PART IV.

Considerations on Localized Velocity Fields in Stellar Atmospheres: Prototype — The Solar Atmosphere.

D. - Collision-Free Shock-Waves.

Summary-Introduction: Collision-Free Plasmas (*).

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1. – Introduction.

In many cases, the outer atmosphere or corona regions of stars are not dominated by the ordinary process of collisions between ions. Very roughly, collision-free phenomena become of interest when the mean free path for interparticle collisions is larger than the region of space being considered. For solar corona conditions, if we consider a density of 10^7 particles per cubic centimeter and a typical particle velocity of 500 kilometer per second, the mean free path is a few times the solar diameter. At the same velocity for a density of 100 particles per cubic centimeter, corresponding to the interplanetary density, the mean free path would be a thousand astronomical units. Since in both of these regions, we are interested in smaller dimensions, it is of interest to consider the properties of a plasma in which collisions are very rare.

A somewhat more precise definition of the region where collisions are unimportant, is obtained by comparing the mean free path to the ion gyro radius. When the mean free path is the larger of these quantities, some important changes in the plasma properties which may be described as collision-free plasma effects become important. A more detailed discussion of the boundaries of such regions was given at the previous Symposium [1]. According to this criterion of gyro radius and mean free path, the photosphere of the sun is definitely collision-dominated, whereas, the solar corona is essentially collision-free.

In this paper, we will consider two aspects of collision-free plasmas—the structure of a shock-wave moving through such a plasma, and the electrical conductivity of such a plasma. As we shall see, the thickness of a shock-wave may be appreciably thinner than the mean free path for interparticle colli-

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sions and therefore, there may be appreciably more abrupt changes in the state of the plasma than one might otherwise have anticipated. The electrical conductivity can be appreciably reduced, as compared to the standard value derived from interparticle collisions, due to the presence of appreciable wave motion in the plasma. This reduction may in some cases be sufficient so that the usual astronomical assumption of the fluid being tied to the magnetic field lines will not always be valid.

2. – Wave turbulence.

One of the striking features which has been discovered in laboratory plasma experiments is that a phenomenon similar to turbulence frequently occurs in collision-free plasmas [2]. Large scale fluctuations in plasma properties appear and considerably increase the rate of dissipation in the plasma. The understanding of this phenomenon seems to be fundamental to a discussion of the transport properties of such a plasma. The use of the term turbulence should not be taken to imply that the mechanism is as complex as it is for ordinary aerodynamic turbulence. As we shall see, these fluctuations in plasmas have a much different physical basis and are easier to understand.

If we examine the small amplitude perturbations which can exist in an otherwise uniform collision-free plasma, we find that most such disturbances can be described as *propagating* waves. The exceptions to this are the very restricted class of disturbances that are completely independent of the co-ordinate along the magnetic field lines. The basic elements which make up plasma turbulence are, therefore, probably propagating waves rather than disturbances which remain stationary in the fluid.

Plasma turbulence may be considered as a distribution of randomly oriented waves propagating through the plasma. As these waves propagate they carry momentum and energy from one place to another. Due to the non-linear interactions between these waves, a particular wave has a finite lifetime at the end of which its momentum and energy has been transferred to other waves. If we visualize the waves as being grouped into wave packets, this picture is closely analogous to the kinetic theory picture of an ordinary gas. The wave packets replace the atoms, and the distance travelled by a wave in its lifetime replaces the mean free path.

The waves which will be of interest to us for the two problems which are considered here, are the magnetohydrodynamic waves at frequencies in the neighborhood of the ion cyclotron and somewhat higher. For these waves, the mean free path has been estimated [3] by considering the random walk of the phase of a wave as it moves through the disturbances in the plasma properties caused by the other waves present in the plasma. The resulting mean free path was

$$\lambda = \frac{4}{\beta \bar{k}}, \tag{1}$$

where $\bar{k}$ is the mean wave number in the distribution of waves and β is the ratio of wave energy to the average magnetic field energy. This result may be interpreted simply as a statement that the scale is determined by the mean wave length of the turbulence and that the number of wavelengths which a wave moves between collisions is inversely proportional to the amount of disturbing wave energy present.

Before developing the consequences of this kinetic theory picture of wave turbulence, it is of interest to compare this picture with the picture of ordinary aerodynamic turbulence. In the latter case, the fundamental element is a vortex which does not propagate through the fluid in the absence of other disturbances. This means that fluid properties are transported from one place to another only through the non-linear interactions of different vortices. The simple picture of an individual element of the turbulence transporting fluid properties is therefore not valid. Furthermore, the fact that vortices do not propagate means that vortices retain the same neighbors, and a high degree of correlation results which must be properly taken into account. In a plasma, waves attain new neighbors due to their propagation, and the simplifying assumption that interacting waves have random phases is justified.

