8

The Minimal Supersymmetric Standard Model

At this point, we have all the ingredients necessary for constructing a supersymmetric version of the Standard Model, complete with explicit soft SUSY breaking terms. The simplest such model, known as the Minimal Supersymmetric Standard Model, or MSSM, is a direct supersymmetrization of the Standard Model (except for the fact that one needs to introduce two Higgs doublet fields). It is minimal in the sense that it contains the smallest number of new particle states and new interactions consistent with phenomenology.

To construct the MSSM, we follow the recipe for the construction of supersymmetric gauge theories at the end of Chapter 6 and proceed as follows:

1. We choose the gauge symmetry group for the theory to be the Standard Model gauge group, $SU(3)_C \times SU(2)_L \times U(1)_Y$.
2. We select the matter content of the theory, to be realized as left-chiral scalar superfields, with gauge quantum numbers exactly as in the Standard Model. The Higgs sector is chosen to consist of two left-chiral scalar superfields with opposite hypercharge.
3. We choose the form of the superpotential.
4. Finally, we compute the supersymmetric Lagrangian using the master formula Eq. (6.44), and augment it by all possible soft SUSY breaking terms consistent with the gauge and Poincaré symmetries as discussed in Chapter 7.

8.1 Constructing the MSSM

As mentioned, we choose the gauge symmetry of the Standard Model: $SU(3)_C \times SU(2)_L \times U(1)_Y$. The gauge bosons of the SM are promoted to gauge superfields. In the Wess–Zumino gauge,

$$\begin{aligned} B_\mu &\to \hat{B} \ni (\lambda_0, B_\mu, \mathcal{D}_B), \\ W_{A\mu} &\to \hat{W}_A \ni (\lambda_A, W_{A\mu}, \mathcal{D}_{WA}),\ \ A = 1, 2, 3,\ \text{and} \\ g_{A\mu} &\to \hat{g}_A \ni (\tilde{g}_A, G_{A\mu}, \mathcal{D}_{gA}),\ \ A = 1, \ldots, 8. \end{aligned}$$

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The second step is to stipulate the matter content of the MSSM. The fermion fields of the SM are promoted to chiral scalar superfields, with one superfield for each chirality of every SM fermion. Since the superpotential must be a function of just *left*-chiral superfields, instead of using the right-handed fermions as the building blocks, we will, as mentioned in Chapter 1, use their left-handed charge conjugates. The matter superfields then consist of,

$$\begin{aligned}
\begin{pmatrix} \nu_{i\mathrm{L}} \\ e_{i\mathrm{L}} \end{pmatrix} &\rightarrow \hat{L}_i \equiv \begin{pmatrix} \hat{\nu}_i \\ \hat{e}_i \end{pmatrix}, \\
(e_{i\mathrm{R}})^c &\rightarrow \hat{E}_i^c, \\
\begin{pmatrix} u_{i\mathrm{L}} \\ d_{i\mathrm{L}} \end{pmatrix} &\rightarrow \hat{Q}_i \equiv \begin{pmatrix} \hat{u}_i \\ \hat{d}_i \end{pmatrix}, \\
(u_{i\mathrm{R}})^c &\rightarrow \hat{U}_i^c, \\
(d_{i\mathrm{R}})^c &\rightarrow \hat{D}_i^c,
\end{aligned}$$

where $i = 1, 2, 3$ refers to the *generation* of each field, i.e. $\hat{u}_3$ contains the dynamical fields $\tilde{t}_\mathrm{L}$ and $\psi_{t\mathrm{L}}$ (in addition to the corresponding auxiliary field).[1] To be explicit, we write down the superfield expansions which contain the electron fields:

$$\hat{e} = \tilde{e}_\mathrm{L}(\hat{x}) + \mathrm{i}\sqrt{2}\bar{\theta}\psi_{e\mathrm{L}}(\hat{x}) + \mathrm{i}\bar{\theta}\theta_\mathrm{L}\mathcal{F}_e(\hat{x}) \tag{8.1}$$

while

$$\hat{E}^c = \tilde{e}_\mathrm{R}^\dagger(\hat{x}) + \mathrm{i}\sqrt{2}\bar{\theta}\psi_{E^c\mathrm{L}}(\hat{x}) + \mathrm{i}\bar{\theta}\theta_\mathrm{L}\mathcal{F}_{E^c}(\hat{x}). \tag{8.2}$$

