CORRESPONDENCE.

RETIRANTS IN THE EIGHTH YEAR.

To the Editor of the Journal of the Institute of Actuaries.

DEAR SIR,—I would draw the attention of your readers to a rather interesting point. I refer to the withdrawals from life companies in the eighth year of insurance, or as it is called in England the seventh year. The matter is mentioned by Mr. King on p. 392, vol. xix of the Institute *Journal*. I extract a table from his paper:—

Discontinuance per-cent.

Year of Assurance.	17 OFFICES' EXPERIENCE. Ages at Entry 40 to 50.	20 OFFICES' EXPERIENCE. Ages at Entry		
		20 to 35.	40 to 50.	55 to 65.
0	·95	3·04	2·06	1·64
1	9·01	7·47	5·98	6·75
2	5·74	5·35	4·29	4·17
3	4·87	4·60	3·42	2·92
4	4·15	3·58	2·84	2·48
5	3·64	2·95	2·57	2·50
6	3·30	2·64	2·11	2·20
7	7·89	3·61	3·53	4·29
8	1·73	2·03	1·59	1·29
9	1·75	1·57	1·53	1·53

What is the cause of this rise at the seventh, or more properly the eighth, year of insurance? Mr. King has shown conclusively that term policies cannot be it. He says:—"Some other cause must have been at work, but I have looked in vain through the Blue-books to detect it." I believe that the seven-years half-credit system is at the bottom of the matter. Under this supposition, we can easily account for both the large number of lapses in the eighth year and the fact that in recent tables the peculiarity is not so noticeable. The explanation would appear perfect, but for the appearance of the anomaly in the experience of the Mutual Life of New York. This company never issued half-credit policies, so far as I am aware. I quote their figures:—

Year of Insurance.	Ratios of Retirants to Exposures.
1	·054
2	·100
3	·068
4	·048
5	·038
6	·033
7	·026
8	·038
9	·020
10	·022

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The rise at the eighth year here is almost precisely the same as in the twenty offices—the ratios at the seventh, eighth, and ninth years in the total experience of both being respectively ·026 : ·038 : ·020 and ·025 : ·038 : ·019.

Professor Bartlett, of the Mutual Life, wrote me that he could give no satisfactory explanation. If, as I have suggested, we assume that the half-credit system is the root of the matter as regards the English statistics, what are we to think of the American ones? We have the option of two conclusions. We have here either a very remarkable coincidence or the indication of the working of some law. Which is it? We incline to the former opinion. Perhaps some of your readers will give us their views.

Yours respectfully,

T. B. MACAULAY.

Montreal,
164 *St. James' Street,*
May 8th 1879.

ON THE RELATION BETWEEN THE VALUE OF A POLICY AND THE RATE OF INTEREST.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—In vol. xvii of the *Journal of the Institute*, both Mr. Macfadyen and Mr. Sutton prove satisfactorily that, under certain limitations, the value of a policy by a higher rate of interest is less than the value by a lower rate. As the following demonstration is possibly more simple than what these gentlemen give, it may not be uninteresting to some of your readers. In the cases referred to, the proof proceeds from the formula

$$1-\frac{1+a_{x+1}}{1+a_x},$$

but without reference to the duration of the policy. The value may be also expressed by

$$\frac{A_{x+n}-A_x}{1-A_x}.$$

Now, assuming the dash to indicate values relating to a higher rate of interest, we have

$${}_nV_x > {}_nV'_x$$

when $$\frac{A_{x+n}-A_n}{1-A_x} > \frac{A'_{x+n}-A'_x}{1-A'_x};$$

that is, when $$\frac{A_{x+n}-A_x}{A'_{x+n}-A'_x} > \frac{1-A_x}{1-A'_x}.$$

But $$A_{x+n}-A_x = v\left(\frac{d_{x+n}}{l_{x+n}}-\frac{d_x}{l_x}\right)+v^2\left(\frac{d_{x+n+1}}{l_{x+n}}-\frac{d_{x+1}}{l_x}\right)+\ldots\ldots$$

and $$A'_{x+n}-A'_x = v'\left(\frac{d_{x+n}}{l_{x+n}}-\frac{d_x}{l_x}\right)+v'^2\left(\frac{d_{x+n+1}}{l_{x+n}}-\frac{d_{x+1}}{l_x}\right)+\ldots\ldots$$

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