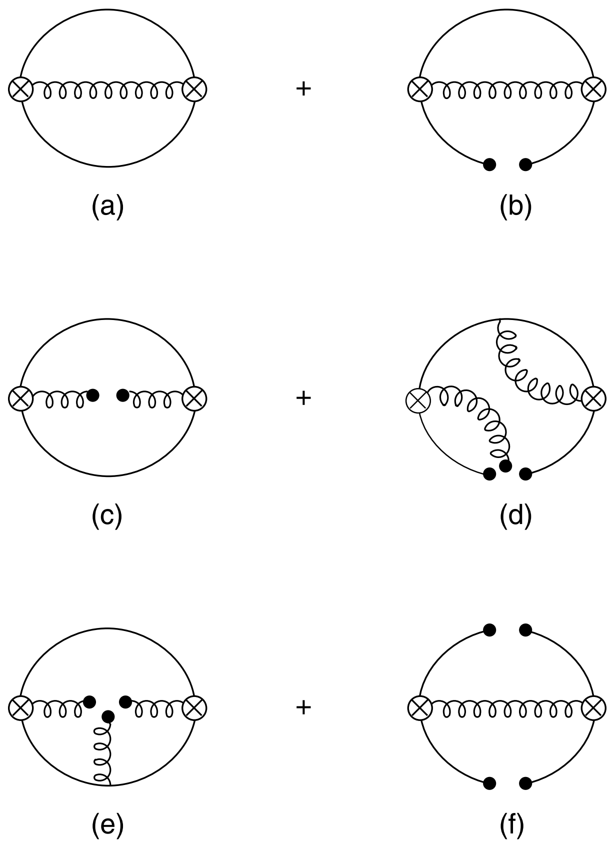

Fig. 38.1. Feynman diagrams corresponding to the OPE of the hybrid correlator: (a) perturbative; (b) quark condensate; (c) gluon condensate; (d) mixed condensate; (e) three-gluon condensate; (f) four-quark condensate.

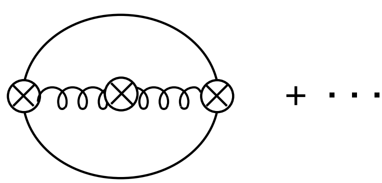

Fig. 38.2. Lowest order tachyonic gluon contribution to the hybrid correlator. The cross in the internal gluon propagator corresponds to the tachyonic gluon mass insertion λ^2.

The (corrected) contributions of the dimension-four and -six terms have been obtained by [461] and reads in the limit $m^2 = 0$:

$$\Pi_V^{(1)}(q^2)_{NP} = -\frac{1}{9\pi}[\alpha_s\langle G^2\rangle + 8\alpha_s m\langle\bar{\psi}\psi\rangle]\log -\frac{q^2}{\nu^2} + \frac{1}{q^2}\left[\frac{16\pi}{9}\alpha_s\langle\bar{\psi}\psi\rangle^2 + \frac{1}{48\pi^2}g^3\langle G^3\rangle - \frac{83}{432}\frac{\alpha_s}{\pi}mg\langle\bar{\psi}G\psi\rangle\right]$$

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$$\Pi_A^{(0)}(q^2)_{NP} = -\left[\frac{1}{6\pi}[\alpha_s\langle G^2\rangle - 8\alpha_s m\langle\bar\psi\psi\rangle] - \frac{11}{18}\frac{\alpha_s}{\pi}\frac{1}{q^2}mg\langle\bar\psi G\psi\rangle + \mathcal{O}\left(\frac{1}{q^2}\right)\right]\log -\frac{q^2}{\nu^2}\ , \tag{38.6}$$

where one can notice from [461] the miraculous cancellation of the log-coefficient of the $D = 6$ condensates in $\Pi_V^{(1)}$.

38.2 Heavy hybrid correlators

Analogous hybrid correlators but for heavy quarks have been evaluated in [463] for unequal masses and for the (axial-)vector channels. In the following, we shall present the results for the equal mass case m in the vector channel which has been checked and completed in [464]. Using the same normalization of currents as in the case of light quarks, one obtains the perturbative spectral functions [464]:

$$\begin{aligned}\frac{1}{\pi}\mathrm{Im}\Pi_{V,\mathrm{pert}}^{(1)} &= \frac{m^6\alpha_s N C_F}{16\pi^3}\frac{1}{t}\left(\frac{7}{3} + \frac{1}{60z^2} - \frac{5z}{3} - \frac{3z^2}{4} + \frac{z^3}{15} + \ln z + 2z\ln z\right)\\ \frac{1}{\pi}\mathrm{Im}\Pi_{V,\mathrm{pert}}^{(1+0)} &= -\frac{m^4\alpha_s N C_F}{16\pi^3}\left(\frac{2}{3} - \frac{1}{15z^3} + \frac{1}{2z^2} - \frac{2}{z} + z - \frac{z^2}{10} - 2\ln z\right)\ ,\end{aligned} \tag{38.7}$$

where $z = t/m^2$. Note that, in [463], the result is given in integral forms. The contributions of the tachyonic gluon with a mass squared $-\lambda^2$ is [464]:

$$\begin{aligned}\frac{1}{\pi}\mathrm{Im}\Pi_{V,\lambda}^{(1)} &= \frac{m^4\lambda^2\alpha_s N C_F}{16\pi^3}\frac{1}{t}\left(-2 - \frac{1}{12z^2} - \frac{2}{3z} + \frac{10z}{3} - \frac{7z^2}{12} - 3\ln z\right)\\ \frac{1}{\pi}\mathrm{Im}\Pi_{V,\lambda}^{(1+0)} &= -\frac{m^2\lambda^2\alpha_s N C_F}{16\pi^3}\left(-\frac{4}{3} + \frac{1}{3z^3} - \frac{4}{3z^2} + \frac{2}{z} + \frac{z}{3}\right)\ .\end{aligned} \tag{38.8}$$

The contributions of the gluon condensate have been obtained in [463] and expressed in terms of the correlators of bilinear quark currents:

$$\begin{aligned}\frac{1}{\pi}\mathrm{Im}\Pi_{V,G^2}^{(1)} &= \frac{4\pi}{9}\langle\alpha_s G^2\rangle t^2\mathrm{Im}\Pi_V(t)\\ \frac{1}{\pi}\mathrm{Im}\Pi_{V,G^2}^{(0)} &= -\frac{2\pi}{3}\langle\alpha_s G^2\rangle t^2\mathrm{Im}\Pi_V(t)\ ,\end{aligned} \tag{38.9}$$

where:

$$\mathrm{Im}\Pi_V(t) = \frac{N}{24\pi}v(3 - v^2) \tag{38.10}$$

is the vector bilinear current spectral function and where $v^2 = 1 - 4m^2/t$ is the square of the heavy quark velocity.