

Correspondence

A review of finite-element modelling in snow mechanics

The finite-element method (FEM) is one of the main numerical analysis methods in continuum mechanics and mechanics of solids (Huebner and others, 2001). Through mesh discretization of a given continuous domain into a finite number of sub-domains, or elements, the method finds approximate solutions to sets of simultaneous partial differential equations, which express the behavior of the elements and the entire system. For decades this methodology has played an accelerated role in mechanical engineering, structural analysis and, in particular, snow mechanics. To the best of our knowledge, the application of finite-element analysis in snow mechanics has never been summarized. Therefore, in this correspondence we provide a table with a detailed review of the main FEM studies on snow mechanics performed from 1971 to 2012 (40 papers), for facilitating comparison between different mechanical approaches, outlining numerical recipes and for future reference. We believe that this kind of compact review in a tabulated form will produce a snapshot of the state of the art, and thus become an appropriate, timely and beneficial reference for any relevant follow-up research, including, for example, not only snow avalanche questions, but also modeling of snow microstructure and tire–snow interaction. To that end, this correspondence is organized according to the following structure. Table 1 includes all essential information about previously published FEM studies originally developed to investigate stresses in snow with all corresponding mechanical and numerical parameters. Columns in Table 1 provide references to particular studies, placed in chronological order. Rows correspond to the main model parameters and other details of each considered case.

In order to give an overview of the studies covered by this review, we briefly summarize them below. Previously considered physical and engineering problems in snow mechanics can be roughly separated into several major categories, namely:

state of strain and stress in snowpack on slope, snow creep and compaction (Smith and others, 1971; Smith, 1972; Curtis and Smith, 1974; Smith and Curtis, 1975; Lang and Sommerfeld, 1977; Johnson, 1998; Bartelt and Christen, 1999; Bartelt and others, 2000; Teufelsbauer, 2009, 2011);

influence of snow weak layer and subcritical weak spots on the mechanical state of snowpack and slab avalanche release (McClung, 1979; Singh, 1980; Bader and Salm, 1990; Bartelt and Christen, 1999; Stoffel and Bartelt, 2003; Stoffel, 2005; Gaume and others, 2011, 2012);

skier loadings on inclined snowpack (Schweizer, 1993; Wilson and others, 1999; Jones and others, 2006; Habermann and others, 2008; Mahajan and others, 2010);

shock loading and explosive loading on snowpack (Johnson and others, 1993; Miller and others, 2011);

reproduction of mechanical experiments for studying fundamental rheological properties of snow (Mohamed and others, 1993; Meschke and others, 1996; Jamieson

and Johnston, 2001; Haehnel and Shoop, 2004; Cresseri and Jommi, 2005; Cresseri and others, 2010);

forces exerted by snow cover on avalanche defense structures (Bartelt and Christen, 1999; Bartelt and others, 2000; Stoffel, 2005; Teufelsbauer, 2011);

fracture properties of snow and snow slabs, crack propagation (Bažant and others, 2003; Mahajan and Senthil, 2004; Stoffel, 2005; Sigrist and others, 2006; Sigrist and Schweizer, 2007; Heierli and others, 2008; Mahajan and Joshi, 2008);

tire/wheel–snow interaction (Haehnel and Shoop, 2004; Lee, 2009);

microstructure studies of snow volume obtained from X-ray microtomography (Pieritz and others, 2004; Schneebeli, 2004; Srivastava and others, 2010; Theile, 2010; Hagenmuller, 2011).

We have omitted some studies from Table 1 because sufficient detail was not available to us (e.g. Navarre and Desrues, 1980; Singh, 1980). Others are omitted because they did not focus purely on mechanics; for example, some studies using FEM for snow or firn studies were mainly dedicated to heat transfer (at the microstructural level or at the snow–permafrost boundary), air ventilation within pore space, water infiltration or metamorphism (Christen and others, 1994; Tseng and others, 1994; Meussen and others, 1999; Phillips and others, 2000; Pielmeier and others, 2001; Albert, 2002; Bartelt and others, 2004; Kaempfer and others, 2005). Still others focused on the transition from solid to fluid (Daudon and Dufour, 2011) or the phase-tracking snow microstructure model (Slaughter and Zabaraz, 2012). Finally, FEM papers on tire–snow interaction may be found in references within Haehnel and Shoop (2004) and Lee (2009).