3. – Shock structure.

Before discussing the structure of a shock-wave in a collision-free plasma, we will review very briefly some of the gross features of shock-waves in a neutral gas where collisions dominate. The first point to be made is that a pressure pulse whose initial dimensions are large compared to the mean free path will tend to steepen to form a shock front. In other words, if viscous forces and heat conduction may be neglected, such a pressure pulse will tend to steepen towards a discontinuity. When such a discontinuity is reached, it is apparent that the neglect of dissipative effects is no longer valid. If one is interested in the large scale properties of the flow, one may calculate the conditions across the discontinuity (shock-wave) from the laws of conservation of mass, energy, and momentum. If, on the other hand, one is interested in resolving the discontinuity, one must calculate the local region of the flow including the effects of viscosity and heat conduction.

A very rough estimate of the resulting thickness of a shock-wave may be obtained in the following manner. In the absence of viscosity, the pressure rise is unbalanced for a steady flow as indicated by the fact that pulse steep-

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ening occurs. In order to maintan a steady shock structure, this excess pressure must be balanced by viscous stresses. For a strong shock wave, the magnitude of the unbalance is of the order of the gas pressure. If the viscous stresses are to be this large, the distribution function of the particles must be appreciably distorted from a Maxwellian. In order to achieve this, there must be appreciable gradients per mean free path. The resulting shock thickness is therefore of the order of a mean free path. More detailed considerations, both experimental and theoretical, show that the shock thickness for a strong shock in an ordinary gas is of the order of two mean free paths in the gas ahead of the shock [4].

If we now consider the case of a plasma (in the absence of turbulence) in which the mean free path is appreciably longer than the ion gyro radius, the viscosity is reduced by the square of this ratio since the ions are no longer free to move with a constant velocity between collisions. This means that the viscous forces are appreciably reduced and cannot balance the excess pressure when the pulse thickness is of the order of the mean free path. The steepening therefore proceeds to a thickness appreciably less than the mean free path. In this case, very few collisions between particles will occur inside the shock font, and we may consider interparticle collisions to be completely absent in the shock front.

If we now want to consider the structure of such a shock-wave from the point of view of kinetic theory of waves, we must first show that large wave amplitudes can be built up by the shock wave [5]. This can be done by considering a small amplitude, small scale disturbance superposed on the velocity gradient which is present in a shock-wave. This small disturbance or wave packet will exert a pressure on the fluid. In the presence of a compressive velocity gradient, there will be a net amount of work done on the wave packet by the flow, and the energy of the wave packet will increase. This compression work is the source of wave energy in the shock front. This mechanism will be most effective for those waves which can spend a long time in the compressive region of the shock. Thus, waves with a group velocity comparable to the flow velocity will predominate. For shock-waves moving perpendicular to the magnetic field at shock speeds not too much above the Alfvén speed, the waves which will be selected on this criterion are the magnetohydrodynamic waves at frequencies somewhat above the ion cyclotron frequency.

As in the case of an ordinary gas, we must have an appreciable distortion of the distribution of waves to give appreciable viscous stress and heat conduction. This implies that the shock thickness will again be a few mean free paths. In the present case, of course, it is a few mean free paths for the waves. The mean free path for the waves as given by eq. (1) depends on the wave energy as well as the mean wave number. The mean wave number is determined from the above criterion of the appropriate wave velocity. The wave energy is determined from the conservation equations. All of the energy which would normally be thermal energy behind the shock is wave energy in this case. The

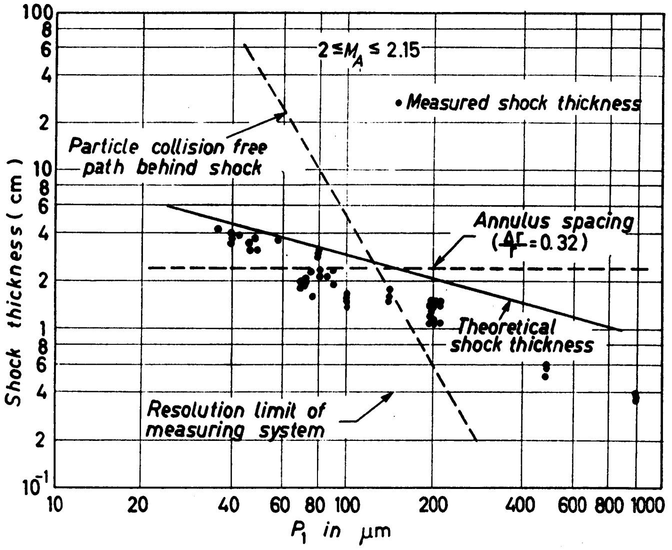

Fig. 1. – Dependence of shock thickness on density. At low densities, the shock thickness is less than the interparticle collision-free path and agrees with the predicted collision-free shock thickness. At high densities, the mean free path is reduced and one would expect a shock thickness several times the interparticle mean free path.

resulting estimate of shoch thickness and its dependence on density and Alfvén-Mach number [6] are given in Fig. 1 and 2.

