3. Evolution of Peculiar Red Giant Stars

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EVOLUTION AND MIXING ON THE AGB

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Abstract It is now well known that Nature can make Carbon stars at lower luminosities than can (human) theorists. A number of workers, stimulated by this challenge, have been attracted to the problem. In this paper I review recent evolutionary models of relatively low mass AGB stars, with emphasis placed on the mixing of carbon to the stellar surface. In particular I discuss some recent improvements in the physics used to construct stellar models. These topics include: breathing pulses of the convective core found during core helium exhaustion; the effects of carbon recombination; the occurrence of semiconvection in the region between the two nuclear burning shells; and the importance of mass loss. Recent calculations have successfully produced models of low luminosity Carbon stars. The strengths and weaknesses of these models will be contrasted.

INTRODUCTION

An asymptotic giant branch (AGB) star is one which has recently exhausted its core helium supply. Outside the carbon-oxygen core (the most recent $^{12}C(\alpha,\gamma)^{16}O$ rate predicts about 80% ^{16}O and 20% ^{12}C in this core) is a helium burning shell. On top of this is material which has been processed by the hydrogen burning shell, and thus contains primarily helium, with enhancements of ^{14}N from CN(O) cycling. Surrounding this is the hydrogen burning shell itself, which is eating its way into the envelope, whose composition is that of the ZAMS star, with abundance changes due to the first and, possibly, second dredge-up (for details see Becker & Iben 1979, 1980, Iben & Renzini 1983, hereafter IR83).

Pioneering studies by Iben (1975a, 1975b, 1976) initiated a systematic investigation of AGB evolution. It was known that the helium burning shell suffered periodic thermal instabilities, called "shell flashes" or "thermal pulses". During these the luminosity from helium burning, L_{He}, reaches $\sim 10^7 L_\odot$. The deposition of such large amounts of energy causes a convective zone to form at the base of the helium burning shell, and this intershell convection extends almost to the hydrogen burning shell. This convective pocket contains about 20% carbon by

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mass. During later pulses the temperature at the base of the convective region exceeds 300×10^6K, and the $^{14}N(\alpha,\gamma)^{18}F(\beta^+\nu)^{18}O(\alpha,\gamma)^{22}Ne(\alpha,n)^{25}Mg$ reactions take place, forming free neutrons which can be captured by ^{56}Fe, and result in the formation of s-process elements (*e.g.* Iben 1975a,b). Following the pulse, rapid expansion extinguishes hydrogen burning. As the envelope cools, convection reaches inward, often beyond the extinct hydrogen shell, with the result that fresh helium is "dredged" to the surface. This deep convective envelope can also reach into the ^{12}C-rich region formed by the erstwhile intershell convection, and carbon can be mixed to the surface. This is called the third dredge-up. These models appear, qualititatively, to provide a natural explanation for carbon stars (whose atmospheres show $n(^{12}C)/n(^{16}O) > 1.0$), together with MS and S stars which show enhancements of s-process elements. Indeed, it was shown by Iben (1975a, 1975b) that AGB stars with hydrogen depleted core masses $M_H \gtrsim 0.95$ could form s-process elements in the same relative distribution as seen in the solar system.

The problems came when a quantitative comparison was made between theoretical AGB star distributions (Iben 1981; Renzini & Voli 1981) and observations of Magellanic Cloud stars (Blanco *et al.* 1978, 1980). Theory predicted no carbon stars less luminous than $M_{bol} \simeq -5$ or brighter than -7.5. This contradicts the observed range in M_{bol} of -3.5 to -6. It was clear that carbon stars must be produced at lower luminosities than predicted. Since there is a (linear) relation between M_H and the total quiescent luminosity before a pulse, this is equivalent to saying that the third dredge-up must operate at lower M_H. Although the luminosity varies slightly during a pulse cycle, the "extended post-flash dip" is a little under one magnitude in size, and cannot account for all of the discrepancy, especially since this dip lasts for only about 20% of the pulse cycle.

Later observations by Wood *et al.* (1983) showed that very bright AGB M stars ($M_{bol} < -6.5$) do exist. Possibly hydrogen burning at the base of the convective envelope is turning carbon stars back into M stars at these luminosities (*e.g.* Renzini & Voli 1981). But the number of these stars is small, and it is believed that most AGB stars reduce their envelope mass $M_e (= M - M_H)$ via stellar winds as they ascend the AGB. As M_e approaches zero the star leaves the AGB to become a white dwarf. This may remove from the AGB the only stars shown capable of producing a solar system distribution of s-process elements, *i.e.* those with $M_H \gtrsim 0.95$. The burden then falls on AGB stars of low M_H (and other sources ?) to produce s-process elements *and* low luminosity carbon stars. Recent calculations addressing these problems will be the topic of this review.