In Eq. (8.2), the scalar component destroys the superpartner of the $SU(2)$ singlet (left-handed) positron, or creates the superpartner of the $SU(2)$ singlet (right-handed) electron, and so is written as $\tilde{e}_\mathrm{R}^\dagger$.

The familiar four-component Dirac spinor for the massive electron is built from the two Majorana spinors ψ_e and ψ_{E^c}. Since $\psi_{e\mathrm{L}}$ and $\psi_{E^c\mathrm{R}}$ have the same electric charge (see the discussion immediately following Eq. (6.38b) of Chapter 6), we may write this Dirac field as,

$$e = P_\mathrm{L}\psi_e + P_\mathrm{R}\psi_{E^c}. \tag{8.3}$$

The other massive matter fermions of the SM are similarly constructed.

[1] We do not introduce fields for the right-handed neutrinos. Although such fields are very likely to be present in nature, they will be part of some extension of the MSSM.

Exercise *The construction of the massive Dirac spinor in terms of two Majorana spinors can be most easily seen in the chiral representation for γ matrices, with*

$$\gamma^5 = \begin{pmatrix} -\mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix},$$

(the bold-face entries are 2×2 matrices). Check that the Majorana spinors

$$\psi_e = \begin{pmatrix} e_1 \\ e_2 \\ -e_2^* \\ e_1^* \end{pmatrix} \text{ and } \psi_{E^c} = \begin{pmatrix} e_4^* \\ -e_3^* \\ e_3 \\ e_4 \end{pmatrix}.$$

can be combined via Eq. (8.3) into an arbitrary Dirac spinor.

Exercise *Check that the kinetic energy terms for the Majorana spinors ψ_e and ψ_{E^c} in our master formula yield the kinetic energy term for the Dirac spinor e; i.e. verify that (up to a total derivative),*

$$\frac{\mathrm{i}}{2}\bar{\psi}_e \partial\!\!\!/ \psi_e + \frac{\mathrm{i}}{2}\bar{\psi}_{E^c} \partial\!\!\!/ \psi_{E^c} = \mathrm{i}\bar{e}\partial\!\!\!/ e. \tag{8.4}$$

The reader will have noticed that in promoting the SM fields to superfields, we have introduced many new particles, in order to complete the multiplets of supersymmetry. The existence of these new states is a prediction of supersymmetry, in exactly the same way the existence of the Ω^- was the prediction of flavor $SU(3)$ way back in the 1960s, or the existence of the Z^0 boson is a prediction of the SM symmetries. The superpartners of matter fermions are spin zero particles, known as sfermions. There is a sfermion pair (the spin zero particle and its antiparticle) for each chiral fermion in the SM, with the same internal quantum numbers as the fermion. The spin zero partners of quarks are the scalar quarks, or *squarks* for short. Likewise, the spin zero partners of the leptons are the scalar leptons or *sleptons*. Other s-words such as *selectron*, *smuon*, and *stau* are analogously defined. The subscripts L and R on the scalar fields in (8.1) and (8.2) refer to the chirality of the corresponding *electron*. These selectrons are referred to as left- (right-)selectrons, and sometimes loosely referred to as left-handed (right-handed) selectrons. It should, of course, be clear that selectrons, being spinless, cannot have handedness or chirality. Left- and right-squarks, sleptons, smuons, staus are similarly defined. The Higgs and gauge fields have fermionic superpartners respectively known as higgsinos and gauginos.