We hope that the papers collected in this review will serve to facilitate comparison between and assimilation of different mechanical approaches or numerical recipes, and that they will be useful for solving the many remaining problems.

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Table 1. Finite-element method studies on snow mechanics, 1971–2012

| Parameter | Smith and others (1971) | Smith (1972) | Curtis and Smith (1974) | Smith and Curtis (1975) | Lang and Sommerfeld (1977) | McClung (1979) | Bader and Salm (1990) | Schweizer (1993) | Johnson and others (1993) |
|---|--|---------------------------|----------------------------------|--|--|--|---|--|---------------------------|
| Constitutive law/ Approach | Linear elasticity | Linear elasticity | Linear elasticity | Linear elasticity and nonlinear behavior | Viscoelasticity with orthotropic snow properties | Linear elastic slab with nonlinear basal softening weak layer | Steady-state linear viscoelasticity | Linear viscosity | Linear elasticity |
| Young's modulus (10^6 N m^{-2}) | $(0.2 \times 10^3) - (1.2 \times 10^3)$ | 0.2×10^3 | 20, 200, 2000 | 20, 40, 100, 150, 260 (after Nakaya, 1959) | – | 3.95 (shear modulus, G), same for slab and weak layer; for $\nu = 0.25$, $E = 9.875$, where $E = 2G(1 + \nu)$ | – | – | – |
| Poisson's ratio | 0.10–0.30 | 0.20 | 0.00, 0.25, 0.45 | 0.25 | – | 0.25 | – | 0.10–0.40 viscous analogue of Poisson's ratio; selected arbitrarily (Mellor, 1975) | – |
| Viscosity (Pa s) | – | – | – | – | – | – | – | $(3.1 \times 10^7) - (2.4 \times 10^9) \text{ Pa s}$ | – |
| Strain rates (s^{-1}) | – | – | – | – | $< 0.5 \times 10^{-7}$ | – | – | – | $> 10^3$ |
| Density or density range (kg m^{-3}) | 100–400 | 127–400 | 210–400 | 210–400 | $\sim 175\text{--}418$ | 300 | 430 | 100–300 | 100–520 |
| Snow depth (slab or sample thickness) (m) | 0.96 | $< \sim 2.0$ | $< \sim 2.0$ | – | 1.3–1.8 | (1) | 2 (1) | < 1.0 (0.4) | (0.08) |
| Fracture energy of the weak layer (J m^{-2}) | – | – | – | – | – | – | – | – | – |
| Weak-layer thickness (m) | No weak layer; single-layer homogeneous snow | not specified | not specified | not specified | 0.1 | 0 | 0–0.02 | 0.02 | – |
| Weak-layer length (m) | – | 12 (same as model length) | ~ 19 (same as model length) | not specified | ~ 10 (same as model length) | ~ 5 (same as model length) | ∞ (same as model length), weak/gliding zone 0.2–80.0 | 10 (same as model length) | – |

