Effects of climatic perturbations on the equilibrium-line altitude, West Greenland

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ABSTRACT. Perturbation analysis is applied to the heat balance at the EGIG line (West Greenland) with respect to climatic changes. The shift of the equilibrium-line altitude is obtained for perturbations of temperature, absolute humidity, cloudiness and annual accumulation. The total net ice ablation at the EGIG line increases by 45% with a perturbation of $+1\,K$. Applying this figure to the entire ice sheet, the total ablation equals the total accumulation at a warming of $+2\,K$, if refreezing of melt water is neglected.

LIST OF SYMBOLS

A Total net ablation
C Total net accumulation
c Annual cumulative accumulation
H Heat-flux density
h Altitude related to the equilibrium line
Δh Difference in altitude related to the present equilibrium-line altitude
k Non-dimensional number
L Specific heat of melting
T_a Air temperature
w Cloudiness
z Altitude
γ Factor of proportionality
μ Factor of proportionality
ρ_v Absolute humidity
τ Number of ablation days

1. INTRODUCTION

The response of the Greenland ice sheet to the increase of CO_2 and other atmospheric greenhouse gases is of recent interest (United States. Department of Energy, 1985). Climatic warming is closely connected with alterations in the heat balance and the shift of the equilibrium-line altitude. For the calculation of the shift of the equilibrium-line altitude due to climatic perturbations, a concept introduced by Kuhn (1981) was applied and adjusted to the conditions on the Greenland ice sheet (Ambach, 1989). The equilibrium-line altitude is essentially determined by the heat balance and the annual accumulation. At the equilibrium-line altitude, the heat of melting, supplied by the heat balance during the entire ablation season, equals the heat consumed for melting the annual accumulation including superimposed ice. The heat balance reads

$$\tau_0 H_0 = kLc_0. \tag{1}$$

τ is the number of ablation days, H is the daily heat-flux density of melting averaged over the ablation period, k is a factor related to the formation of superimposed ice, i.e. $1 \le k \le 5/3$ (Ambach, 1985), L is the specific heat of melting and c is the annual accumulation. The subscript zero refers to the steady-state conditions. τ, H and c are functions of altitude obtained from measurements, and affected by climatic perturbations.

2. CLIMATIC PERTURBATIONS

In Kuhn's concept, the following climatic perturbations were introduced (Ambach, 1989):

Perturbations of air temperature (δT_a) with effects on H and τ.

Perturbations of the absolute humidity ($\delta \rho_v$) with effects on H.

Perturbations of the cloudiness (δw) with effects on H.

Perturbations of the annual accumulation (δc).

It is true that

$$\delta H = \mu_1 \delta T_a + \mu_2 \delta \rho_v + \mu_3 \delta w \tag{2}$$

$$\delta \tau = \gamma \delta T_a. \tag{3}$$

The numerical values of μ_1, μ_2, μ_3 and γ are contained in previous work, when μ_1, μ_2 and μ_3 were introduced as constants and γ as a function of altitude (Ambach, 1989). The data are based on measurements of the heat balance, carried out at 1013 m a.s.l. during the International Glaciological Greenland Expedition (EGIG–1959;

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Ambach, 1963). Therefore, the results of the perturbation analysis are strictly valid for the EGIG line only.

Applying perturbation analysis, Equation (1) reads

$$[\tau_0 + \delta\tau + (\partial\tau/\partial z)\Delta h][H_0 + \delta H + (\partial H/\partial z)\Delta h]$$

$$= kL[c_0 + \delta c + (\partial c/\partial z)\Delta h]. \quad (4)$$

This formulation contains the climatic perturbations $\delta\tau$, δH, δc and the respective alterations $(\partial\tau/\partial z)\Delta h$, $(\partial H/\partial z)\Delta h$ and $(\partial c/\partial z)\Delta h$ due to the shift of the equilibrium line. Here z is the altitude and Δh is the shift of the equilibrium-line altitude.