Next, we introduce the Higgs multiplets of the theory. The usual Higgs doublet of the SM is promoted to a doublet of left-chiral superfields:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \to \hat{H}_u = \begin{pmatrix} \hat{h}_u^+ \\ \hat{h}_u^0 \end{pmatrix}. \tag{8.5}$$

It transforms as a doublet **2** under $SU(2)_L$ and carries weak hypercharge $Y = 1$. The usual Yukawa interactions of its scalar component with matter fermions must arise via the superpotential, since our list of soft SUSY breaking interactions does not include interactions of chiral fermions. The VEV of the scalar component of $\hat{h}_u^0$ gives mass to up-type quarks but, unlike in the SM, cannot give a mass to the $T_3 = -1/2$ fermions. This is because the $Y = -1$ field needed to give mass to these would have to be the scalar component of the right-chiral superfield $\hat{h}_u^{0\dagger}$, and so, not allowed in the superpotential. We contrast this with the situation in the SM where the charge conjugate field $\phi^c = \mathrm{i}\sigma_2\phi^*$ with weak hypercharge $Y = -1$ could be responsible also for the mass of the down-type fermions. We are thus forced to introduce a *second* left-chiral scalar doublet superfield,

$$\hat{H}_d = \begin{pmatrix} \hat{h}_d^- \\ \hat{h}_d^0 \end{pmatrix}, \tag{8.6}$$

which transforms as a $\mathbf{2}^*$ under $SU(2)_L$ and has weak hypercharge $Y = -1$. The VEV of $\hat{h}_d^0$ can give mass to the down-type quarks and the charged leptons.

Remarkably, the introduction of this second doublet also solves another problem that we have unwittingly created. In promoting $\phi \to \hat{H}_u$, we have introduced new fermions, the hypercharge $Y = 1$ higgsinos $\psi_{h_u^+}$ and $\psi_{h_u^0}$ into the theory, which upsets the successful cancellation of triangle anomalies in the SM. However, the higgsinos in the $Y = -1$ doublet have just the right quantum numbers to restore the anomaly cancellation.

The third step in our construction procedure is to choose a superpotential to describe the interactions between the various chiral superfields. For the MSSM, we take this to be,

$$\hat{f} = \mu \hat{H}_u^a \hat{H}_{da} + \sum_{i,j=1,3} \left[(\mathbf{f}_u)_{ij} \epsilon_{ab} \hat{Q}_i^a \hat{H}_u^b \hat{U}_j^c + (\mathbf{f}_d)_{ij} \hat{Q}_i^a \hat{H}_{da} \hat{D}_j^c + (\mathbf{f}_e)_{ij} \hat{L}_i^a \hat{H}_{da} \hat{E}_j^c \right]. \tag{8.7}$$

The indices a and b are $SU(2)$ doublet indices, and explicitly exhibit the contractions needed for the invariance of the superpotential under $SU(2)_L$ transformations. In all but the second term, a doublet **2** is contracted with an antidoublet $\mathbf{2}^*$, and this contraction is trivial. In the second term, ϵ_{ab} is the completely antisymmetric $SU(2)$ tensor with $\epsilon_{12} = 1$. Its presence reflects the fact (familiar from elementary quantum mechanics) that it is the antisymmetric combination of two doublets that is an $SU(2)$ singlet. The color indices on the triplet (antitriplet) superfields $\hat{Q}$ ($\hat{U}^c$, $\hat{D}^c$) contract

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Table 8.1 *The matter and Higgs superfield content of the MSSM along with gauge transformation properties and weak hypercharge assignments, for a single generation.*