| | | | | | | | | | |
|---|--|--|--|--|--|--|---|---|---|
| Weak-layer properties | – | Bottom layer with lower Young's modulus ($0.2 \times 10^8 \text{ N m}^{-2}$) | Bottom layer with lower Young's modulus ($0.2 \times 10^8 \text{ N m}^{-2}$) | Sub-layer with nonlinear stress-strain curve | Basal layer with densities 245–255 and 257–348 kg m^{-3} | 1-D line spring elements at the base of the slab allowing strain softening | Thin middle layer with low viscosity (0.5×10^8 or 10^8 Pa s) containing zone with zero viscosity (ideal gliding), which was replaced by corresponding interface conditions for the top and bottom layers | Soft layer 50 kg m^{-3} , with low viscosity ($3.1 \times 10^7 \text{ Pa s}$) | – |
| Temperature or temperature range ($^{\circ}\text{C}$) | – | – | – | – | – | – | – | –5.15 | –23.0 to –2.0 |
| Calculation times | – | – | – | – | 3 or 5 days | – | 0–60 min | – | <200 μs |
| 1-D, 2-D or 3-D mesh | 2-D | 2-D | 2-D | 2-D | 2-D | 2-D | 2-D | 2-D | 1-D and 2-D |
| Element shape | Triangular | Triangular | Triangular | Triangular | 4-node quadrilateral | not specified | 4-node element | 4-node quadrilateral | – |
| FEM code or software used | Program by Wilson and Clough (1963) | Program by Wilson and Clough (1963) | Program by Wilson and Clough (1963) | Program by Wilson and Clough (1963) | FE code by Goudreau and others (1967) | – | TPS10 | TPS10 | PRONTO 2-D |
| Issues addressed | State of stress in homogeneous snow on slope | State of stress in layered snow on slope, effect of new layer and weak sub-layer | State of stress in layered snow on slope, refinement of bottom weak sub-layer | State of stress in layered snow on slope, with nonlinear stress analysis | Study of snow deformation on avalanche slope with FE five- or seven-layer model corresponding to local stratigraphy in the experimental area, compared to measured deformation rates | Calculation of snow slab stress conditions prior to shear band propagation | Influence of weak layer on stress and strain distributions in a layered snowpack with consideration of snow creep: conditions and speed of fracture propagation | Skier-induced stress and snow layering | Shock loading of snow, pressure-density relationships, edge wave superposition on primary shock wave, impedance mismatch between stress gauges and snow |
| Laboratory or field studies used as references for validation | – | Mellor (1966, 1968) | Mellor (1966) | – | Experimental measured deformation of a sloping snowpack as part of the same study | – | – | – | – |
| Sites investigated for validation | Lift Gully, Berthoud Pass, CO, USA | Lift Gully, Berthoud Pass, CO, USA | Lift Gully, Berthoud Pass, CO, USA (1967/68) | Lift Gully, Berthoud Pass, CO, USA (1965/66) | Avalanche slope at Berthoud Pass, CO, USA | – | – | – | – |

Table 1. (continued)