3. SHIFT OF THE EQUILIBRIUM-LINE ALTITUDE BY CLIMATIC PERTURBATIONS

By applying the numerical values of $\partial T_a/\partial z$, $\partial\rho_v/\partial z$, $\partial w/\partial z$, $\partial\tau/\partial z$ and $\partial c/\partial z$ (Ambach, 1989), the shift of the equilibrium-line altitude is obtained for various perturbations (Table 1). The perturbation $\delta T_a = +1\,\text{K}$ has the biggest effect on the shift of the equilibrium-line altitude, since a rise in temperature leads to an increase in the heat-flux density of melting and a prolongation of the ablation period. Model calculations regarding the greenhouse effect have shown that in higher latitudes a warming of the lower atmosphere by a few degrees is likely (Manabe and Wetherald, 1975). The perturbation $\delta\rho_v = +0.25\,\text{g m}^{-3}$ corresponds to a 5% increase in

Table 1. Shift of the equilibrium-line altitude by climatic perturbations at the EGIG-line with $\partial c/\partial z = 0.55\,kg\,m^{-2}\,m^{-1}$ and $\partial T_a/\partial z = -0.0073\,K\,m^{-1}$

δT_a	$= +1\,\text{K}$	Δh	$= +87.5\,\text{m}$
$\delta\rho_v$	$= +0.25\,\text{g m}^{-3}$	Δh	$= +9.8\,\text{m}$
δc	$= +50\,\text{kg m}^{-2}$	Δh	$= -24.3\,\text{m}$
δw	$= +1/10$	Δh	$= -4.9\,\text{m}$

relative humidity at 0°C. The effects of changes in cloudiness are small and are therefore neglected in the following. The inconsiderable influence of cloudiness on the shift of the equilibrium-line altitude is due to the "radiation paradox" (Ambach, 1974). It implies that shortwave radiation decreases with increasing cloudiness, whereas longwave radiation increases so that a compensation effect occurs. Thus, the net radiation balance increases with cloudiness at an albedo larger than 0.66 and decreases at an albedo smaller than 0.66. The perturbation $\delta c = +50\,\text{kg m}^{-2}$ is approximately 10% of the annual accumulation at the present equilibrium-line altitude. All perturbations occur with mutual feed-back mechanisms. Very likely, an increase in temperature is coupled with an increase in absolute humidity, cloudiness and precipitation (Letréguilly and others, 1991a).
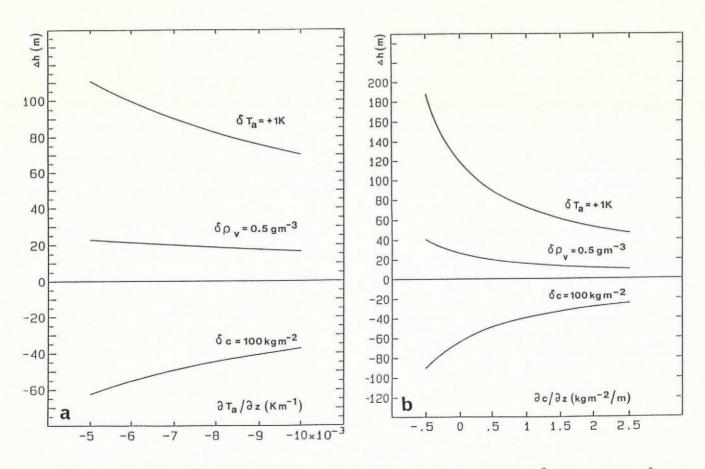

Fig. 1. a. Shift of the equilibrium-line altitude by perturbations $\delta T_a = +1\,K$, $\delta\rho_v = +0.5\,g\,m^{-3}$ and $\delta c = 100\,kg\,m^{-2}$ as a function of $\partial T_a/\partial z$. b. Shift of the equilibrium-line altitude by perturbations $\delta T_a = +1\,K$, $\delta\rho_v = +0.5\,g\,m^{-3}$ and $\delta c = 100\,kg\,m^{-2}$ as a function of $\partial c/\partial z$.

4. SHIFT OF THE EQUILIBRIUM-LINE ALTITUDE AND REGIONAL PARAMETERS

The gradients $\partial T_a/\partial z$ and $\partial c/\partial z$ are regarded as regional parameters. They are hardly influenced by climatic changes. On the Greenland ice sheet, various ablation regimes occur, which are characterized by the values of $\partial T_a/\partial z$ and $\partial c/\partial z$. In the following, the limiting values $-0.0073 \leq \partial T_a/\partial z \leq -0.0057\,\text{K m}^{-1}$ and $-0.32 \leq \partial c/\partial z \leq +0.55\,\text{kg m}^{-2}\,\text{m}^{-1}$ are introduced. With these limiting values and with $\delta T_a = 1\,\text{K}$, the shift of the equilibrium-line altitude is shown in Table 2. Alterations of $\partial c/\partial z$ have a far stronger effect on Δh than alterations of $\partial T_a/\partial z$. Negative values of $\partial c/\partial z$ result in a particularly strong shift of the equilibrium-line altitude. Such negative values of $\partial c/\partial z$ on the Greenland ice sheet have been reported by Benson (1962). In Figure 1a and b the shift of the equilibrium-line altitude as a function of $\partial T_a/\partial z$ and $\partial c/\partial z$ is compared for equal perturbations.