Field	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$\hat{L} = \begin{pmatrix} \hat{\nu}_{eL} \\ \hat{e}_L \end{pmatrix}$	**1**	**2**	-1
$\hat{E}^c$	**1**	**1**	2
$\hat{Q} = \begin{pmatrix} \hat{u}_L \\ \hat{d}_L \end{pmatrix}$	**3**	**2**	$\frac{1}{3}$
$\hat{U}^c$	$\mathbf{3}^*$	**1**	$-\frac{4}{3}$
$\hat{D}^c$	$\mathbf{3}^*$	**1**	$\frac{2}{3}$
$\hat{H}_u = \begin{pmatrix} \hat{h}_u^+ \\ \hat{h}_u^0 \end{pmatrix}$	**1**	**2**	1
$\hat{H}_d = \begin{pmatrix} \hat{h}_d^- \\ \hat{h}_d^0 \end{pmatrix}$	**1**	$\mathbf{2}^*$	-1

trivially, and have been suppressed. Also, it is easily checked that the hypercharge of each term sums to zero, so the superpotential is invariant under $U(1)_Y$. The **f** terms are elements of 3×3 Yukawa coupling matrices with indices $i, j = 1$–3 corresponding to the various generations. In general, the $(\mathbf{f})_{ij}$ as well as μ are complex numbers.

The reader can easily check that the superpotential in Eq. (8.7) respects baryon and lepton number conservation, where these are defined in their natural manner: $B = 1/3$ $(-1/3)$ for quark (antiquark) superfields, $L = 1$ (-1) for the lepton (antilepton) superfields, and zero for the Higgs and gauge superfields. The gauge (and gaugino) interactions on the first three lines of our master formula (6.44) obviously conserve B and L also.

Within the SM, the requirement of gauge invariance automatically guarantees baryon and lepton number conservation for all renormalizable interactions. Unfortunately, this is not the case in the MSSM. Because there are scalar fields that carry baryon or lepton number (the scalar components of quark and lepton superfields), it is possible to write down renormalizable operators that do not conserve B or L that are consistent with the SM gauge symmetries as well as supersymmetry. To see this, we simply note that the additional superpotential interactions, the terms

$$\hat{f}_{\not{L}} = \sum_{i,j,k} \left[\lambda_{ijk}\epsilon_{ab}\hat{L}_i^a\hat{L}_j^b\hat{E}_k^c + \lambda'_{ijk}\epsilon_{ab}\hat{L}_i^a\hat{Q}_j^b\hat{D}_k^c\right] + \sum_i \mu'_i\epsilon_{ab}\hat{L}_i^a\hat{H}_u^b, \tag{8.8a}$$

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and

$$\hat{f}_{\not{B}} = \sum_{i,j,k} \lambda''_{ijk} \hat{U}^c_i \hat{D}^c_j \hat{D}^c_k, \tag{8.8b}$$

are consistent with $SU(3)_C \times SU(2)_L \times U(1)_Y$ symmetry, but violate the conservation of lepton and baryon number, respectively. Since the superpotential terms (8.8a) and (8.8b) are at most cubic in the superfields, they result in renormalizable interactions that do not conserve L or B.

Obviously, the presence of such terms is dangerous since B- or L-violating processes are strongly constrained by experiment. For instance, if conservation of baryon number and lepton number are both violated, protons will decay at extremely rapid rates. In the spirit of minimality of new interactions, we will insist upon B and L conservation, and set these terms to zero. The SUSY non-renormalization theorem then ensures that these will not be radiatively generated.

Before proceeding to construct the Lagrangian for the MSSM, let us digress to discuss alternative symmetries that can be invoked to justify the absence of these terms. After all, the conservation of baryon number and lepton number are broken by non-perturbative effects, and so cannot be exact. The unwanted terms can also be forbidden by requiring that the superpotential be invariant under a new type of parity (often referred to as matter parity), where quark and lepton superfields are odd, while the gauge and Higgs superfields are even. This requirement then allows the superpotential terms in (8.7), while forbidding those in (8.8a) and (8.8b).