| Parameter | Mohamed and others (1991) | Meschke and others (1996) | Johnson (1998) | Wilson and others (1999) | Bartelt and Christen (1999) | Bartelt and others (2000) | Jamieson and Johnston (2001) | Stoifel and Bartelt (2003) | Bažant and others (2003) |
|---|---|---|---|---|--|---|--|--|---|
| Constitutive law/Approach | Linear-plastic behavior with nonlinear material | Modified nonlinear elastic Cam-clay and Drucker-Prager models | Dynamic, with roughly spherical elastic-plastic particles | Linear elasticity | Viscoelasticity with microplane N -directional 1-D material laws ($N=10$ was found to be optimal) | Viscoelasticity with nonlinear and instationary material with macroscopic and microscopic parameters used in constitutive laws (Lagrangian coordinate system) | Linear elasticity | Temperature-dependent viscoelasticity | Linear elasticity with equivalent linear elastic fracture mechanics |
| Young's modulus (10^6 N m^{-2}) | – | 7.5 (shear modulus) assuming $\nu=0.25$, $E=18.75$ | – | 0.25–10.0 (after Mellor, 1975; Shapiro, 1997) | – | – | 0.2–1.0 | $E_0 = f(\rho)$, after von Moos and others (2003), which agrees well with Mellor (1975); Voytkovskiy (1977) | 0.9875 (after McClung, 1977, 1979) |
| Poisson's ratio | 0.00 | – | – | 0.25 | 0.25 ($300 \leq p \leq 500 \text{ kg m}^{-3}$) (after Mellor, 1975) | – | 0.25 | 0.00 | 0.25 (after McClung, 1977, 1979) |
| Viscosity (Pa s) | – | – | – | – | $\eta = f(\rho, T)$ Empirical laws after many authors; see original text | $\eta = f(\rho, T)$ Empirical laws after many authors; see original text | – | – | – |
| Strain rates (s^{-1}) | 3×10^{-4} (for compression tests) | – | $0.4\text{--}20 \text{ m s}^{-1}$ | $>10^{-4}$ | $<10^{-3}$ | – | $>2.5 \times 10^{-3}$ | $10^{-9}\text{--}10^{-4}$ | – |
| Density or density range (kg m^{-3}) | 350–600 (initial and final) | 400, 520 | – | not specified | 115, 138–237 (initial) | 213–242, 300 (initial) | 160–200 | 180–350 | – |
| Snow depth (slab or sample thickness) (m) | 0.178, 0.79 | (0.076, 0.04, 0.08) | – | 1.0 (0.3, 0.5) | 0.9 or 1.0 (initial) | 0.24, 3.0 | (~ 0.04) | <2.0 | <3.5 |
| Fracture energy of the weak layer (J m^{-2}) | – | – | – | – | – | – | – | – | 2.51, 2.73, 2.97 |
| Weak-layer thickness (m) | – | – | – | 0.005 | – | – | 0.002 | 0.0 | – |
| Weak-layer length (m) | – | – | – | 6 (same as model length) | – | – | 0.156/0.208 (below shear frame/ including substratum) | 0–20 | ∞ (same as model length) |
| Weak-layer properties | – | – | – | E_0 lower than neighboring snow layers (0.25 MPa) | – | – | 2 mm soft layer (160 kg m^{-3}) with lower E_0 (0.2 MPa) | Springs transferring normal stress with shear resistance = 0 | Crack propagates at or near underlying rigid-elastic interface |

| | | | | | | | | |
|---|--|---|--|---|--|---|--|---|
| Temperature or temperature range (°C) | - | - | -15.0 to -5.0 | -10.0 | -10 to -2 | - | -12.0 to 0.0 | - |
| Calculation times | - | - | - | 40 days | 3.5 and 20 days | - | 24–48 hours | - |
| 1-D, 2-D or 3-D mesh | 2-D | 2-D | 2-D | 2-D | 2-D | 2-D | 2-D | 1-D, 2-D |
| Element shape | Triangular and 1-D joint elements | Single finite and bilinear plane-strain elements | 3-D quadrilateral or smoothed particle hydro-dynamic elements | Triangular | 3-node triangular | 8-node quadrilateral | 4-node quadrilateral | - |
| FEM code or software used | FE program by Hanna (1975) | MARC | PRONTO 3-D (Taylor and Flanagan, 1989) | - | - | - | - | - |
| Issues addressed | Modeling plate penetration test (with snow properties obtained by confined compression and direct shear tests) | Modified Cam-clay plasticity model and Drucker–Prager plasticity model validated on hydrostatic and shear-box tests for snow and used for simulations of interactions between a single rubber thread block and snow-covered surface | Constant-speed uniaxial strain compaction of snow at different rates to study stages of compaction | Stationary viscoelastic creep in snow (including large volumetric and shear strains, temperature-dependent behavior with phase change, fracture or progressive creep damage of weak layers, forces exerted on defense structures) | Development of FE program <i>Haefeli</i> solving instationary heat transfer and creep (viscoelastic deformation), its validation for temperature drop imposed on a snow block and forces exerted on avalanche defense structures to compare with Swiss Guideline calculation procedure | Determination of shear stress distribution in the weak layer caused by shear frame (inserted into weak layer or above it) | Study of effects of weak-layer length, deformation rate, temperature variation | Formulation of size effect law for fracture triggering in dry snow slabs for studying material fracture parameters (fracture toughness and snow slab thickness) |
| Laboratory or field studies used as references for validation | Part of the same study | Part of the same study and Meschke and others (1993) | - | De Quervain (1945); Kojima (1974) | Cold laboratory experiments on snow metamorphism by M. Schneebeli and B. Brown, in 1998 | Part of the same study | - | Perla (1977) |
| Sites investigated for validation | - | - | Compared to Byrd Station polar snow pressure-density profile (Gow, 1968) | - | - | - | - | - |