INPUT PHYSICS

Core Helium Burning

Before discussing AGB evolution it is prudent to backstep to the core helium burning phase, which immediately precedes the AGB. Calculations by Lattanzio (1986, hereafter JL1) showed that a semiconvective region formed at the edge of the convective core in the (1-3 $M_\odot$) models of that study. This is analogous to the situation known to exist for $M \lesssim M_\odot$, but often ignored in more massive stars (*e.g.* Becker & Iben 1979). Briefly, a semiconvective zone is one which is marginally unstable to convection, but where the act of mixing creates a structure which is radiatively stable (according to the Schwarzschild criterion; for details see Castellani *et al.* 1971a, 1971b). The result is a region where the abundances are adjusted to give precise convective neutrality. The consequences of semiconvection are twofold. Firstly more helium is mixed into the convective core, and upon core helium exhaustion the helium depleted core is larger. Secondly, as a consequence of the increased helium supply, the time spent burning helium is significantly larger ($\sim$ 50% or more), so that the hydrogen burning shell has time to eat further into the envelope. Thus the hydrogen exhausted core is also larger, upon helium depletion, than in models which ignore semiconvection. Obviously, semiconvection will greatly alter the structure of the star at the start of the AGB.

Core Breathing Pulses

An instablility has been found to occur during the final stages of core helium burning (*e.g.* Sweigart & Demarque 1972, 1973; Gingold 1976; JL1). It has been seen that the convective zone grows rapidly, mixing large amounts of helium into the burning region. Sweigart & Demarque (1973) provided a linear stability analysis of the phenomenon, known as "core breathing pulses", and found that the instability is due to the strong dependence of the triple alpha reactions on the helium abundance. A small growth in the size of the convective core causes an increase δY_c in the central helium content Y_c. When $Y_c \lesssim 0.12$ even a small δY_c can result in a large increase in the energy generation, due to the extreme sensitivity of helium burning to the helium content. This extra energy results in the growth of the convective core, which continues the runaway. The instability is quenched when either Y_c is sufficiently large that a small δY_c has a negligible effect, or when the helium stratification in the surrounding layers is no longer capable of providing sufficiently large values of δY_c in response to growth of the convective core. Thus we see that the instability has a physical basis. One should note that it appears in different formulations of semiconvection (*e.g.* Gingold 1976; JL1; Castellani *et al.* 1985a). Note also that it occurs both when convective (and semiconvective) boundaries are obtained implicitly, as in the Robertson & Faulkner (1972) method used by Gingold and others, or explicitly (as in JL1 and Castellani *et al.* 1985a).

A detailed study by Castellani *et al.* (1985a) shows that a model typically experiences three core breathing pulses before finally exhausting its core helium supply. This agrees with JL1 (see Lattanzio 1984 for details) and Mazzitelli & D'Antona (1986). The effects of these convective pulses are analogous to those of semiconvection: an increase in the amount of helium burnt, and consequently an increase in both the hydrogen and helium exhausted cores at the start of AGB evolution. A further consequence of each breathing pulse is a rapid blueward loop in the HR diagram. Typically $\Delta log(L/L_\odot) \simeq \Delta log T_e \simeq 0.1$, with the loop taking some 10^5 years. Such small variation is not open to observational detection, unfortunately.

Convective Overshooting

Almost all stellar structure codes use a mixing length formulation of convection. This "local" theory makes no allowance for the kinematics of the convective motions (velocity, momentum *etc.*), and consequently ignores the possibility of overshoot beyond the formally convective region. Recently Bressan *et al.* (1981) addressed this problem, and included overshooting in their models (see Chiosi *et al.* 1987 for a summary). They find that their models develop neither semiconvection nor core breathing pulses, which they claim are due to using "local" theories of convection. Interestingly, their models complete core helium burning with hydrogen and helium depleted core masses which are very similar in size to models which include both semiconvection and convective pulses. The models seem to be demanding a certain structure. In any event, these models have not yet been evolved into the thermally pulsing regime, and will not be discussed further.

