ABSTRACT OF THESIS

J. S. Taylor, Functions, Uniformities and Extension Spaces, presented at McMaster University, (Supervisor, Bernhard Banaschewski).

The basic theme of this thesis is the construction of topological spaces and the application of specific construction methods to problems dealing with the extensions of a completely regular space.

If ξ denotes a particular kind of structure that can be attached to an arbitrary set E let Ξ denote the following category: the objects of Ξ are the pairs (E, X) where E is a set and X is a ξ-structure for E; and the maps are a suitable class of functions α: E $\to$ E' that preserve the structure. A ξ-process p on a subcategory Ξ_o of Ξ consists of a covariant functor $P: \Xi_o \to \Sigma$, the category of topological spaces, and a family (p_X), (E, X) in Ξ_o, of functions p_X: E $\to$ P(E, X) such that

(ξP_1) $p_X E$ is dense in P(E, X); and

(ξP_2) if α is a map of Ξ_o in Hom((E, X), (E', X')) then

$$P(\alpha) \circ p_X = p_{X'} \circ \alpha.$$

Two ξ-processes are isomorphic if their associated functors are naturally equivalent in a way that is compatible with the corresponding families.

In chapter one, function processes are considered (i. e. X is always a set S of real-valued functions on E and α: E $\to$ E' is a map if $S' \circ \alpha \subseteq S$). A theoretical introduction is followed by a detailed discussion of several examples (e. g. the process $\mathcal{H}$ which associates with each function algebra S on E the space of multiplicative linear functionals and the obvious map of E into that space) which culminates in an isomorphism theorem for four of the examples. This theorem is then used to discuss the problem:

(EA): Characterize the subalgebras S of $\mathcal{C}_E$ for which there is an extension F of E such that $S = \mathcal{C}_F | E$ (where $\mathcal{C}_E$ is the algebra of continuous real-valued functions on E). A solution is given in terms of internal conditions on S and an external factoring condition.

Chapter two considers the totality of extensions of a completely regular space E. Another solution to (EA) is stated in terms of those "extension algebras" S containing $\mathcal{C}^*_E$ and the uniformly closed unitary subalgebras $\mathfrak{A}$ of $\mathcal{C}^*_E$. The translation lattices

$\mathcal{L}(\mathfrak{A}) = \{f \mid \forall\lambda \geq 0\ (f \frown \lambda) \smile (-\lambda) \in \mathfrak{A}\}$ associated with each $\mathfrak{A}$ are characterized by internal properties. The following problem is shown to be non-trivial. For which Q-spaces E is it true that $\mathcal{C}_E$ is the sole lattice $\mathcal{L}$ of this form satisfying:

(1) $\mathcal{L}$ contains an unbounded function if E is not compact;
(2) $\mathcal{L}$ is closed under addition; and
(3) the weak topology associated with $\mathcal{L}$ is the topology of E?

Chapters three and four are roughly analogous to the first two with sets of functions replaced by uniformities.

The last chapter discusses various methods of compactification in the context of the theory of processes. A topological process p is said to be continuous if each of the functions in the family given by p is continuous. A compactification is defined to be a continuous compact topological process. It is shown that every compactification p may be uniquely decomposed into the function process $\mathcal{H}$ and a suitable functor A: $\Sigma_0 \to \Phi$, the category with objects (E, S), S a set of real-valued functions on E. The functor A is described explicitly for the following compactifications: the Stone-Čech compactification; Alexandroff's one-point compactification; Banaschewski's zero-dimensional compactification; Freudenthal's rim-compact process; and Freudenthal's Λ-compactification.