Table 1. (continued)

| Parameter | Mahajan and Senthil (2004) | Haehnel and Shoop (2004) | Schneebeil (2004) | Pieritz and others (2004) | Stoffel (2005) | Jones and others (2006) | Sigrist and others (2006) | Sigrist and Schweizer (2007) | Mahajan and Joshi (2008) |
|--|--|---|---|--|---|--|---|---|---|
| Constitutive law/Approach | Static viscoelasticity with phenomenological law for cohesive surface elements (Needleman, 1990), which create a new surface | Capped Drucker-Prager constitutive law for low-density snow | Linear elasticity (snow fabric obtained from X-ray microtomography) | Linear elasticity under isothermal conditions (for raw snow volume data obtained from X-ray microtomography) | Temperature-dependent viscoelasticity/Tensorial constitutive law with linear fracture mechanics (2-D); N-directional approach with damage mechanics (3-D) (used for the first time on a geophysical material) | Linear elasticity for compressible material | Linear elasticity | Linear elasticity | Static nonlinear plane strain; dynamic linear elasticity with elastic constitutive law for cohesive surface elements (Xu and Needleman, 1994), which create a new surface |
| Young's modulus (10^6 N m^{-2}) | 3, 10 | 1.379–13.79 (after Shapiro and others, 1997) | $E_f = 9.5 \times 10^3$ 61.8–226 (simulated) | $E_f = 9.0 \times 10^3$ | $E_0 = f(\rho) = 0.1873e^{0.0149\rho}$ ($r^2 = 0.928$), for fine-grained well-bonded snow with $\rho = 180\text{--}450 \text{ kg m}^{-3}$, temp. = -20°C to -2°C (after von Moos and others, 2003) | 0.3, 1.5, 7.5 (after Mellor, 1975; Shapiro and others, 1997) | 0.3, 1.5, 7.5 (after Mellor, 1975; micro-penetrometer (SMP) penetration resistance signal made with SMP) | 5.0–10.0 (determined via penetration resistance profile) | 0.1–10.0 |
| Poisson's ratio | 0.23 | 0.30 | $\nu_f = 0.30$ | $\nu_f = 0.33$ | 0.00 | 0.25, 0.49 | $\nu = \nu_0 + c(\rho - \rho_0)$, where $\nu_0 = 0.2$, $\rho_0 = 300 \text{ kg m}^{-3}$, $c = 5 \times 10^{-4} \text{ m}^3 \text{ kg}^{-1}$ (after Mellor, 1975) | $\nu = \nu_0 + c(\rho - \rho_0)$, where $\nu_0 = 0.2$, $\rho_0 = 300 \text{ kg m}^{-3}$, $c = 5 \times 10^{-4} \text{ m}^3 \text{ kg}^{-1}$ (after Mellor, 1975) | 0.23 |
| Viscosity (Pa s) | $10^8, 5 \times 10^{10}$ | – | – | – | – | – | – | – | – |
| Strain rates (s^{-1}) | – | – | – | – | $10^{-10}\text{--}10^{-3}$ | $>10^{-4}$ | – | – | – |
| Density or density range (kg m^{-3}) | 100, 300 | 150–250 (average 200) | 243, 308 | ~260 | 180–300 | 105, 185, 275 | ~234–348 | 173, 187, 202 | 100, 300 |
| Snow depth (slab or sample thickness) (m) | 2.0 assumed, bottom 0.105 modeled | 0.392–0.514 | 0.0036, 0.00756 | 0.0015 (diameter) | <3.0 | 2.1–2.7 (0.1–0.7) | <0.2 | (0.21–0.3) | <1.0 |
| Fracture energy of the weak layer or interface (J m^{-2}) | 0.05 | – | – | – | – | – | 0.04 ± 0.02 (critical energy release rate) | 0.07 (critical energy release rate; determined by the study) | $\phi_f = 0.01\text{--}0.02$ $\phi_n = 0.05$ |
| Weak-layer thickness (m) | 0.002–0.01 | – | – | – | 0.0 | 0.003 | 0.0 | – | 0.005–0.0113 |
| Weak-layer length (m) | 2 (same as model length) | – | – | – | 0–20 | 10 (same as model length) | 0.5 (same as specimen length) | 0.6–1.8 | 2 (same as model length) |