Observations

One can appeal to observations in an attempt to discriminate between various mixing scenarios. A simple test is to determine the ratios of the lifetimes on the AGB and the horizontal branch. This ratio should be equivalent to ratio of the number of stars in these two phases. As summarized by Renzini and Fusi Pecci (1988), the observations seem to favor semiconvection without core breathing pulses, or possibly the overshooting models of Chiosi and co-workers (see Chiosi *et al.* 1987). Why breathing pulses seem to be required, yet do not match the observations, is unknown. Perhaps a better test would be to construct luminosity functions for models with the various forms of convection, and compare these with the observations. This work is in progress (Bertelli *et al.* 1988). Note that it has been recently suggested that core breathing pulses are caused by the assumption of instantaneous mixing (Chieffi and Renzini, private communication).

Carbon Recombination

Thermal pulse calculations by Sackmann (1980) found that the expansion engendered by the thermal pulse could push the carbon-rich region out to very low temperatures ($\sim 10^4$K in that study). The older opacity tables of Cox & Stewart (1970) were then widely in use, yet these only provided opacity of carbon-rich regions for $T > 10^6$ K. Carbon begins to recombine a little below this temperature (*e.g.* Sackmann & Boothroyd 1985), and this would greatly increase the opacity beyond estimates based on the Cox & Stewart tables.

Iben & Renzini (1982a) investigated the effect of this recombination on the electron pressure P_e and the adiabatic gradient. They found P_e decreased by 9% at $T = 2 \times 10^5$K, but by only 1% at $T = 4 \times 10^5$K, with variation being insignificant for higher temperatures. Thus it appears that the effect of recombination in the equation of state may only be important for $T \lesssim 4 \times 10^5$K. The changes in ∇_{ad} were even smaller than the changes in P_e, and can be safely ignored.

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Iben & Renzini included, in an approximate way, recent opacity calculations by Art Cox for carbon-rich mixtures of low temperatures. They found that when the carbon-rich pocket (formed by flash-driven convection during the previous pulse) experienced temperatures below $\sim 5 \times 10^6$K, a semiconvective zone appeared at the outer edge of the carbon-rich zone. Note that for this temperature (and density $\sim$few$\times 10^{-3}$g cm^{-3}) we are still in a region of the (T, ρ) plane which is covered by Cox & Stewart tables. Presumably an important effect operating here is the increase in recent opacity calculations of $\sim$ 50% compared to the Cox & Stewart values (Magee *et al.* 1975). Also, Cox & Stewart give tables for only three mixtures relevant to the carbon-rich region: $(Y, ^{12}\mathrm{C})$=(0.0,1.0), (0.5,0.5), (1.0,0.0). Interploation in these sparse values of the carbon content might also be responsible for previous investigators missing this phenomenon. It is clear that although carbon recombination may be important at later times (the carbon-rich material does cool to few $\times 10^5$K), the initial appearance of semiconvection *may not* be due to the recombination of carbon. Iben and Renzini (1982a) showed that 99.97% of the carbon is still fully ionized at the temperatures and densities where semiconvection first appears. Of course, although the amount of carbon recombined is small, it may still be the primary opacity source.

In any event, the semiconvection mixes carbon outward and hydrogen inward, the latter being the more extensive. Some carbon is mixed outward by $\sim 2\times 10^{-5} M_\odot$, sufficiently far to make contact with the (inward moving) convective envelope, with the result that carbon is mixed to the surface by the usual dredge-up (Iben & Renzini 1982a,b; hereafter IR82a, and IR82b, respectively). The entry of hydrogen into a region rich in carbon causes the $^{12}\mathrm{C}(p,\gamma)^{13}\mathrm{N}(\beta^+\nu)^{13}\mathrm{C}(p,\gamma)^{14}\mathrm{N}$ reactions to consume all the hydrogen when this region is heated later in the pulse cycle. As a consequence, a few $\times 10^{-4} M_\odot$ now shows the abundance of ^{13}C exceeding that of ^{14}N. During the next thermal pulse temperatures rise enough to ignite $^{13}\mathrm{C}(\alpha,n)^{16}\mathrm{O}$, which releases free neutrons for capture by ^{56}Fe with the potential for forming s-process elements (*e.g.* IR82b).

Unfortunately, Iben & Renzini were forced to terminate their calculations because of convergence difficulties caused by instantaneous mixing of regions of vastly different composition. Hollowell (1987, 1988, hereafter DH1 and DH2, respectively) has repeated these calculations with detailed opacity and an algorithm designed to overcome the mixing problems. This work will be discussed below.

