GRAVITATIONAL LENSES AND DARK MATTER: THEORY

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ABSTRACT: In principal, gravitational lenses can be used to study dark matter in a variety of ways. They can provide information on masses of lensing galaxies, circular velocities in halos, the value of Ω in condensed objects, and the constitution of heavy halos (whether they are made of low mass stars or not). They can tell us whether mass and light are distributed equally, provide insights on mass in groups of galaxies and on exotic dark matter such as strings and non zero Λ. As an example, the existance of the normal lens case QSO 2016 with z_Q = 3.27 allows one to set the limit $q_o > -2.3$.

1. INTRODUCTION

Gravitational lenses are an important new tool for studying dark matter in the Universe. I will discuss a number of ways in which these systems may be used in principal.

2. MASSES OF LENSING GALAXIES

Image separations may be used to put lower limits on the mass associated with lensing galaxies. Suppose the lensing galaxy lies at a distance d from us and is about halfway between us and the lensed QSO. The galaxy produces a bend angle of $\alpha \sim 4GM/rc^2$ where r is the impact parameter. Typically two relatively bright QSO images will be seen on opposite sides of the lensing galaxy with a third faint image of the QSO lying near the center of the galaxy. The separation of the two bright images is approximately $\Delta\theta \sim (2r/d) \sim \alpha \sim [8GM(1+z_L)/dc^2]^{1/2}$. With $\Omega_o \sim 0$, $H_o \sim 50$ km s^{-1} Mpc^{-1}, z_Q large, $z_L \sim 0.4$, d $\sim$ 1500 Mpc one finds

$$\Delta\theta \sim 4'' \ (M/10^{12}\ M_\odot)^{1/2}$$

Since the observed cases have separations of 2" to 7" this shows immediately that the masses of lensing galaxies (the mass contained within the two QSO images) is surprisingly large, implying some dark

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matter. E.L. Turner, in the following companion paper, presents detailed lower mass limits of this type for each case.

3. CIRCULAR VELOCITIES IN HALOS

If we model galaxies as singular isothermal spheres then we find that the two bright images should have

$$\overline{\Delta\theta} \sim 4GM/rc^2 \sim 2\pi(V_{CIR}/c)^2 \sim 0''.6\ (V_{CIR}/200\ \text{km s}^{-1})^2$$

where V_{CIR} is the circular velocity in the lensing galaxy halo [cf. Turner, Ostriker, Gott (1)]. This shows that the observed lensing galaxies either have larger circular velocities than average or are being helped by additional matter in the beam most likely from the group or cluster of which the galaxy is a member (1). This also prompts us to search for lensing cases with small angular separation. Tyson _et al._ (2) have shown that measurements of the lensing distortion of background galaxies by foreground galaxies can be used to set limits on V_{CIR} for typical galaxies. They find $V_{CIR} \lesssim 190$ km s^{-1} out to ~ 120 Kpc which compares with values of $V_{CIR} \sim 130$ to 220 km s^{-1} at these separations deduced from binary galaxies [Turner (3), Yahil (4), White (5)].

4. Ω IN COMPACT DARK OBJECTS

Press and Gunn (6) showed that out to a redshift of $z_Q \sim 2$ the optical depth to lensing is of order

$$\tau \sim 0.3\ \Omega_{LENS}\ .$$

Separations of order $\Delta\theta \sim 0''.001\ (M/10^5\ M_\odot)^{1/2}$ would be seen. Since VLBI studies show that most QSO's are not multiply lensed on milli-arcsecond scales there can not be a closure density of compact objects with masses of $\gtrsim 10^5\ M_\odot$. Using the mini-lensing effect [twinkling of the compact QSO continuum region produced by the lensing action of small masses (7,8)] Canizares (9) has shown that $\Omega < 0.1$ in objects of Jupiter mass or larger or otherwise QSO's would show more line versus continuum variations than they do.

5. ARE MASS AND LIGHT DISTRIBUTED EQUALLY?

Lenses provide a particularly clear test of this question. As E. L. Turner will discuss in the next paper the observed cases rather strongly suggest that the mass and light are not distributed equally.

