THE DISTRIBUTION OF ASTEROIDS IN THE DIRECTION PERPENDICULAR TO THE ECLIPTIC PLANE

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For examining the steady-state distribution of asteroids in the direction perpendicular to the ecliptic plane (the z distribution), we shall assume all orbits to be circular. This assumption is incompatible with the north-south asymmetry found by Nairn (1965); but Kresak (1967) has shown that the asymmetry is caused by a combination of cosmic and human factors and is present only among fainter asteroids, $B(a, 0) > 16$, where the discovery is grossly incomplete. There is another perhaps even more cogent reason for using only the brighter asteroids: The easily understandable practice of confining asteroid hunting close to the ecliptic plane has meant that among the fainter objects, orbits with high inclinations are underrepresented (Kiang, 1966). Actually, in the range $14 < B(a, 0) < 15$ where, I estimate, the discovery is 95 percent complete, the sample of inclinations may already be somewhat biased in the same sense. One has to balance this risk, however, with the advantage of a much greater data size; and I shall use all the numbered asteroids with $B(a, 0) < 15$ as given in the 1962 *Ephemeris* volume (excluding 13 that are regarded as "lost").

A very welcome new set of data is provided by the Palomar-Leiden survey (PLS) (van Houten et al., 1970). In this case, important selection effects should and can easily be made. According to the authors (van Houten et al., 1970, p. 360), the area searched extends to a height of $5\overset{\circ}{.}9$ from the ecliptic. Consider all the orbits with the same radius a; for these, the search extends to a heliocentric latitude of $b = 5\overset{\circ}{.}9(a - 1)/a$. Although an orbit with inclination $i < b$ lies entirely within the latitudes $\pm b$, an orbit with $i > b$ has only the fraction

$$f_1 = f_1(i, a) = \frac{2}{\pi} \arcsin\left[\sin i \sin \frac{5\overset{\circ}{.}9(a - 1)}{a}\right] \tag{1}$$

lying in the same range. Hence, as far as the shape of the distribution of i at a given a (in practice, within a small range Δa) is concerned, the correction factor is simply $1/f_1$. Expression (1) differs a little from expression (3) in the PLS paper (van Houten et al., 1970, p. 361), but appears to be more in line with the assumption of circular orbits.

Among numbered asteroids known at a given time, one always finds a positive correlation between a and i; this feature has been reported repeatedly, but the question whether this is due, at least in part, simply to the fact that Earth is inside the ring of asteroids has never been examined. Of course, even if the distribution of i is the same for all a, there will still be a systematic increase of the thickness of the system with increasing distance from the Sun. Here we shall concentrate on the z distribution at different intervals of a.

As may be seen from figure 1, the well-known Kirkwood gaps and other commensurability points divide the main belt $(2.0 < a < 3.8)$ quite naturally into nine zones. These will be labeled zones 0 to 8 inclusive. The Hilda group, the Trojans, and the range $1.0 < a < 2.0$ will be labeled zones 9, T, and M (for Mars), respectively. Table I lists some statistics of the zones. The next-to-the-last column refers to the numbers of the largest asteroids $(B(1, 0) < 10)$ found in the sample. These numbers are very likely to be complete, except the one in zone T. The last column gives the numbers of these objects per unit circle (in AU) of the ecliptic plane. These areal densities are only approximations to the average state of affairs at the corresponding distances from the Sun because of the strong radial asymmetry in the distribution in the ecliptic plane (Kresák, 1967). Because resonance effects obviously dominate the orbits and thus the