While discussing opacity, it should be noted that dredge-up of carbon significantly alters the composition of the envelope. One can find the "metallicity" $Z = 1 - X - Y$ increasing by a factor of 2 (*e.g.* Boothroyd & Sackmann 1988d). In calculating the opacity of these mixtures the significant abundance of carbon should be included. One should also check on the effects of carbon recombination on the equation of state. We have seen that this can be ignored for mixtures containing as much as $\sim$ 20% carbon provided that temperatures remain above a few $\times 10^5$K (IR82a). But the envelope will reach down to a few thousand degrees. Of course, although carbon may be the main contributor to Z, it is still a small amount of the total mass of the envelope ($Z \approx 10^{-3}$).

EVOLUTIONARY RESULTS

A brief description of the structure of an AGB star and the evolution through one pulse was given in the Introduction. Because this evolution is well understood, the reader is referred to other reviews for details (*e.g.* IR83; Iben & Renzini 1984; Iben 1984, 1987). We will concentrate on the results of the recent calculations of Lattanzio (1986, 1987a, 1987b, 1988, hereafter JL1–4, respectively), Boothroyd & Sackmann (1988a–d, hereafter BS1–4, respectively), and Hollowell (DH1 and DH2). Differences between the physics used by these investigators will be discussed below, as will the differences in the carbon stars that resulted.

One very important parameter is the mass of the hydrogen exhausted core at the first thermal pulse, M_H^{TP}. The models of JL, BS and Mazzitelli & D'Antona (1986) are all in agreement, showing that M_H^{TP} is approximately independent of mass for solar metallicity and masses in the range 1–3$M_\odot$, as stressed by JL1. A second important result is the strong dependence on total mass for the $Z = 0.001$ models (JL1, BS3). Note also that attempts by IR83 to estimate M_H^{TP} for stellar masses in the range 1–3$M_\odot$ based on the results for more massive and less massive stars fail to give accurate results (JL3, JL4, BS3). Since this is an important parameter in synthetic AGB star distributions (Iben 1981; Renzini & Voli 1981) these calculations will be significantly altered because of the new results.

Another critical number is the maximum (quiescent) luminosity L_{TP} reached by the models just prior to the first thermal pulse (see figures in JL2, JL3 and BS3). The new determinations are 1 to 2 magnitudes below the IR83 estimate, based on the only models available at that time, which were outside the required mass range. This is encouraging, as it allows some time for the models to experience dredge-up before reaching the luminosity of carbon stars. Again, synthetic distributions using these results should be better able to explain the observations.

An important effect included in the BS models is mass loss. During the latter stages of AGB evolution mass loss is more important in reducing the envelope mass than is the advance of the hydrogen shell (Schonberner 1979), even before the "superwind" phase (*e.g.* IR83). The reduction of envelope mass complicates the prediction of evolutionary behaviour. It has been shown (*e.g.* Wood 1981) that models with smaller envelope masses are less likely to experience dredge-up than are stars with a larger M_e (given the same M_H). But when/if carbon dredge-up begins, the reduced envelope mass means a smaller dilution of the added carbon. Hence less carbon is needed to form a carbon star, and thus fewer pulses are necessary than when mass loss is ignored.

A second significant effect is that a star can only remain on the AGB provided $M_e > 0$! Consequently mass loss will completely terminate the AGB phase, for a given mass, much earlier than when mass loss is ignored. BS3 note that their 3$M_\odot$ model with $Z = 0.001$ shows M_H^{TP} larger than the expected final mass for such a star, based on the observed initial–final mass relation. Obviously, if this is correct, such a star should not experience the third dredge-up at all. BS3 believe, however, that they have overestimated the rate of mass loss, probably by as much as a factor of 2. BS used the Reimers formula for mass loss, and took 0.4 as the value of the parameter η, which enters this formula (the larger value of 1.4 was used for the $3M_\odot$ models). This was determined by previous calculations as the value needed to match the mass loss occurring during the ascent of the giant branch. But this calibration was made for an α, the ratio of the mixing length to the pressure scale height, of 1.5, whereas BS used $\alpha = 1.0$. Perhaps more importantly, BS included the effects of some molecules in the envelope opacities which they used. Both of these effects will directly alter the stellar radius, which enters the Reimers formula. It seems probable that, under these circumstances, a smaller value of η would be needed.

In summary, it is clear that mass loss is important for (at least) three reasons: 1) the reduction in M_e makes dredge-up less likely; 2) with a smaller envelope mass fewer pulses are needed to produce a carbon star; 3) mass loss terminates the AGB evolution at some stage, thus limiting the number of thermal pulses which a star can experience. We should also note the apparent dependence of some characteristics of the evolution on the total mass and past history of the star (BS3, DH2). This makes it dangerous to neglect mass loss and artificially alter the envelope mass (for a given core mass) in an attempt to explore parameter space.