6. ARE HEAVY HALOS MADE OF LOW MASS STARS?

Gott (7) and Chang and Refsdal (8) showed that for a double lensed QSO like 0957 if the heavy halo of the lensing galaxy were made of low mass

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stars of mass M_S then there would be an uncorrelated twinkling in the images with timescales of

$$\Delta t \sim 1.4 \text{ yrs } (M_S/M_{JUPITER})^{1/2} .$$

For 0957 the two images have optical depths to such mini-lensing of 0.25 and 3. Long term monotoring of lensed QSO's is needed to do this test for low mass stars. Further detailed studies of such mini-lensing effects have been carried out by Young (10), Narasimha et al. (11), Ostriker and Vietri (12), and Paczynski (13).

7. ARE THERE BLACK HOLES OR CUSPS IN GALACTIC NUCLEI?

Chang and Refsdal (14) showed that if a black hole is placed in a galactic nucleus it can block completely the central faint overfocused image. A similar effect occurs if there is a cusp in the center as in a singular isothermal sphere. If the mass distribution is smooth, there is a theorem by Burke (15) that there must be an odd number of images. Black holes or cusps in galactic nuclei swallowing the faint central image may be responsible for the fact that all the observed cases show an even number of images.

8. ARE HALOS NON-BARYONIC?

If $H_o = 50$ km s^{-1} Mpc, $\Omega_{baryon} \sim 0.1 \sim \Omega_{HALOS}$ Gott, Gunn, Schramm, Tinsley (16) but if $H_o = 100$ km s^{-1} Mpc, $\Omega_{baryon} \sim 0.025 < \Omega_{HALOS}$. Refsdal (17) has shown that in a double lensed QSO intrinsic variations in the QSO will appear in the two images with a time delay of order

$$\Delta t \sim \Delta\theta^2 d/c \sim 1 \text{ yr } (\Delta\theta/3'')^2 \; (50/H_o).$$

With good modeling we can get H_o [Borgeest and Refsdal (18), Young et al. (19)]. We really only get upper limits on H_o because smooth matter in the beam can artifically lower the time delay [Falco et al. (20), Borgeest and Refsdal (18), Alcock and Anderson (21)]. Long term monotoring is needed to measure Δt.

9. DARK MATTER IN GROUPS AND CLUSTERS

Dense groups and clusters can serve as gravitational lenses (1,22). An interesting possible example of this is VV172 which is being investigated by a group of us at Princeton and by Hammer and Nottale (23). The group VV172, at a redshift of $\sim 15{,}800$ km s^{-1}, may be lensing the background galaxy at a redshift of $\sim 36{,}900$ km s^{-1}. This could explain why the background galaxy has a high apparent luminosity for its color.

10. EXOTIC DARK MATTER-STRINGS

The exact exterior metric for a string is

$$ds^2 = -dt^2 + dz^2 + dr^2 + (1-4\mu)^2 r^2 d\phi^2$$

[Gott (24)] where μ is the mass per unit length in the string in Planck masses per Planck length. For string assisted galaxy formation $\mu \sim 10^{-5}$. This is a conical space with an angle deficit $D = 8\pi\mu$. A string can produce double lens images of distant QSO's with separations of

$$\Delta\theta \leq 8\pi\mu \sim 50''$$

which should be observable. For futher studies on strings see Vilenkin (25), Kaiser and Stebbins (26) and Hogan and Rees (27). Moving strings produce fluctuations in the cosmic microwave background of order $\Delta T/T \sim 8\pi\mu \sim 10^{-4}$ (24, 26).

11. MEASURING Ω

Several methods are available for measuring Ω. After many cases have been discovered we may infer a curve of optical depth versus z_Q. The optical depth due to galaxies in an Ω=0 model rises more steeply with z_Q than in an Ω=1 model (1) being a factor of 2 higher at at $z_Q \sim 3$. We can plot $\Delta\theta$ versus z_Q; in the Ω=1 case this is flat but in the Ω=0 case it falls by about 20% by a redshift of $z_Q \sim 3$. The ultimate method wuld be to measure H_o using time delays from a low redshift set of QSO's and compare it with the value of H_o deduced from a high redshift set of QSO's, thus measuring the decelleration of the universe directly.