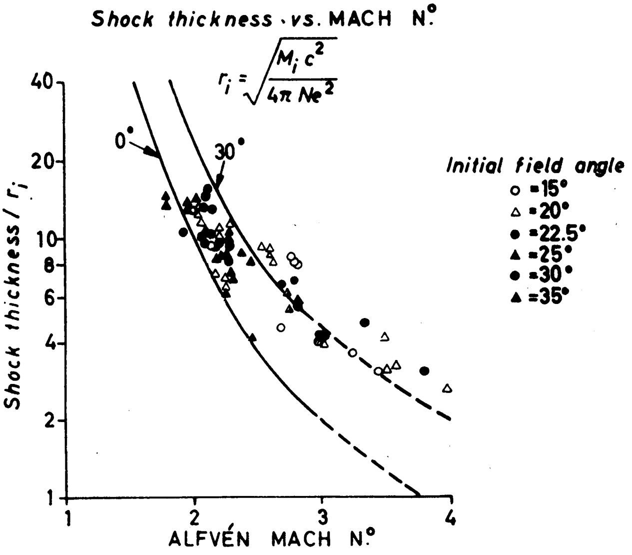

Fig. 2. – Dependence of shock thickness on the ratio of shock speed to Alfvén speed ahead of the shock. With increasing Alfvén Mach number the shock thickness decreases roughly as predicted. The ordinary particle collision shock thickness would increase. The angle referred to is the angle between the magnetic field ahead of the shock and the plane of the shock. Over the range covered, no experimental dependence is observed. The theoretical variation is, however, also rather small.

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The experimental data shown in these figures was obtained by PATRICK [7] in a magnetic annular shock tube. This is a device in which the test gas occupies the annular space between two concentric cylinders. Current from a condenser discharge between the inner and the outer cylinders creates a magnetic field which acts as a piston. This piston pushes the gas down the annulus to form a shock-wave. These shock waves were observed principally by recording the emitted bremsstrahlung from the hot gas with a collimated photomultiplier system looking across the annulus.

Shock speeds of the order of 300 km per second were obtained at the densities used. The absolute value of the radiated intensity varies as the square of the electron density and is almost independent of the temperature. Its measurement can therefore be used to determine the density behind the shock-wave. In Fig. 3, a comparison is given of the theoretical and experimental light intensities which are seen to be in very good agreement. This agreement shows that the interactions with the wall do not appreciably change the conservation relations across the shock. That is, the effects of the wall are presumably confined to a narrow region near the wall.

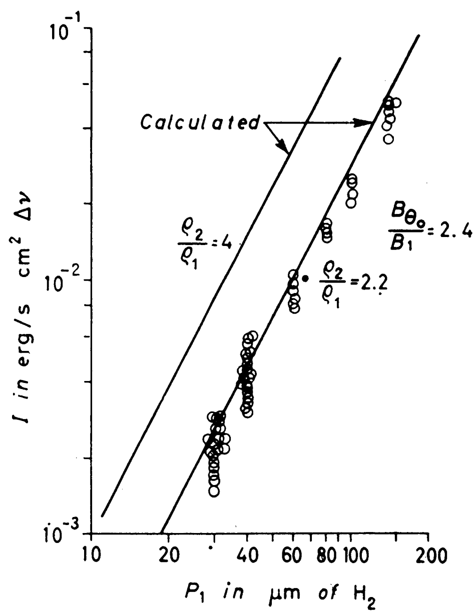

Fig. 3. – Measured light intensity behind the shock wave as a function of density. The expected value for the initial magnetic field used corresponds to $\varrho_2/\varrho_1 = 2.2$

The experimental shock thicknesses were obtained by measuring the rise time of the light intensity as the shock passes the collimating slits. The resulting agreement between theory and experiment is somewhat better than one might expect from the present crude form of the theory.