DREDGE-UP AND CARBON STARS

Before discussing the dredge-up found in recent models we should recall some basics already well understood. For example, dredge-up is more likely to occur in models with higher core masses. For a given core mass, dredge-up is favored by lower Z, larger α, or increased envelope mass (*e.g.* Wood 1981, IR83).

The Models of Lattanzio

These included semiconvection and core breathing pulses during the core helium burning phase. The opacities are from Huebner *et al.* (1977). No allowance is made for the effect of any extra carbon which may be dredged into the envelope. No opacities are calculated for carbon-rich mixtures below 10^6K, but the models of JL never entered this regime. The opacity of the carbon-rich matter is calculated with the Huebner *et al.* code, but at the same (T, ρ) and abundances as the older Cox & Stewart values (see above). Care was taken to accurately obtain the core masses at the core helium flash (see JL1 for details).

It was found that many variables previously thought to depend only on core mass (*e.g.* luminosity, interpulse period) also showed a dependence on abundance. For example, M_H^{TP} increases by $\sim 0.05M_\odot$ when Y increases from 0.2 to 0.3. Including these effects in calculations of AGB star luminosity functions will be necessary.

The encouraging results from these models include the reduction of L_{TP} and the new M_H^{TP}, discussed in detail above. Calculations of the thermally pulsing evolution of $1.5M_\odot$ models, without mass loss, showed dredge-up at core masses as low as $0.62M_\odot$ (JL3). Three of the four models of that study ($Z = 0.003$ and 0.006, each with $Y = 0.2$ and 0.3, $M = 1.5, \alpha = 1.5$) were found to dredge carbon to their surfaces. Calculations were stopped when one of the models became a carbon star, with M_{bol} dropping to -4.4 after the pulse, and $M_H = 0.65$ at this time. This luminosity agrees very well with estimates of the transition luminosity between M and C stars (see JL3 for details). (There is no reason to believe that the fourth model, whose quiescent luminosity had reached $M_{bol} = -4.8$ when calculations were stopped, would not also have experienced dredge-up in the future.)

These models may be criticized on a number of counts. Firstly the effective temperature of JL3's carbon star is $\log T_e \simeq 3.56$, but the observations show $\log T_e \lesssim 3.5$ (*e.g.* Richer 1981). Ignoring uncertainties in the temperature calibration, agreement could be forced if $\alpha \simeq 1.25$. Note that JL1 found no dredge-up for $\alpha = 1.0$, while the carbon star of JL3 used $\alpha = 1.5$. It is unknown if dredge-up would occur at this intermediate value of α. Of course, the inclusion of molecular opacities and other neglected envelope effects, including the enhanced carbon abundance, may aid this situation (JL4), so perhaps the disagreement is not serious.

Secondly, these models ignored mass loss. Consequently the large envelope mass will be acting in favour of dredge-up at lower luminosities. But we have shown earlier how the larger M_e also hinders carbon star formation because of the much larger dilution factor. It is not clear which of these effects dominates.

Thirdly, temperatures in the carbon rich region were always $< 200 \times 10^6$K, and hence too low to ignite the ^{22}Ne source. No semiconvection has been found, and thus the Iben/Renzini/Hollowell mechanism is not operating. Consequently the ^{13}C neutron source is not operating in these models either. Thus these calculations would produce carbon stars without s-process elements, contrary to the (available) observations (Smith & Lambert 1986). The resolution of this discrepancy (if, indeed, there are no C-rich stars without s-process element enhancements) is unclear. We have already discussed the possibility that inaccuracies in interpolation within opacity tables may be responsible. Also, it may be that stronger pulses are needed to push the carbon rich mixtures out to temperatures low enough for semiconvection to occur. That Wood and Lattanzio, using essentially the same code, obtain less violent pulses than do Iben and Hollowell, using essentially the same code, indicates that a detailed comparison between codes may be required. Thermal pulses are a demanding phase of stellar evolution, and small differences in codes may cause large differences in calculated behaviour. Note also that it appears that semiconvection does not occur over a wide range of mass and composition (Iben 1983), and seems to require total masses < 1.0 and $Z \simeq 0.001$ (see also DH2).

It is clear that a larger survey of parameter space is required to make definite conclusions. Note also that JL1 found no dredge-up for the $1.5M_\odot$ models over a wide range of Z, with $\alpha = 1.0$. It seems likely that these models would experience

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the third dredge-up for $\alpha = 1.5$, at least for the lower metallicity models.