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|---|---|--|---|--|--|---|--|--|---|
| Weak-layer properties | Soft layer (100 kg m ⁻³) with lower E ₀ and viscosity (3 MPa; 10 ⁸ Pa s) with normal and shear strength of interface as 0.00184 and 0.002–0.008 MPa | – | – | – | Springs transferring normal stress with shear resistance = 0 | Low-density layer (105 kg m ⁻³) with E ₀ = 0.3 MPa | Interface, E ₀ = 4–20 MPa (average = 11.6 MPa) | Assumed as the same material as the adjacent upper slab (observed: faceted crystals and depth hoar overlain by a 2 mm crust) | Soft layer (100 kg m ⁻³) with lower E ₀ (0.05, 0.1, 0.75, 1.0 MPa) with normal and shear strength of interface as 0.00184 and 0.00085–0.0017 MPa |
| Temperature or temperature range (°C) | – | –10 to –1 | (Temperature gradient 100 °C m ⁻¹) | –5 (for E _i and v _i) | –32.5 to –1.0 | – | – | – | – |
| Calculation times | 10 hours | – | – | – | – | – | – | – | <30 ms |
| 1-D, 2-D or 3-D mesh | 2-D | 1-D, 2-D, 3-D | 3-D | 3-D | 2-D, 3-D | 2-D | 2-D | 2-D | 2-D |
| Element shape | 4-node quadrilateral | Single- and multi-element (no details) | 8-node brick element | 4-node tetrahedral | Quadrilateral (2-D) Tetrahedron (3-D) | 4-node quadrilateral | not specified | – | 4-node quadrilateral, triangular |
| FEM code or software used | ABAQUS | ABAQUS | FE program by Van Rietbergen and others (1996) | Package (RCFEA) developed as objective of the study (C++ with OOP) | – | PATRAN, ABAQUS | ANSYS | – | ABAQUS |
| Issues addressed | Interface crack propagation in layered snow (length of crack, slope angle, interface properties, weak-layer thickness) | Simulating radially confined uniaxial compression test, plate-sinkage test, wheel rolling through fresh snow (for homogeneous and multilayer snow) | Determination of elastic modulus and its dependence on microstructure, stress distribution in snow volume obtained from X-ray microtomography | Development of code for micromechanics modeling with 3-D geometry obtained from X-ray microtomography; simulation of uniaxial compression text | Snow creep, snow glide, stress on defense structure, avalanche formation (stress intensity factor at crack, weak-layer length, fresh snow loading, temperature rise) | Layered snowpack loaded by skier (slab thickness, relative stiffness of layers and stress concentrations) | Evaluate energy release rate in mode II fracture in layered snow | Determination of critical energy release rate for fracture propagation in a weak layer | Interface crack propagation in layered snow (rapid crack growth, crack velocity under skier loads) |
| Laboratory or field studies used as references for validation | – | Abele and Gow (1975); Alger and Osborne (1989); Shoop and Alger (1998); Blaisdell and others (1990); Green and Blaisdell (1991); Richmond (1995) | Part of the same study | – | De Quervain (1945); Kirchner, (2002a,b); Baggi (2003) | – | Part of the same study | Part of the same study | Part of the same study |
| Sites investigated for validation | – | – | – | – | Vallée de la Sionne, 'Matte'-Frauenkirch (Switzerland) | – | – | – | – |