The theoretical shock thickness at a fixed Alfvén-Mach number varies inversely as the square root of the density. For a shock speed of twice the Alfvén speed and a density of 100 particles per cubic centimeter corresponding to the interplanetary gas, the predicted thickness is still only 200 km.

This scale is still reasonably small compared to the length scales which are of interest. We may therefore conclude that shock waves of almost negligible thickness can exist in the interplanetary medium in spite of the extremely long mean free path for interparticle collisions.

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Another type of approach to the collision-free shock which has been attempted in several variations is to assume that the flow is completely one-dimensional—that is, independent of the co-ordinates in the plane of the shock front. This approach is somewhat questionable in view of the argument for the growth of wave energy which was given. This argument would imply that such solutions might be unstable to three-dimensional disturbances. These treatments will not be considered in this paper since some of the proponents of these theories are present and will bring them up in the discussion.

4. – Electrical conductivity.

The suggestion that small-scale turbulence could be responsible for an enhanced diffusion of plasma across a magnetic field was first made by Böhm [8]. Buneman [9] considered the reduction in electrical conductivity associated with the build-up of waves by the two-stream instability. His mechanism however, requires currents corresponding to electron velocities greater than the electron thermal velocities. Such large values of current will not occur in general. Spitzer [10] has considered the random walk diffusion of particles due to the presence of a wave field. His results, however, would seem to be applicable only to the small number of particles which move in phase with the waves. His diffusion rate applied to the plasma as a whole is, therefore, probably a gross overestimate.

In an ordinary plasma in which collisions dominate, the electrical conductivity is determined by the friction force which results from collisions between electrons and ions when the electron stream has a velocity relative to the ions. If the electrical conductivity is to be reduced by the presence of wave turbulence, the waves must be more effective in producing such a friction force than the interparticle collisions are. In order to see how a wave field could give rise to such a force, let us assume that a wave field exists such that the electric field associated with the waves has a component which is in phase with the fluctuations in the mass density. A net force would result on the relative motion between the electrons and ions, since in the high density regions there are more electrons than ions present to be accelerated in one direction than are present in the low density region where the acceleration is in the opposite direction. A similar term would also arise if there were a correlation between the mass velocity of the fluid and the perturbed magnetic field. If we consider both of these terms, the momentum balance for the electrons for a quasi-steady state current may be written as

$$\varrho_0 E_0 = -\overline{\varrho' E'} - \varrho_0 \overline{v' \times B'}, \tag{2}$$

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where ϱ is the mass density of the plasma, E the electric field, v the mass velocity, and B the magnetic field, the subscript 0 indicates the undisturbed conditions, the prime indicates quantities which fluctuate due to the turbulence and the bar indicates an average over a region large compared to the wavelength of the turbulence. The left-hand side indicates the acceleration due to the applied static electric field and the right-hand side represents the friction associated with the fluctuating fields. It is important to note that this equation suggests that very high frequency waves which oscillate too rapidly to produce changes in the mass density will not be effective in producing the required friction force.

The friction force resulting from a linear wave of constant amplitude is zero, since a wave which proceeds unchanged cannot continually supply momentum to the relative motion between electrons and ions. This may be regarded as the statement that the effective electric field and the density are exactly 90 degrees out of phase for a wave of constant amplitude. If, however, one considers a wave whose amplitude is changing with time, the phase angle between these quantities will change slightly and an in-phase component will appear. There is then a net force associated with a wave whose amplitude is changing with time. The order of magnitude of this force has been estimated by considering the linear dispersion relation for a zero temperature plasma in which the wave is artificially excited by the presence of a hypothetical oscillating body force. For the case of the magnetohydrodynamic waves at frequencies much smaller than the electron cyclotron frequency the friction force is of the order of magnitude of the rate of change of momentum of the wave (rate of change of energy divided by phase velocity).

Unless strong damping or strong instabilities exist, the time rate of change of the energy of a particular wave is probably determined by collisions with other waves. Roughly speaking, the friction force resulting from a collision between waves will be of the order of magnitude of the momentum interchange in the collision.

