5
Target response surfaces

As we shall see, the target response in inclusive electron scattering $(\mathrm{e}, \mathrm{e}')$ is summarized in the following Lorentz tensor

$$\begin{aligned} W_{\mu\nu} &= \frac{(2\pi)^3}{\hbar c} \overline{\sum_i} \sum_f \langle i|\hat{J}_\nu(0)|f\rangle\langle f|\hat{J}_\mu(0)|i\rangle(\Omega E)\delta^{(4)}(p'-p-k) \\ &= W_1(k^2, k\cdot p)\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) \\ &\quad + W_2(k^2, k\cdot p)\frac{1}{M_T^2}\left(p_\mu - \frac{p\cdot k}{k^2}k_\mu\right)\left(p_\nu - \frac{p\cdot k}{k^2}k_\nu\right) \end{aligned} \tag{5.1}$$

The cross section is expressed in terms of the two, two-dimensional response surfaces as

$$\begin{aligned} \frac{d^2\sigma}{d\Omega_2 dk_2} &= \sigma_M \frac{1}{M_T}\left[W_2(k^2,\nu) + 2W_1(k^2,\nu)\tan^2\frac{\theta}{2}\right] \\ \sigma_M &= \frac{\alpha^2\cos^2\theta/2}{4k_1^2\sin^4\theta/2} \end{aligned} \tag{5.2}$$

Here $k_1 = |\mathbf{k}_1|$. The square of the four-momentum transfer is given for a relativistic electron by

$$\begin{aligned} k^2 &= (\mathbf{k}_1 - \mathbf{k}_2)^2 - (k_1 - k_2)^2 \\ &= 4k_1k_2\sin^2\theta/2 \qquad ; \text{ in laboratory} \end{aligned} \tag{5.3}$$

The second Lorentz scalar is written as the kinematic variable

$$\begin{aligned} \nu &\equiv -k\cdot p/M_T \\ &= k_1 - k_2 \qquad ; \text{ in laboratory} \end{aligned} \tag{5.4}$$

There are three lepton variables in electron scattering, the initial and final electron energies ε_1 and ε_2 and the scattering angle θ, or equivalently

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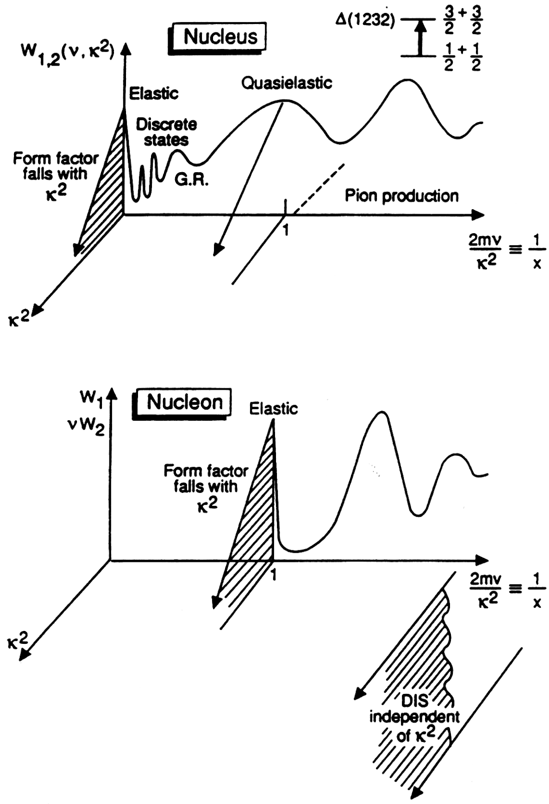

Fig. 5.1. Qualitative sketch of response surfaces $W_{1,2}(\nu,k^2)$ for nuclei and nucleons. One axis is the square of the four-momentum transfer k^2 (denoted in this figure by κ^2), the other is $2M\nu/k^2 = 1/x$.

(k^2,ν,θ). The two response surfaces can be separated by varying the electron scattering angle θ at fixed ν and k^2. Alternatively, one can work at back angles $\theta = \pi$ where only the term in W_1 contributes.

For orientation, a qualitative sketch of the response surfaces $W_{1,2}(\nu,k^2)$ for electron scattering (e,e') from both nuclei and nucleons is given in Fig. 5.1.[1] For a nucleus, one has the following features. First there is elastic scattering with a form factor that falls in the k^2 direction indicating the extended charge distribution in the target. One then sees inelastic scattering leading to excitation of discrete nuclear levels. The form factors for these inelastic transitions characterize the spatial distribution of the *transition* charge and current densities. At higher energy loss, above particle

[1] Electrons are light and radiate as they scatter; these *radiative corrections* must always be unfolded from the data before one gets at the underlying nuclear physics. We go into this in some detail in the section on QED.

emission threshold, one observes nuclear giant resonances (GR) with broader widths. The subsequent quasielastic peak is essentially scattering from a free nucleon, Doppler broadened by the Fermi motion of the nucleons in the nucleus. At energy losses higher than the pion mass, pion production occurs. At still higher energy loss, one observes production of the internal excitations of the nucleon itself, the first and most prominent being the $\Delta(1232)$ with $(J^\pi, T) = (3/2^+, 3/2)$. The k^2 dependence of the form factors for the excitation of nucleon resonances characterizes the spatial distribution of the transition charge and current densities in the nucleon.

For a single nucleon target, one sees first elastic scattering with a form factor which falls with k^2, again indicating a spatially extended structure in the nucleon — Robert Hofstadter won the Nobel prize for the measurement of the charge distributions of nuclei and the charge and magnetization distributions of the nucleon. At sufficiently high energy loss there is production of the nucleon resonances with the characteristic k^2 dependence of the inelastic form factors. Since all the nucleon resonances lie above the pion production threshold, they have strong-interaction widths. While the $\Delta(1232)$ appears as a distinct isolated peak, the higher nucleon resonances, as with giant resonances in nuclei, present multiple broad overlapping structures.

With higher energy accelerators, one can push into the region of deep-inelastic scattering (DIS) where the electron energy loss gets very large $\nu \to \infty$ and the four-momentum transfer also grows very large $k^2 \to \infty$ but where the ratio of these quantities $x \equiv k^2/2M\nu$ is fixed at a finite value

$$\begin{aligned} \nu \equiv -k \cdot p/M &\to \infty \\ k^2 &\to \infty \\ x &\equiv k^2/2M\nu \qquad ; \text{ fixed in DIS} \end{aligned} \tag{5.5}$$

In DIS something quite remarkable happens. The two response surfaces are *independent of* k^2 and satisfy *Bjorken scaling*, becoming finite functions of the single variable x [Bj69, Fr72]

$$\begin{aligned} \frac{\nu}{M} W_2(k^2, \nu) &\to F_2(x) \\ 2W_1(k^2, \nu) &\to F_1(x) \qquad ; \text{ Bjorken scaling in DIS} \end{aligned} \tag{5.6}$$

The fact that the structure functions become independent of k^2 indicates that the objects inside the nucleon from which one is scattering have no spatially extended structure, that is, one is scattering from *point-like constituents*. Friedman, Kendall, and Taylor won the Nobel prize for their

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discovery of this dynamic evidence for a point-like quark substructure of the nucleon.

As we shall see, scattering of polarized electrons on polarized targets allows one to access additional spin structure functions.