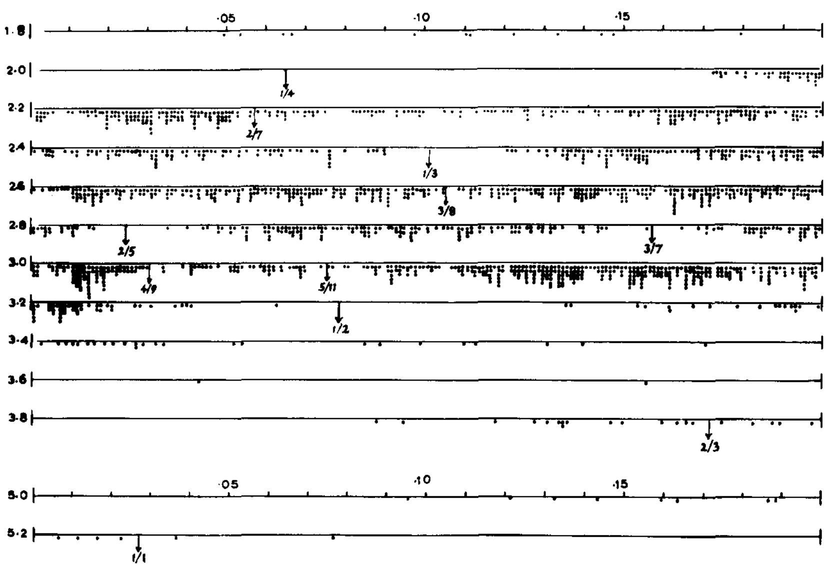

Figure 1.—Frequency distribution of semimajor axes of asteroids in intervals of 0.001 AU. Sample consists of the 1647 numbered asteroids given in the 1962 *Ephemeris* volume minus the 13 asteroids that are marked as "lost." The following five fall outside the diagram: 1566 Icarus, 1620 Geographos, 433 Eros, 279 Thule, and 944 Hidalgo with orbital radii a of 1.077, 1.244, 1.458, 4.282, and 5.794 AU, respectively. Commensurability points are marked with arrows, together with the ratios of periods (asteroid/Jupiter).

TABLE I.–*Division of the Ring of Asteroids Into 12 Zones*

Zone	Number of asteroids[a]	Observed limits, AU	Median a, AU	Median $B(a, 0) - B(1, 0)$	$B(1, 0) < 10$	
					Number	Number per unit circle of ecliptic plane
M	15	1.077 to 1.979	1.879	1.39	0	–
0	150	2.153 to 2.256	2.225	2.17	3	7
1	240	2.257 to 2.490	2.385	2.59	39	35
2	310	2.520 to 2.705	2.618	3.13	67	69
3	200	2.708 to 2.816	2.762	3.43	58	95
4	129	2.834 to 2.956	2.887	3.68	23	30
5	122	2.964 to 3.030	3.011	3.91	17	44
6	41	3.033 to 3.074	3.056	3.99	13	52
7	345	3.075 to 3.263	3.151	4.16	74	62
8	46	3.280 to 3.756	3.411	4.57	18	5
9	20	3.887 to 3.998	3.948	5.33	8	8
T	14	5.095 to 5.277	5.188	6.68	[b]11	[b]6
Total	1632	...	...	...	331	...

[a]Sample consists of all numbered asteroids given in the 1962 *Ephemeris* volume except 13 marked there as "lost," 279 Thule at $a = 4.282$ AU, and 944 Hidalgo at $a = 5.794$ AU.

[b]Multiply by 3.5 to correct for incompleteness of discovery.

spatial distribution of the Hilda and Trojan asteroids, and the asteroids in zone M are also rather special, the following discussion on the z distribution will be confined to zones 0 to 8 of the main belt.

The frequency distributions of i in zones 0 to 8 observed in the adopted sample of numbered asteroids are listed in table II. Let $n(\Delta i)$ be the number of objects in the interval Δi in a given zone; then the average number $n(\Delta b)$ of objects in an interval Δb of the heliocentric latitude, taken without regard to sign, in the same zone is calculated, in practice, according to the formula

$$n(\Delta b) = \sum_{\text{all } i} n(\Delta i) \cdot P(\Delta b, i) \tag{2}$$

where

$$P(\Delta b, i) = \Delta\left(\frac{2}{\pi} \arcsin \frac{\sin b}{\sin i}\right) \tag{3}$$

is the fraction of a circular orbit of inclination i that is included in the interval Δb. Now let $b_{0.5}$ and $b_{0.95}$ denote, respectively, the 50 percent point (the median) and the 95 percent point in the resulting b distribution; then the same percentage points in the corresponding z distribution are given by

$$z_{0.5} = a \sin b_{0.5} \tag{4}$$

$$z_{0.95} = a \sin b_{0.95} \tag{5}$$

In these expressions it is sufficiently accurate to set a equal to the appropriate median value shown in table I. The values $z_{0.5}$ and $z_{0.95}$ for each zone, together with their standard errors (s.e.), are listed in table II and are shown in figure 2 (the solid lines). Both show a steady increase with increasing distance from the Sun. Actually, $b_{0.5}$ and $b_{0.95}$ also show some increase with increasing a; but, of course, these increases are much less rapid and steady than the ones for $z_{0.5}$ and $z_{0.95}$.

We now examine the data of fainter asteroids provided by the PLS. The observed frequency distributions are listed in table III. They are corrected for the latitude cutoff as outlined above. The corrected individual frequencies are not shown; only their marginal totals S_1 are given. From the corrected i distribution in each zone except zone 8, which has too few objects for a proper determination, the values of $z_{0.5}$ and $z_{0.95}$ are derived as before; they are shown as broken lines in figure 2. Two effects are apparent: (1) The thickness of the system of fainter asteroids sampled by PLS also increases with increasing distance from the Sun, and (2) the thickness is noticeably less than that of the system of brighter asteroids at the same distance.

