# A question concerning Aronhold's Theorems on Bitangents

By J. H. GRACE.

(*Received* 23*rd October* 1946. *Read* 6*th December* 1946.)

The question to be discussed is this:—

*Can Aronhold's theorems on the bitangents of a quartic curve of genus three be extended to the tritangent planes of a space sextic of genus four?*

The answer is "No": decisiveness arises from the fact that the gist of Aronhold's results can be stated in a very simple form, namely:—

($\alpha$) *Given seven lines in a plane it is possible to derive from them uniquely and symmetrically a quartic curve that has them for bitangents.*

The analogous statement for genus four is

($\beta$) *Given eight planes in space it is possible to derive from them uniquely and symmetrically a sextic space curve of genus four for which they are tritangent planes.*

The falsity of ($\beta$) may (if the reader pleases) be regarded as the meaning of the "No" above; the assertion ($\beta$) is not *a priori* unreasonable because a tritangent plane imposes three conditions and the curve (the intersection of a cubic surface and a quadric) depends on twenty-four constants.

The conditions of tritangency are complicated enough, however, to make a closer approach to the question of ($\beta$) precarious; accordingly I generalize and restate as follows:—

($\gamma$) *A sextic space curve of genus four cannot stand in unique, symmetric and projective relation to eight given planes.*

Since tritangency is projective the truth of ($\gamma$) will involve the negation of ($\beta$).

To justify ($\gamma$) I observe that the space curve, if there is one, will lie on a unique quadric $Q$ and this in turn will also stand in the three-fold relation above to the eight planes.

Now introduce an ordinary set of coordinates ($x$) and let the equations of the quadric and the planes be

$$Q = 0 \qquad p_r = 0 \qquad r = 1, 2, \ldots 8.$$

Since the relation of $Q$ to the $p_r$ is unique and projective, $Q$ must be a rational covariant of the linear forms $p_r$. Suppose that it is of degree $L_r$ in the coefficients of $p_r$ for each $r$, then from elementary Invariant Theory (or the symbolical notation) we have

$$\Sigma L_r = 4\mu + 2$$

where $\mu$ is the multiplying power of the determinant belonging to the covariant $Q$.

Further, since the relation of $Q$ to the $p_r$ is also symmetrical the various $L_r$ are equal, say to $L$, and hence

$$8L = 4\mu + 2.$$

Such a relation between the positive integers $L$ and $\mu$ is out of the question, accordingly the covariant $Q$ cannot exist and neither can the sextic curve of $(\gamma)$.

The falsity of $(\beta)$ follows at once and this is my thesis.