Table 2. Shift of the equilibrium-line altitude for $\delta T_a = 1\,K$ and various combinations of $\partial c/\partial z$ and $\partial T_a/\partial z$

$\partial c/\partial z$	$\partial T_a/\partial z$	Δh
$\text{kg m}^{-2}\,\text{m}^{-1}$	K m^{-1}	m
$+0.55$	-0.0073	87.5
$+0.55$	-0.0057	102.5
-0.32	-0.0073	153.6
-0.32	-0.0057	211.4

5. ALTITUDINAL PROFILE OF NET ABLATION

The heat of melting of snow, superimposed ice and glacier ice is represented as a function of altitude at the EGIG line without any perturbation (Fig. 2a). The difference between the dashed line and the solid line corresponds to the melting of snow and superimposed ice, whereas the solid line stands for the altitudinal profile of the heat of melting of glacier ice, corresponding to the net ice ablation. The net ice ablation starts at the equilibrium-line altitude $(\Delta h = 0)$ and increases towards the ice margin $(\Delta h = -600\,\text{m})$ to about $3000\,\text{kg m}^{-2}$. The non-linear profile results from both the increase in the heat-flux density of melting and the longer ablation period at lower altitudes.

For comparison, the altitudinal profile of the heat of melting was calculated for the perturbation $\delta T_a = +1\,\text{K}$ (Fig. 2b, dashed line). The corresponding shift of the equilibrium-line altitude is indicated on the horizontal axis and the augmented ice net ablation due to this perturbation is shown by arrows. The increase in ice net ablation varies along the profile between $300\,\text{kg m}^{-2}$ $(\Delta h = 0)$ and $750\,\text{kg m}^{-2}$ $(\Delta h = -600\,\text{m})$. Assuming a constant slope along the profile, the averaged relative increase in net ice ablation amounts to 45%.

6. FINAL REMARKS

Th extension of the perturbation analysis to the entire Greenland ice sheet is of speculative character, since the required data are broadly unknown. Equilibrium-line altitude, ablation area and climatological conditions vary considerably from one region to another. Therefore, it is proposed to subdivide the Greenland ice sheet into four climatic zones (Oerlemans, 1991; Oerlemans and others, in press).

The gradient $\partial c/\partial z$ is of decisive importance for the stability of the mass balance (Ambach, 1988). Negative values of $\partial c/\partial z$ which are significant for the sensitive response of the equilibrium line on the Greenland ice sheet have been reported by Benson (1962). Along a profile at $70°\,\text{N}$ between $2000\,\text{m} \leq z \leq 3100\,\text{m}$ a.s.l. the averaged value amounts to $\partial c/\partial z = -0.26\,\text{kg m}^{-2}\,\text{m}^{-1}$, along a profile at $77°\,\text{N}$ between $1000\,\text{m} \leq z \leq 2100\,\text{m}$ a.s.l. it amounts to $\partial c/\partial z = -0.32\,\text{kg m}^{-2}\,\text{m}^{-1}$. The latter value is of particular importance since the

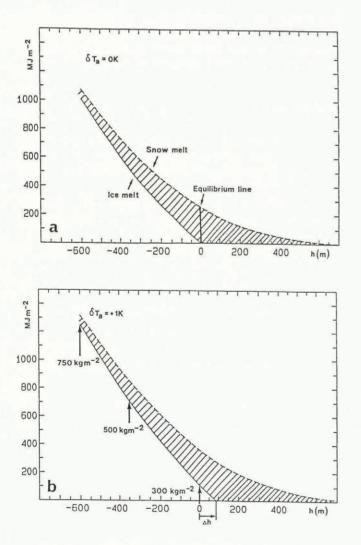*Fig. 2. a. Heat of melting as a function of altitude with $\delta T_a = 0$ for the entire ablation period. The dashed range corresponds to melting of snow and superimposed ice, the solid line corresponds to net ablation. b. As Figure 2a with $\delta T_a = +1\,K$. Arrows indicate the increase in net ablation.*

present-day equilibrium line in the Thule area is situated only slightly below this range of altitude. When the equilibrium line is shifted by climatic warming into an area of negative values of $\partial c/\partial z$, the instability criterion becomes effective. In this case, the shift of the equilibrium line due to perturbations in temperature is significantly enlarged by a factor of 1.8 compared with the condition at the EGIG line.