The Models of Boothroyd & Sackmann

BS1–4 investigate AGB evolution of models with $Z = 0.001$ and 0.02, and initial masses of 1–3 $M_\odot$. These calculations included semiconvection during the core helium-burning stage, but experienced convergence difficulties during the core breathing pulses. Consequently these were suppressed in most calculations. They used the latest Los Alamos opacities, with effects of some molecules included at the lowest temperatures (see Sackmann & Boothroyd 1985). The opacity of any carbon dredged to the surface was included approximately by using an opacity table for a metallicity $Z = 1 - X - Y$, even though much of the "Z" is pure carbon. Nevertheless, this is a better approximation than ignoring the effect, as everyone else has done. Opacities for carbon-rich mixtures were included for a wide range of temperatures, thus allowing for carbon recombination during the expansion following a thermal pulse. (Again, the opacity tables used here were available for a much denser sampling in carbon abundance than used by Lattanzio and Wood.) Likewise, the effect of carbon recombination on the equation of state has been included in the carbon-rich region (although it is believed to be small, IR82a). A Reimers mass-loss formula was included, although the value of η used is believed to be too large, possibly by as much as a factor of two. As in the calculations of JL2 (and some of JL1), the models are evolved from the ZAMS. Note that BS evolved their models through the core helium flash, whereas JL "jumped over" it, after obtaining the core mass and envelope abundance changes. This difference is negligible. Incidentally, it is worth noting that BS3 find that the helium flash is ignited in the center of their models, contrary to all other calculations including neutrino losses (except for Mazzitelli & D'Antona 1986). Although in subsequent calculations one model ignited carbon off-center (Boothroyd, private communication), the reason for this difference with virtually all other codes is unknown. (Unpublished calculations by Lattanzio also showed off-center ignition.) BS3 state that a possible reason is the use of different screening corrections.

The calculations of BS show the importance of mass loss, as mentioned earlier. Even though they have overestimated its effectiveness, the reduction of the envelope mass severely limits the total number of pulses experienced on the AGB. They showed that only models with initial masses $\lesssim 2M_\odot$ can become low luminosity carbon stars and still satisfy the observed initial-final mass relation of Weidemann & Koester (1983). They also show that the flash strength, as measured by the maximum luminosity due to helium burning, L_{He}^{max}, is not simply a function of M_H, but depends on the total mass and the composition. This, unfortunately, means one cannot adjust the envelope mass of a given model in the hope that the result will accurately reflect the effects of mass loss. This is especially true of the dredge-up phase, which depends very sensitively on the flash strength.

BS3 showed that the composition of the carbon rich region is virtually independent of both stellar mass and composition, being approximately 20% carbon and only

$\sim 2\%$ ^{16}O, even with the increased rate for the $^{12}C(\alpha,\gamma)^{16}O$ reaction. For models with $Z = 0.001$ a semiconvective region formed at the top of the carbon pocket, as found by IR82a,b and DH1,2. However the models of BS lacked the fine resolution (in mass co-ordinate) obtained by Hollowell, and this may be why they find an order of magnitude less ^{13}C than DH2.

For models with $\alpha = 1.0$ BS4 find no dredge-up of carbon. For their model with $(M, Y, Z) = (2.0, 0.24, 0.001)$ on the ZAMS it was necessary to increase α to 1.5 to obtain dredge-up. This change was made between the 9^{th} and 10^{th} pulses, and resulted in a carbon star of $1.72 M_\odot$ with a minimum luminosity of $M_{bol} = -4.68$ during the extended post-flash dip. Interestingly, and possibly quite importantly, no subsequent pulses produced dredge-up. BS claim that this is due to the increased metallicity (carbon) in the envelope, and also the reduction of envelope mass due to mass loss. Yet neither of these effects were included in the models of Iben (1983), who observed similar behaviour.

No dredge-up had been found during the evolution of the model with ZAMS values $(M, Y, Z) = (1.2, 0.24, 0.001)$. The evolution was repeated with α=1.5, 2.0 and 3.0, the change to new values of α being made between the fifth and sixth pulses. Dredge-up was obtained only for $\alpha = 3.0$, with the model becoming a carbon star on the next pulse and with M_{bol} dropping to -3.59. Again, further dredge-up did not occur. It is worth noting that in both of the sequences which became carbon stars, no semiconvection was seen when carbon was actually dredged to the surface, although it was seen in both subsequent and previous pulses.