Table 1. (continued)

| Parameter | Heierli and others (2008) | Habermann and others (2008) | Teufelsbauer (2009) | Lee (2009) | Mahajan and others (2010) | Srivastava and others (2010) | Cresseri and others (2010); Cresseri and Jommi (2005) | Theile (2010) |
|--|---|---|---|---|---|--|--|--|
| Constitutive law/ Approach | Linear elasticity | Static linear elasticity | Viscoelasticity | Drucker-Prager cap-hardening model for fresh snow | Static nonlinear plane strain; dynamic linear elasticity with elastic constitutive law for cohesive surface elements (Xu and Needleman, 1994), which create a new surface | Linear elasticity (snow fabric obtained from X-ray microtomography) | Elasto-viscoplastic law with analytical expression for sintering | Linear elasticity (snow fabric obtained from X-ray microtomography) with maximal principal stress failure criterion; Glen's creep laws for mono- or polycrystalline models |
| Young's modulus (10^6 N m^{-2}) | 7.5 ± 2.5 1.5 ± 0.8 5.0 (Shapiro, 1997) | 0.15–7.5 Also use $(E_0(\rho) = A(\rho/\rho_0)^{2.94}$ where $A = 968 \text{ MPa}$, $\rho_0 = 917 \text{ kg m}^{-3}$ after Sigrist (2006) | – | 13.79 | 0.1, 1.0 | $E_f = 9.5 \times 10^3$ (Sanderson, 1988) ~ 460 to 1007 (calculated primary Young's moduli, E_1) | 2.1–12.4 (shear modulus) (various sources) assuming $\nu = 0.25$, $E = 5.25\text{--}31.0$ | $E_f = 10 \times 10^3$ |
| Poisson's ratio | 0.20 | 0.25 | – | 0.30 | 0.23 | $\nu_j = 0.30$ (after Sanderson, 1988) | – | 0.3 |
| Viscosity (Pa s) | – | – | – | – | – | – | – | – |
| Strain rates (s^{-1}) | – | – | – | – | – | – | – | – |
| Density or density range (kg m^{-3}) | 187, 134, 200 | 100–270 | Measured for initialization ($\sim 40\text{--}360$) | 200 | 100, 300 | $\sim 368\text{--}428$ | 10^{-7} to 10^{-4} | 10^{-8} to 10^{-6} , $>10^{-4}$ |
| Snow depth (slab or sample thickness) (m) | (0.26, 0.11) | <2.2 (0.2–1.2) | Lidar data | 0.05–0.60 | 0.605 (0.1) | (0.00257) | – | (<0.004) |
| Fracture energy of the weak layer or interface (J m^{-2}) | 0.07 ± 0.02 0.03 (specific fracture energy per unit of crack surface) | – | – | – | $\phi_t = 0.01, 0.02$ $\phi_{h,wl} = 0.05$ | – | – | – |
| Weak-layer thickness (m) | 0.01 | 0.001–0.011 | – | – | 0.005 | – | – | – |
| Weak-layer length (m) | ∞ | 10 (same as model length) | – | 1.46 (domain length) | 8 (same as model length) | – | – | – |

Table 1. (continued)