12. DARKEST MATTER - Λ

Paczynski and Gorski (28) and Alcock and Anderson (29) have noted that a positive cosmological constant can produce large image separations. Gott and Park (30) have found an interesting effect that occurs in closed universes with $\Lambda > 0$. Here the cosmological metric is:

$$ds^2 = -dt^2 + a^2(t)\ [d\chi^2 + \sin\chi^2\ (d\theta^2 + \sin^2\theta d\phi^2)]$$

They find that the displacement in the sky produced by a singular isothermal sphere galaxy is

$$\beta_{cr} = \alpha \sin(\chi_Q - \chi_L)/\sin(\chi_Q)$$

where $\alpha = 2\pi\ (V_{CIR}/c)^2$ = const. and χ_L and χ_Q are the co-moving distances to the lens and QSO respectively. The antipode is at $\chi = \pi$. If $\chi_L < \chi_Q < \pi$ then $\beta_{cr} > 0$ and three images are formed if the QSO's true position is within β_{cr} of the center of the lensing galaxy. A bright double with separation $\Delta\theta = 2\beta_{cr}$ is formed plus a third extremely faint image between them which is located at the position of the lensing galaxy nucleus (see fig. 1). As $\chi_Q \to \pi, \beta_{cr}$ blows up; the lensing cross sections, image magnifications and separations also blow up. Thus if the antipodal redshift were within the observed range of QSO's we would expect to see a large number of bright double lensed

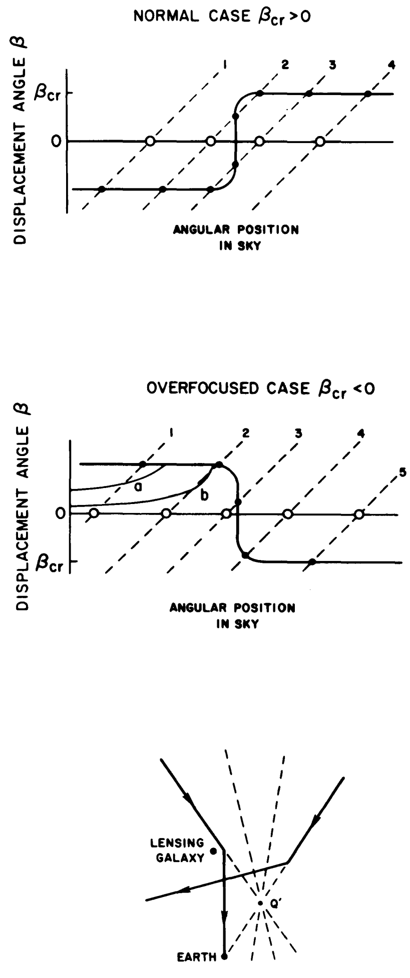

Figure 1. Normal and overfocused lensing cases for a galaxy with an isothermal sphere heavy halo and small core radius (nearly a singular isothermal sphere). Image displacement is plotted as a function of angular position in the sky. The true unlensed position of the QSO is shown as an open circle, the positions of the images are shown as filled circles. In the normal case ($\beta_{cr} > 0$), if the QSO is at positions 1 or 4 only one image will be formed, but if it is at positions 2 or 3, three images will be formed. In the overfocused case ($\beta_{cr} < 0$) one image is formed at positions 1 through 5. If the rotation curve falls slowly as in curve a still only one image will be formed, but if the rotation curve falls rapidly enough as in curve b, three images will be formed for a QSO at position 2. In the overfocused case the universe as a whole forms a real image of the QSO at Q' as illustrated. The bottom illustration shows how light beams are bent in the overfocused case when QSO is at position 4.