It should be remembered that the samples used here are, in one case strictly and in the other approximately, limited by $B(a, 0)$. If we note the actual differences between the solid and broken lines in figure 2 and the differences

TABLE II.–*Frequency Distribution of Inclinations and 50 and 95 Percent Points of* z *in Separate Zones*

[Data from numbered asteroids with $B(a, 0) < 15$]

i	*i* distributions for zones 0 to 8									Total
	0	1	2	3	4	5	6	7	8	
0° to 1°	2	1	–	2	–	1	–	3	–	9
1 to 2	2	5	3	2	5	–	1	5	1	24
2 to 3	2	8	8	5	5	1	–	10	1	40
3 to 4	3	9	7	6	5	1	2	10	1	44
4 to 5	4	7	7	9	3	2	–	7	3	42
5 to 7	15	21	27	17	6	6	3	8	2	105
7 to 9	5	18	16	24	6	7	5	21	1	103
9 to 11	–	19	15	27	9	16	2	19	4	111
11 to 13	–	5	28	8	9	7	3	13	1	74
13 to 15	–	5	18	15	8	6	1	16	2	71
15 to 19	–	4	20	15	9	3	10	28	3	92
19 to 23	–	3	2	4	2	2	–	14	7	34
23 to 27	–	7	5	5	2	1	–	8	1	29
27 to 35	–	–	4	1	–	1	–	–	–	6
Total	33	112	160	140	69	54	27	162	27	784
$z_{0.5}$	0.117	0.177	0.260	0.267	0.259	0.328	0.343	0.330	0.378	
s.e., ±	.025	.021	.024	.026	.048	.044	.074	.038	.117	
$z_{0.95}$	.269	.665	.770	.790	.802	.819	.894	1.007	1.235	
s.e., ±	.027	.114	.077	.081	.080	.122	.085	.095	.205	

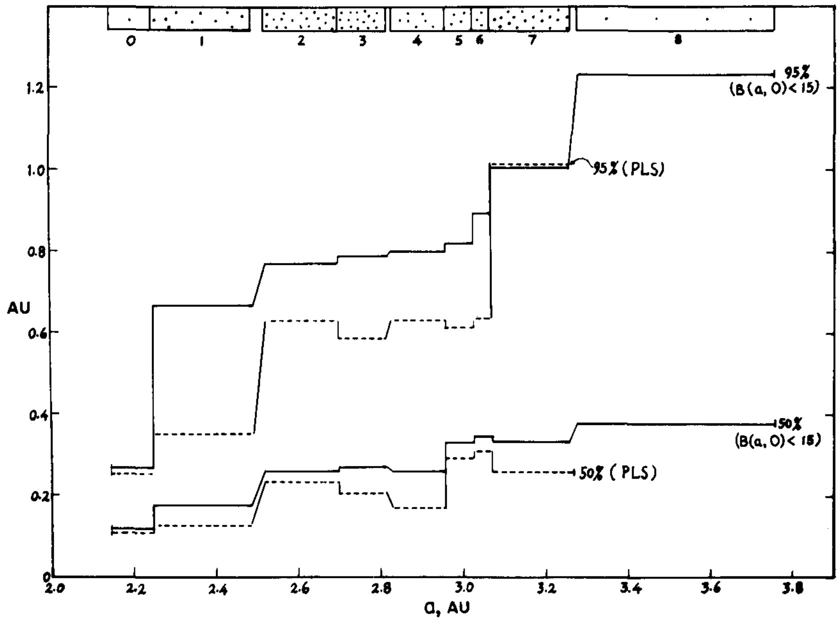

Figure 2.–The 50 and 95 percent points in the z distribution in nine intervals of the semimajor axis. Solid lines refer to numbered asteroids with $B(a, 0) < 15$, and broken lines to those found in the PLS. The "dominoes" along the top edge illustrate the areal densities of the largest asteroids given in the last column of table I.

$B(a, 0) - B(1, 0)$ listed in table I, we find that for a system of asteroids limited by constant $B(1, 0)$ (i.e., effectively down to certain physical size) there will be a rather reduced rate of increase in thickness with increasing distance. This, of course, assumes that effect (2) is real, which is by no means certain. The correction for the latitude cutoff is based on the value 5°9 for the extension of the search area in PLS. Because vignetting is certainly present on the plates used in PLS, the effective extension may be less than 5°9; thus the i distributions were undercorrected, leading to thickness estimates that are too low. This point should be examined in greater detail.