Although BS succeeded in making low luminosity carbon stars, the requirement of α as large as 3.0 is disturbing. It seems that $\alpha \simeq 1.5$ is capable of matching models to observations over a surprisingly large range of evolutionary stages (*e.g.* Wood 1981, JL1). Nevertheless, BS4 remind us that various constants of order unity enter the mixing length theory of convection, and different implementations may not match exactly. BS4 state that $\alpha_{Iben} = 1.5$ is equivalent to $\alpha = 2.5$–3.0 in "all other codes that we are aware of". Yet Hollowell, using an Iben code, does not find that $\alpha_{Iben} = 1.5$ is *sufficient* to induce dredge-up in a model similar to the BS4 model which required $\alpha = 3$ (actually, all one can say is that the critical value of α is between 2 and 3). In summary, there does seem to be some variation in models constructed with different codes but "identical" α's. A source of calibration could be the construction of a standard solar model. Lattanzio needs $\alpha = 1.42$ (Lattanzio 1984), BS3 needs $\alpha \simeq 2$, while it is unknown what value the current Iben code would require. VandenBerg (1983) needs α =1.4–1.5. Perhaps a calibration with the same opacities and abundances would be a worthwhile exercise.

Encouraging results of the BS study include confirmation of the composition dependence found by Lattanzio. Also note that the models of JL1 which were claimed to have reached full flash amplitude probably have not, as noted by BS2 and BS3. BS find L_{TP} values in good agreement with JL1 and JL2, and the importance of this has been discussed above. On the negative side is the fact that BS may need large values of α to obtain dredge-up in lower mass stars (but see below). Finally, the importance of mass loss has been stressed by BS, although this can work both for and against carbon star formation.

The Models of Hollowell

Hollowell (see DH1 and DH2) has carefully investigated the effect of carbon recombination on opacities and the semiconvection first obtained by IR82a,b. These models use the latest Los Alamos opacities, including (fits to) tables for carbon-rich mixtures (which are provided for many carbon abundances, probably allowing for more accurate determination of the carbon dependence than in Lattanzio's models). No allowance is made for the effect of carbon recombination on the equation of state (believed to be small, IR82a), nor for any carbon added to the envelope as a result of dredge-up. Mass loss is not included. The model studied by Hollowell was previously studied by Iben (1982, 1983), where it is stated that the model had been evolved from the ZAMS by Despain, although we are not told if semiconvection (or core breathing pulses) were found. Note that Despain (1981) did include semiconvection in calculations of a 0.6 $M_\odot$ model, and it would be expected to occur in a $0.7 M_\odot$ star also.

DH2 shows the opacity for carbon rich mixtures for $Z = 0.001$ and $Z = 0.02$. The opacity bump due to recombination of carbon is seen near 10^6K in the $Z = 0.001$ mixture but is not obvious in the more metal rich mixture. This explains why semiconvection has so far only been seen in models with metallicity $Z = 0.001$.

In an attempt to minimize convergence difficulties found by IR82a, due to instantaneous mixing of large regions of very different abundance, Hollowell has developed a "random walk" model for time-dependent convection. In this formulation convection mixes abundances only over a finite region L_{mix}, determined by the time-step and the convective velocity (see DH2 for details), rather than instantaneously throughout the entire convective zone. While one may criticize this (or any) particular formulation, it is physically motivated and not unreasonable.

Basically, Hollowell confirms the picture painted by Iben & Renzini. After a pulse the top of the carbon rich pocket becomes semiconvective (at $T \simeq 5 \times 10^6$K), mixing carbon outward by a small distance (in mass). Hollowell does not find that carbon is mixed sufficiently far for the inner edge of the convective envelope to penetrate the carbon enhanced zones, as necessary for the third dredge-up. Note that this is despite using $\alpha_{Iben} = 1.5$. The semiconvection does, however, mix hydrogen inward quite a distance (see DH1 and DH2) This results in the ignition of the ^{13}C neutron source, and the formation of s-process elements during subsequent AGB evolution. (Note that DH2 and Hollowell & Iben 1988 provide a detailed analysis of the nucleosynthesis occurring in these models, including the formation of s-process elements. This will not be discussed in this paper.)

Motivated by the fact that low luminosity carbon stars do exist, DH2 then repeated the calculations but with convective motions overshooting by a distance L_{mix}, which was never allowed to exceed one pressure scale height. In this case Hollowell obtained dredge-up of both carbon and s-process elements. His model became a carbon star with a post-flash luminosity reaching down to $M_{bol} = -4.3$ and $M_H = 0.639$. While it seems unfortunate that some convective overshooting is required to obtain dredge-up, there can be little doubt that this is a real phenomenon. Only the precise extent and details are unknown. Nevertheless, it is encouraging that stars of such low mass ($0.7M_\odot$) can become carbon stars at the luminosities required by the observations, and can indeed be a source of s-process elements.