With the perturbation $\delta T_a = +1$ K, an increase of the net ice ablation of 45% was obtained at the EGIG line. If this figure is considered to be representative for the entire Greenland ice sheet, the annual increase of total net ablation results in $0.45 A_0$ km^3 year^{-1} water equivalent, where A_0 is the present annual total net ablation of the entire Greenland ice sheet. With $A_0 = 300$ km^3 year^{-1} (Weidick, unpublished), the annual change in total net ablation amounts to 135 km^3 year^{-1} water equivalent for $\delta T_a = +1$ K. The related sea-level rise depends significantly on the share of refreezing of meltwater in the ablation area. If no refreezing occurs, the sea-level rise results in 0.37 mm year^{-1}, which is the same figure obtained by Oerlemans and others (in press) by means of a more detailed analysis. A previous work by Ambach and Kuhn (1989) yielded a value of 0.34 mm year^{-1}. Bindschadler (1985) took into consideration the response of the ice margin, when a perfect plastic ice sheet is assumed. For two scenarios, the corresponding sea-level rise results in 0.40 and 0.54 mm year^{-1}. However, a considerable degree of uncertainty must be accepted.

Another criterion for the stability of the Greenland ice sheet is the condition, when the total net ablation (A) equals the total net accumulation (C). Neglecting possible refreezing of meltwater and applying data from the EGIG-line on to the entire Greenland ice sheet, the condition $A = C$ is fulfilled at a warming of $\delta T_a \sim +2$ K (Ambach and Kuhn, 1989). In this case, the surface balance is zero; due to continuous iceberg discharge, the shrinking of the Greenland ice sheet is accelerated. Even if

some parameters in the model are varied, the condition $A = C$ results in between 1 and 3 K (Table 3). Thus, the Greenland ice sheet reacts very sensitively to climatic warming.

In view of a three-dimensional thermomechanic model applied by Huybrechts and others (1991), the run-off is increased by 40% (exactly 42%) for $T_a = +1$ K as compared to 45% obtained in the present paper. The surface balance becomes negative for a warming above $+2.7$ K. This conclusion is in good agreement with the condition $A = C$ for $\delta T_a \approx +2$ K, which is derived in the present paper. The entire Greenland ice sheet will disappear for $\delta T_a = +6$ K after 20 000 years (Letréguilly and others, 1991a). Koerner (1989) described observational evidence for intensive melting of the Greenland ice sheet in the last interglacial, although, according to Reeh and others (1991) and Letréguilly and others (1991b), the summit of the central Greenland ice sheet seems to have survived the warm climate due to isostatic uplift.

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Table 3. Perturbations in temperature which yield the condition $A = C$, dependent upon various combinations of gradients $\partial c/\partial z$ and $\partial T_a/\partial z$ as well as initial values A_0, C_0

A_0 (km^3 year^{-1} w.e.)	300	320	390
C_0 (km^3 year^{-1} w.e.)	500	600	620
$\partial c/\partial z = +0.55$ kg m^{-2} m^{-1} $\partial T_a/\partial z = -0.0073$ K m^{-1}	2.3 K	3.0 K	2.2 K
$\partial c/\partial z = +0.55$ kg m^{-2} m^{-1} $\partial T_a/\partial z = -0.0057$ K m^{-1}	2.0 K	2.6 K	1.9 K
$\partial c/\partial z = -0.32$ kg m^{-2} m^{-1} $\partial T_a/\partial z = -0.0073$ K m^{-1}	1.3 K	1.7 K	1.3 K
$\partial c/\partial z = -0.32$ kg m^{-2} m^{-1} $\partial T_a/\partial z = -0.0057$ K m^{-1}	1.0 K	1.2 K	0.9 K

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