Exercise *Convince yourself that all the kinetic terms as well as the non-superpotential interactions in our master formula (6.44) conserve matter parity. It may be simplest to do so by observing that, except for the term involving the superpotential, all terms in (6.47) are manifestly invariant under the matter parity transformation.*

The conservation of matter parity works out to be equivalent to the conservation of R-parity defined (for the component fields) by,

$$R = (-1)^{3(B-L)+2s}, \tag{8.9}$$

where s is the spin of the field. Note that because of the $(-1)^{2s}$ dependence, the scalar and fermion (spinor and vector) components of a chiral scalar (spinor) superfield have opposite R-parities. If we now take the Grassmann co-ordinate θ to be odd under R, we see that R-parity transformation of the superfield is just the matter parity transformation discussed above. A look at (6.47) of Chapter 6 shows that R-parity violation can only come from R-odd B- and L-violating terms in the superpotential.

Exercise *Starting with the definition (8.9) for R-parity, verify that the SM fermions, gauge bosons, and both Higgs doublets are R-even, while their superpartners are R-odd. In models with conserved R-parity, this quantum number therefore provides an unambiguous distinction between "ordinary particles" and superpartners.*

It may appear that the assumption of R-parity conservation is equivalent to the conservation of B and L. This is the case for renormalizable operators in a theory whose field content is that of the MSSM. For higher dimensional operators, this need not be the case as is exemplified by the exercise below.

Exercise *Verify that the low energy superpotential of an effective low energy theory could contain the R-parity invariant operators*

$$\epsilon_{ab}\hat{L}^a\hat{H}_u^b\epsilon_{cd}\hat{L}^c\hat{H}_u^d \text{ or } \hat{U}^c\hat{U}^c\hat{D}^c\hat{E}^c.$$

Observe that the first of these violates the conservation of lepton number while the latter violates both lepton and baryon number conservation. These operators could be responsible for neutrino masses and proton decay, respectively, even if R-parity is conserved.

Can you construct an R-parity and gauge invariant operator that conserves L but not B? Such an operator could, for instance, be responsible for neutron anti-neutron oscillations.

We repeat that our choice of the MSSM superpotential to be that given by (8.7) is dictated only by constraints of minimality of new interactions. The resulting conservation of R-parity has important phenomenological consequences as we will see. We should mention, however, that it is possible to construct phenomenologically viable models in which R-parity is not conserved. Indeed, we will discuss such models in Chapter 16, but now we proceed with the construction of the MSSM.

Up to this point, we have stipulated the symmetries, field content, and superpotential of the MSSM. We can now use the master formula (6.44) to write down the complete globally supersymmetric Lagrangian. The final step is to write down the various soft SUSY breaking terms for the MSSM.

We may use Eq. (7.37) to write all gauge invariant soft SUSY breaking terms. They are

$$\begin{aligned}\mathcal{L}_{\text{soft}} = -\Big[&\tilde{Q}_i^\dagger \mathbf{m}^2_{\mathbf{Q}ij}\tilde{Q}_j + \tilde{d}^\dagger_{\text{R}i}\mathbf{m}^2_{\mathbf{D}ij}\tilde{d}_{\text{R}j} + \tilde{u}^\dagger_{\text{R}i}\mathbf{m}^2_{\mathbf{U}ij}\tilde{u}_{\text{R}j} \\ &+ \tilde{L}_i^\dagger \mathbf{m}^2_{\mathbf{L}ij}\tilde{L}_j + \tilde{e}^\dagger_{\text{R}i}\mathbf{m}^2_{\mathbf{E}ij}\tilde{e}_{\text{R}j} + m^2_{H_u}|H_u|^2 + m^2_{H_d}|H_d|^2\Big] \\ &-\frac{1}{2}\left[M_1\bar{\lambda}_0\lambda_0 + M_2\bar{\lambda}_A\lambda_A + M_3\bar{\tilde{g}}_B\tilde{g}_B\right] \\ &-\frac{\mathrm{i}}{2}\left[M_1'\bar{\lambda}_0\gamma_5\lambda_0 + M_2'\bar{\lambda}_A\gamma_5\lambda_A + M_3'\bar{\tilde{g}}_B\gamma_5\tilde{g}_B\right] \\ &+\Big[(\mathbf{a_u})_{ij}\epsilon_{ab}\tilde{Q}_i^a H_u^b \tilde{u}^\dagger_{\text{R}j} + (\mathbf{a_d})_{ij}\tilde{Q}_i^a H_{da}\tilde{d}^\dagger_{\text{R}j} + (\mathbf{a_e})_{ij}\tilde{L}_i^a H_{da}\tilde{e}^\dagger_{\text{R}j} + \text{ h.c.}\Big] \\ &+\Big[(\mathbf{c_u})_{ij}\epsilon_{ab}\tilde{Q}_i^a H_d^{*b} \tilde{u}^\dagger_{\text{R}j} + (\mathbf{c_d})_{ij}\tilde{Q}_i^a H^*_{ua}\tilde{d}^\dagger_{\text{R}j} + (\mathbf{c_e})_{ij}\tilde{L}_i^a H^*_{ua}\tilde{e}^\dagger_{\text{R}j} + \text{ h.c.}\Big] \\ &+\left[bH_u^a H_{da} + \text{ h.c.}\right], \end{aligned} \tag{8.10}$$