If the waves present in the plasma have a symmetric distribution of the wave vectors, the net friction force resulting from each of the individual collisions will average to zero. In order to obtain a net friction force when averaged over all the collisions, there must be an asymmetry in the distribution of waves. This asymmetry results from the presence of a current in the plasma. Depending upon the frequency of the wave, the wave may be more coupled to the electron or to the ions. The waves coupled to the ions will tend to move with respect to a co-ordinate system determined by the ions, whereas, those coupled to the electrons move with respect to a co-ordinate system determined by the electron motion. In other words, one would expect that a change in the phase velocity of some waves would be associated with the introduction of a current into the system. If one calculates the dispersion relation for the

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magnetohydrodynamic waves in a zero temperature plasma and in the presence of a steady current, one does find a change in the phase velocity of the waves which is of the order of magnitude of electron velocity associated with the current. This change in phase velocity is a function of the frequency of the wave. It is constant at very low frequencies and at very high frequencies, and it changes in the neighborhood of the ion cyclotron frequency. In the presence of a current, therefore, the high frequency waves will have a net drift velocity relative to the low frequency waves. This gives rise to an asymmetry in the distribution of waves which is proportional to the current. The collisions between the high and low frequency waves result in a transfer of momentum between the two groups of waves. This momentum transfer between the waves is transmitted as a momentum transfer between electrons and ions by the mechanism discussed earlier.

Since the net friction is proportional to the current, we have a linear relation between the current and the electric field. The order of magnitude of the resulting conductivity can be estimated from the above arguments using eq. (1) to determine the mean free time for collisions between waves and taking the mean frequency of the wave distribution to be the ion cyclotron frequency. This choice of mean frequency is dictated by the fact that it separates the regions of different change of the phase velocity of the waves. Waves must be present on both sides of the ion cyclotron frequency in order to have a relative velocity between them.

The resulting conductivity is

$$\sigma = \frac{8Nec}{B}\,\frac{1}{\beta^2}\,, \tag{3}$$

where N is the particle density, B is the average magnetic field, e is the electronic charge, and c the velocity of light. The ratio of this conductivity to the conductivity determined by the interparticle collisions is $8\beta^2/\omega_{ce}\tau_e$, where ω_{ce} is the electron cyclotron frequency and τ_e the electron mean free time. Since $\omega_{ce}\tau_e$ can be very large under high temperature and low density conditions, the reduction in conductivity can be appreciable even for moderate values of β [11].

A very rough comparison of this theory with experiment may be obtained from some stabilized pinch experiments of Colgate [2]. The measured lifetime of the pinch indicates a conductivity of 10^3 mho per cm at a density of $2\cdot 10^{15}/\text{cm}^3$ and a magnetic field strength of $2\cdot 10^4$ gauss. No direct measure of β exists for these experiments. However, the plasma pressure was estimated for magnetic probe measurements to be about one-sixth of the magnetic pressure. If the arbitrary assumption is made that all of this pressure is wave pressure, the predicted conductivity is $5\cdot 10^2$ mho per cm.

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The astronomical implications of this reduced conductivity are not completely clear since the magnitude of β which occurs is generally not known. In principle, of course, this is calculable from an energy balance, consideration of the wave energy. The joule dissipation associated with these waves increases the wave energy and damping of the waves, as well as their diffusion out of the current region decreases their energy. This energy balance has, however, as yet not been considered even for the simpler laboratory cases. It is, however, interesting to note that if a large value of β does occur, the electrical conductivity can become so low that in some astronomical cases, appreciable diffusion of the magnetic field relative to the plasma can occur. For example, for $\beta=1$, a density of 10^8 particles per cubic centimeter, and a magnetic field strength of 10^3 gauss, the diffusion depth is of the order of $3\cdot10^{10}$ cm in one month. This implies that for sufficiently large β, the magnetic field above a sunspot can diffuse across the dimension of a sunspot in a time less than the lifetime of the sunspot. These arguments, of course, do not apply to the currents in the photospherc and below, which are responsible for the magnetic energy of the sunspot. The structure of the field in the corona above the sunspot is, however, of interest in determining the behavior of disturbances. This structure may be appreciably influenced by the diffusion mechanism described above.

5. – Summary.

In the corona regions of stars the ratio of the mean free path for interparticle collision to the gyro radius becomes very large. Under these conditions the transport processes in the plasma become dominated by plasma turbulence. This phenomenon can be described in terms of a kinetic theory of a random distribution of waves. The structure of a shock-wave under these conditions has been considered. Even at densities as low as 10^2 particles per cm^3 the thickness of such a shock-wave should be very thin, less than 100 km. The effects of such a turbulent wave field on reducing the electrical conductivity has also been considered. It seems possible that appreciable diffusion of the magnetic field relative to the plasma could occur under some astronomical conditions.

Note added in proof.

Recent more detailed analysis of the shock structure problem (to be published in *Proc. of the Intern. Atomic Energy Conf. on Plasma Physics and Controlled Nuclear Fusion Research* to be held in Salzburg, Austria, from 4 to 9 September 1961) has indicated that in order to persist the wave distribution must be highly asymmetric. This casts some doubt on the assumption in the estimate of electrical conductivity that a small departure from a symmetric distribution is caused by the current.

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