My conclusions are as follows:

(1) The thickness of the system of asteroids of the main belt increases steadily with increasing distance from the Sun. The 50 percent point in the z distribution increases from 0.12 at 2.2 to 0.38 at 3.5 (all values in AU), and the 95 percent point increases from 0.27 at 2.2 to 1.24 at 3.5.

(2) Less certainly, at a given distance from the Sun, the thickness is less for the system of smaller objects than for the system of larger ones.

These statements corroborate and amplify the conclusions on the proper inclinations reached earlier (Kiang, 1966).

TABLE III.–*Observed Frequency Distribution of Inclinations in Eight Zones*

[Data from class I orbits in the PLS]

i	*i* distributions for zones 0 to 8									S_0	S_1
	0	1	2	3	4	5	6	7	8		
0° to 1°	3	11	1	–	3	2	–	17	1	38	38
1 to 2	13	43	21	7	10	7	1	19	–	121	121
2 to 3	19	68	30	9	14	4	2	26	–	172	172
3 to 4	17	38	37	14	11	4	3	14	1	139	154
4 to 5	25	39	24	17	1	3	1	20	–	130	222
5 to 7	14	70	24	20	1	–	1	12	–	142	351
7 to 9	5	19	12	13	2	7	2	6	–	66	219
9 to 11	–	5	6	8	2	13	6	12	2	54	202
11 to 13	–	5	27	–	7	3	1	5	–	48	237
13 to 15	–	–	22	6	–	–	1	5	–	34	196
15 to 19	–	–	2	–	1	1	–	6	–	10	128
19 to 23	–	–	1	–	–	–	–	5	–	6	49
23 to 27	–	–	–	–	–	–	–	1	–	1	9
27 to 35	–	–	–	–	–	–	–	–	–	–	–
S_0	96	298	207	94	52	44	18	148	4	961	–
S_1	162	532	546	212	157	113	48	319	9	–	2098
$z_{0.5}$	0.105	0.124	0.237	0.206	0.171	0.291	0.307	0.259	–	–	–
$z_{0.95}$	.256	.351	.629	.585	.630	.611	.636	1.011	–	–	–

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DISCUSSION

VAN HOUTEN (submitted after meeting): Formula (3) in the PLS is based on circular orbits. Kiang indeed found an error: $(a \cos i - 1)$ should be $(a - 1)$; fortunately, for most asteroids $\cos i \approx 1$.

Kiang's expression (1) should include the correction for the length of the arc traversed by the asteroid during the observation period, as explained in the PLS. The correct expression is

$$f_1 = \frac{2}{\pi}\left[\arcsin\left(a^{-1} \tan \frac{5^\circ\!.9}{a \sin i}\right) - \frac{1}{2}\Delta tn\right]$$

in which n is the mean daily motion of the asteroid and Δt the period over which the observations extend.

This additional correction term is only important for large inclinations, and therefore it is not certain how this influences the data derived by Kiang. For that reason a comparison is made in table D-I with my own results given in an earlier paper[1] in which $\overline{z_0}$ is given. This can be transformed into $\overline{z}$ by multiplication with a factor of 0.64. If the distribution of z is gaussian,

$$z_{0.5} = 0.84\overline{z} = 0.54\overline{z_0}$$

My values of $z_{0.5}$ were obtained by interpolation in table I of my paper.

It follows that no systematic difference exists between my values and those of Kiang for the first four zones; whereas for zones 5 through 8, Kiang's values appear about 6 percent too low. This difference is hardly meaningful.

Accordingly, Kiang's conclusion that the PLS asteroids are more concentrated toward the ecliptic than the numbered asteroids must be accepted as being correct. His tentative

TABLE D-I.—*Comparison of $z_{0.5}$ in the Eight Kiang Zones*

Kiang zone	van Houten, interpolated	Kiang	Difference
1	0.094	0.105	−0.011
2	.124	.124	.000
3	.248	.237	+.011
4	.205	.206	−.001
5	.226	.171	+.055
6	.258	.291	−.033
7	.270	.307	−.037
8	.329	.259	+.070

[1]See p. 183.

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conclusion that vignetting effects may be the cause of this difference should be rejected on account of the small field effects of the Palomar 122 cm Schmidt plates. More likely the explanation should be sought in the remark made by Kiang (1966): "Large values of i are especially associated with values of the node around 90° . . . ." Because in the PLS the nodal values of the high-inclination asteroids cluster around 0° and 180°, it can be expected that the PLS material is deficient in minor planets with large inclination, which is in agreement with Kiang's conclusion.

DISCUSSION REFERENCE

Kiang, T. 1966, Bias-Free Statistics of Orbital Elements of Asteroids. Icarus 5, 437-449.

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