| Parameter | Teufelsbauer (2011) | Miller and others (2011) | Hagemuller (2011) | Gaume and others (2011) | Gaume and others (2012) |
|--|--|--|---|--|--|
| Constitutive law/Approach | Isotropic viscous fluid | Snow is considered as Lagrangian solid with volumetric and deviatoric constitutive relationships. Volumetric contribution is obtained from an equation of state (with shock Hugoniot). Deviatoric part is modeled as linear elastic with maximum principal stress failure criterion, $f(\rho)$ | Linear elasticity (snow fabric obtained from X-ray microtomography) with maximal principal stress failure criterion | Elasto-plastic (Drucker–Prager model for a slab and interface with shear softening for the weak layer) | Linear elasticity with a quasi-brittle (strain-softening) interfacial law for weak layers |
| Young's modulus (10^6 N m^{-2}) | – | 0.2, 60 (after Mellor, 1975) | $E_f = 10 \times 10^3$ | 1.0 | 1.0 |
| Poisson's ratio | $0.0 < \nu_{\text{viscous}} = f(\rho, T) < 0.49$ (see original) | – | 0.3 | 0.30 | 0.20 |
| Viscosity (Pa s) | $\eta_s = 0.05 \times \rho^{-0.03717T+4.4} \times (10^{-4} e^{0.018\rho} + 1)$, where $\eta_s = 2\eta(1-\nu)/(1-2\nu)$ $< 10^{-5}$ | – | – | – | – |
| Strain rates (s^{-1}) | – | – | $> 10^{-4}$ | – | – |
| Density or density range (kg m^{-3}) | 30–57; 115; 244–449 (initial) | 111, 400 | $\rho_i = 916.7, 324\text{--}621$ | 250 | 250 |
| Snow depth (slab or sample thickness) (m) | 0.36–0.5; ~3.36; terrestrial laser scanning | 1.51 (0.5) | < 0.004 | (0.3–2.1) | < 2.0 |
| Fracture energy of the weak layer or interface (J m^{-2}) | – | – | – | – | – |
| Weak-layer thickness (m) | – | 0.01 | – | 0 | 0 |
| Weak-layer length (m) | – | 10 (same as model diameter) | – | 30 (same as model length) | 50 (same as model length) |
| Weak-layer properties | – | Soft layer (111 kg m^{-3}) with lower E_0 (0.2 MPa) with maximum compressive normal stress, maximum shear stress and maximum octahedral stress = 0.03, 0.04 and 0.015 MPa, respectively | – | Elasto-plastic interface with shear softening (characteristic length = 2 mm; $\tau_{\text{res}} = \tau_{\text{peak}}/2$); Mohr–Coulomb rupture criterion with no cohesion | Elasto-plastic interface with shear softening (characteristic length = 2 mm; $\tau_{\text{res}} = \tau_{\text{peak}}/2$); Mohr–Coulomb rupture criterion with spatially varying cohesion (average = 0.006–0.015 MPa, std dev. = 0.003 MPa), and a friction angle = 30° . |
| Temperature or temperature range ($^\circ\text{C}$) | –32.5 to –1.0 | – | –20 (for experiments) | – | – |
| Calculation times | – | $< \sim 100$ ms | – | – | – |
| 1-D, 2-D or 3-D mesh | 2-D | 3-D | 3-D | 2-D | 2-D |

| Element shape | 2-D axisymmetric square mesh | Beam elements with circular cross section (ANSYS SOLID188); voxel (8-node brick elements – ANSYS SOLID45); tetrahedron elements (ANSYS SOLID185 and 92) | 4-node quadrilateral | 4-node quadrilateral |
|---|--|---|--|---|
| triangular | ANSYS/ AUTODYN | ANSYS | Cast3M (Laborderie and Jeanvoine, 1994) | Cast3M (Laborderie and Jeanvoine, 1994) |
| FEM code or software used | MATLAB/SnowSim | | | |
| Issues addressed | Development and validation of snow creep model with variable temperature and density-dependent Poisson ratio (part of SnowSim model) | Study of snow strength (brittle failure) as a function of snow microstructure (derived from computed tomography scans); mesh type and element number influence on simulated snow strength | Studying heterogeneity of weak layer and avalanche release angle | Obtaining avalanche release depth distributions through coupling of mechanical and meteorological factors |
| Laboratory or field studies used as references for validation | De Quervain (1945); Hiller and Bader (1990); Gubler (1994); Abe (2001) | Part of the same study | | Database of avalanche release depths from La Plagne ski resort, France |
| Sites investigated for validation | Avalanche test site in Lech am Arlberg, Austria | | | |

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