SUMMARY

One thing is clear from the calculations discussed above. Theory is now in a much better position to confront the observations. The parametrized input used in the synthetic AGB star distributions of Iben (1981) and Renzini & Voli (1981) has been shown to be inaccurate for (initial) masses in the range 1–3 $M_\odot$, which probably form the bulk of the Magellanic Cloud carbon stars.

Each of the recent sets of calculations (Lattanzio; Boothroyd & Sackmann; Hollowell) succeeds in producing carbon stars of quite low luminosity. The models of Lattanzio have larger envelope masses than appropriate, however. This favors dredge-up but delays carbon star formation. The Boothroyd & Sackmann models may require large values of α to obtain dredge-up at low luminosities. But recall that BS included molecular opacity sources in their envelopes. This cools the envelope substantially. To return the envelope to the original temperature would require a larger α. That BS need $\alpha \approx 2$ to make a solar model while other codes need 1.4–1.5 is consistent with this picture. Perhaps we should not be too hasty in criticizing larger α in the BS code. Hollowell also forms carbon stars (and s-process elements), but requires some form of convective overshooting. That overshooting is real is not denied, but the extent is unknown and a good understanding is lacking.

Since different evolutionary codes differ in some respects (such as L_{He}^{max}), it may be worth making a detailed comparison between codes. One identical model should be distributed to each investigator, and the subsequent evolution compared. The insights gained would make this a valuable exercise, and aid future comparisons.

Let us now discuss the physics which should be included in the ideal AGB star stellar structure code. Firstly, the models should be evolved from the ZAMS, to accurately find the core mass at ignition of the core helium supply. Mass loss should be included, as should semiconvection. We must determine if core breathing pulses occur, and if so include them. (If not, then we must understand why our present understanding predicts them.) Of course, the most accurate opacities and nuclear reaction rates must be included. One should include opacities for carbon rich regions, and also allow for the opacity of any carbon which is mixed into the envelope. The effect of carbon recombination on the equation of state will be important for the carbon pocket at temperatures below few$\times10^5$K, and must be included if these temperatures are reached. Although the carbon added to the envelope constitutes only a small fraction by mass, it will be experiencing temperatures down to a few thousand degrees. At these temperatures the recombination of carbon may effect the equation of state. The formation of various carbon molecules in the envelope will effect the opacity, and must be included. (With these opacity sources it will probably be necessary to recalibrate the values of α and η needed to match the observations.) From a numerical point of view, it should be noted that some authors allow a model to converge *before* finding the convective boundaries (*e.g.* BS3; Becker & Iben 1979). Mixing of the abundances is then performed between those boundaries. The resultant model is somewhat inconsistent, because the abundances feed into the structure equations, which have been solved for a different composition than is finally indicated after mixing. Although the errors will probably be small for small time-steps, a better procedure (somewhat more expensive in computer time, but not excessively) is to calculate the convective boundaries after each iteration, and perform any mixing at that time (JL1). Thus the model is internally consistent.

To include all these modifications in any stellar structure code would be a laborious exercise, but it is also bound to be fruitful. Note that some of these effects have been included in recent calculations, but no-one has considered all of them. In this sense, the works of Lattanzio, Boothroyd & Sackmann, and Hollowell are complimentary, each addressing some different aspect of AGB star evolution.

Finally, in the hope that we have now solved the low luminosity carbon star problem (or that the "ideal" code described above will solve it), what can we say about the absence of high luminosity carbon stars ? Mass loss has long been suspected to be at least partly to blame, the implication being that the rate of mass loss must be higher than used in the synthetic distributions of Iben (1981) and Renzini & Voli (1981). A recent, and intriguing, development has been the suggestion that previous estimates of $M_{up}(= M_5$ in JL4), the maximum (initial) stellar mass which will develop a degenerate carbon-oxygen core, have been too large (Renzini *et al.* 1985). Because stars more massive than this will ignite their core carbon supply, M_5 is the maximum mass star which can appear on the AGB. Certainly core breathing pulses and convective overshooting both act to reduce M_5 (Castellani *et al.* 1985b, Chiosi *et al.* 1987). Once a star reaches the AGB, of course, it will be mass loss which determines the maximum luminosity which it will attain. But with the most massive stars denied passage to this phase we may now understand why there are so few very bright ($M_{bol} < -6$) AGB stars.

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