where the generation indices i, j, as well as the $SU(2)$ indices a, b, are implicitly summed over. Hermiticity requires that the scalar mass squared matrices are 3×3 Hermitian matrices, each of which can be written in terms of 6 real and 3 imaginary parameters. The six gaugino mass parameters (M_i, M_i') with $i = 1$–3 corresponding to the three factors of the MSSM gauge group, are real. The terms with M's violate CP invariance. The **a** and **c** matrices that describe trilinear scalar interactions are general 3×3 complex matrices, just like the Yukawa matrices. The parameters $m^2_{H_u}$ and $m^2_{H_d}$ are real, while the b bilinear term is, in general, complex. The trilinear interactions involving **c** matrices are frequently not written down because such terms are strongly suppressed in many models, but there is really no reason to exclude these within the MSSM framework.

At this point, we have the complete Lagrangian for the MSSM. Of course, it is written in terms of fields with definite quantum numbers for the gauge group. Upon spontaneous symmetry breaking, fields with the same color, electric charge, and spin may mix. The spectrum and couplings of the mass eigenstates have to be extracted from this Lagrangian.

8.1.1 Parameter space of the MSSM

It is now worthwhile to count the free parameters that enter the MSSM Lagrangian. Recall that the SM has nineteen free parameters: three gauge couplings g_1, g_2, and g_3, the parameter θ_{QCD}, μ and λ from the Higgs potential, six quark and three lepton masses, plus three mixing angles, and one CP-violating phase in the Kobayashi–Maskawa matrix.

In the MSSM, we have in the gauge sector again g_1, g_2, g_3, and θ_{QCD}, plus we have six gaugino masses M_1, M_2, M_3 and M_1', M_2', and M_3'. As noted in the exercise

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below, one of the CP-violating gaugino masses can be removed by performing a chirality transformation of the gaugino field. By convention, we choose this to be M_3'. Thus, we have nine parameters in the gauge sector of the model.

In the Higgs sector, we have the real mass terms $m_{H_u}^2$ and $m_{H_d}^2$, together with μ from the superpotential and its corresponding soft SUSY breaking term b. The latter two are complex but one of the phases, usually taken to be the one associated with b, can be absorbed by redefining the overall phase of one of the Higgs fields. Thus, in the Higgs sector of the MSSM, we have five real parameters.

Finally, we turn to the matter fermions and their superpartners. First, there are five soft SUSY breaking Hermitian mass matrices for the scalar partners of the quarks and leptons, with six real parameters plus three phases each, for a total of 45 parameters. Then, we have three 3×3 complex Yukawa coupling matrices ($18 \times 3 = 54$ parameters). There are another 54 terms in three corresponding **a**-parameter matrices and the same number in the **c** matrices. This gives a total of 207 parameters in the flavor sector, but not all of them are physical.