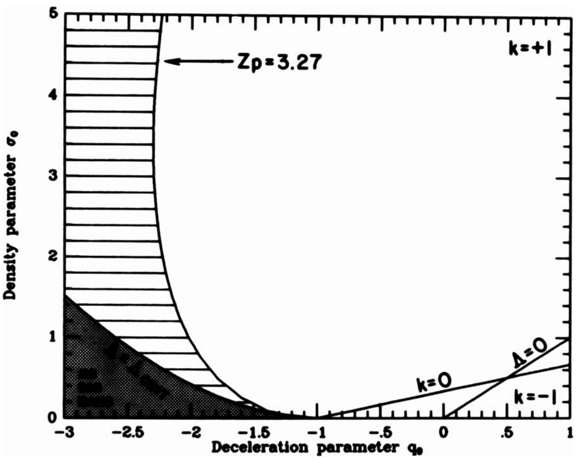

Figure 2. Constraints implied by gravitational lensing are plotted on the σ_0, q_0 plane. The vertical coordinate is the density parameter $\sigma_0 = \Omega_0/2$ and the horizontal coordinate is the decelleration parameter q_0. The region below the line $\Lambda = \Lambda_{CRIT}$ is not allowed because the models have no big bang. The observed lens cases including a normal one with $z_Q = 3.27$ imply that in a closed model the antipodal redshift z_p should be greater than 3.27. The curve showing models with $z_p = 3.27$ is plotted. The hatched region has $z_p < 3.27$ and is not allowed by the lensing criterion. For all σ_0, the allowed value of $q_0 > -2.3$. The graph shows the regions for which the models have positive, negative or zero curvature k = +1, −1, 0 and the line showing models with $\Lambda = 0$.

QSO's of wide separaton which could have hardly escaped notice. For $\chi_Q > \pi$ and $\chi_Q - \chi_L < \pi$, $\beta_{cr} < 0$ and we have the over focused case illustrated in fig. 1. The universe itself forms a real image of the quasar at Q' just in front of us at a distance of $\chi_Q' = \chi_Q - \pi$. The lensing galaxy lies behind this deflecting the light beams out of the way. With a singular isothermal sphere galaxy only one image is formed. And if the QSO's true position is within β_{cr} of the center of the lensing galaxy this one image will be an extremely faint one going through the galactic nucleus. If the galaxy has a rotation curve that falls rapidly enough there is a possibliity of forming three images all to one side of the lensing galaxy (see fig. 1) as Paczynski (31) has noted. Even if galaxies have heavy halos that cut off sharply at 100 kpc, the rotation curves will not fall rapidly enough to allow the offset triple image case unless the galaxy is close enough to the antipode so that $\beta_{cr} < -14''$. As long as the rotation curves are either flat or rising as discussed at this conference we will see only one image in the overfocused case. Davis et. al's (32) data on velocity diferences between pairs of galaxies suggests that this is the case at least out to the correlation length $r \sim 10$ Mpc (H_o=50). In any case for the overfocused case we never see two bright images with the lensing galaxy in between. The lensed QSO 2016 has (33) $z_L \sim 0.8$, $z_Q = 3.27$. It is a bright double with the lensing galaxy roughly between the two images. It is thus an example of a normal lens case where $\beta_{cr} > 0$ and not an example of an overfocused lens case where $\beta_{cr} < 0$. Since the lensing galaxy is roughly halfway between us and the QSO Gott and Park (30) show that the only way to reasonably get $\beta_{cr} > 0$ for this case is if $\chi_Q < \pi$, and the antipodal redshift $z_p > 3.27$. The existence rather simple normally lensed cases with $z_Q = 1.41$, $z_Q = 2.15$, $z_Q = 3.27$ and the observed lack of correlation of $\Delta\theta$ with z_Q strongly implies that $z_p > 3.27$. The regions in the σ_o, q_o plane ruled out by this limit are shown by hatched lines in fig. 2. We see that the only allowed models have $q_o > -2.3$.

13. SUMMARY

We can see that gravitational lenses provide many exciting posibilities for studying dark matter in the universe. A number of these tests are difficult and making them work in practice will require a careful observational treatment with due attention to selection effects and modeling uncertainties. Some may have to await discovery of particularly simple or auspicious lensing cases. E.L.Turner in the next paper will discuss what we have learned so far from the six observed cases and the prospects for the future.

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