To count the number of unphysical parameters, i.e. those parameters that can be removed by field redefinitions, we first note that the kinetic terms and gauge interactions are invariant under a global $U(3)^5$ transformation, one $U(3)$ corresponding to transformations amongst each of the three $\hat{L}_i$, $\hat{E}_i^c$, $\hat{Q}_i$, $\hat{U}_i^c$, and $\hat{D}_i^c$. It is just the superpotential Yukawa terms, and the SUSY breaking **a** and **c** terms that are not invariant under these global chiral transformations, which can thus be used to remove some of these parameters. Since any $U(3)$ can be parametrized by three angles and six phases, $5 \times (3 + 6) = 45$ parameters of the 207 that we obtained above should be removable. However, two of the phases in $U(3)^5$ correspond to the conservation of the total B and L: since the corresponding transformations leave the Lagrangian invariant, they cannot be used to do any useful field redefinitions. Summing up the gauge, Higgs, and matter sectors, we have a model with a total of $9 + 5 + 207 - 43 = 178$ parameters in the MSSM. As we stated above, the 54 **c** parameters are usually not included in the MSSM which is then said to contain 124 parameters.

Presumably, once we understand the mechanism underlying SUSY breaking (including how it is conveyed to the superpartners of the SM particles), it will be possible to reduce this plethora of parameters to a handful of truly fundamental parameters. But until then, one of the principal goals of model builders is to arrive at phenomenologically viable but economic models, based on well-motivated assumptions of physics at high energy scales, each with just a few model parameters. It is reasonable to expect that once sparticles are discovered, a determination of their properties will serve to discriminate between these models, thereby pointing the way to the underlying theory.

Exercise *Show that, for an appropriate choice of* ϕ*, the transformations* $\tilde{g}_L \to e^{-i\phi}\tilde{g}_L$*,* $\tilde{g}_R \to e^{i\phi}\tilde{g}_R$ *(this maintains the Majorana property of* $\tilde{g}$*) can be used to eliminate the* CP*-violating mass parameter* M_3' *of the gluino field. In fact, we used this procedure in our discussion following Eq. (7.39) of the technical note in Chapter 7. The phase* ϕ*, which then shows up in the* $\tilde{g}q\tilde{q}_{L,R}$ *couplings, can be absorbed by redefinition of the squark fields. We should emphasize that this does not mean that the* M_3' *term is irrelevant, but only that this term does not give rise to observable* CP*-violation. The physical mass of the gluino is* $m_{\tilde{g}} = \sqrt{M_3^2 + M_3'^2}$*, and it is this quantity that appears as the coefficient of the "usual gluino mass term" after the* CP*-violating gluino bilinear is rotated away.*

Note also that we cannot simultaneously remove M_1' *or* M_2' *since the phase that needs to be absorbed into the squark field will now be different. That we choose to remove the* CP*-violating mass of the gluino rather than the* $SU(2)$ *or* $U(1)$ *gaugino is, of course, only a convention.*

Exercise *Using arguments similar to the ones for the MSSM, show that the Yukawa sector of the SM with n generations (assuming neutrinos are massless) contains* n *real parameters in the lepton sector and* $n(n+3)/2$ *real parameters and* $n(n-3)/2+1$ *phases in the quark sector.*

Note that, unlike the MSSM, the SM with massless neutrinos separately conserves the lepton number for each generation.

8.1.2 A simplified parameter space

We have just seen that the MSSM contains an intractably large number of parameters for meaningful phenomenological analyses. While we have no direct knowledge of these parameters, we can nonetheless make reasonable simplifying assumptions to facilitate our discussion.

We begin by recalling that our motivation for weak scale supersymmetry was to stabilize the electroweak symmetry breaking sector of the SM which suffered from the presence of quadratic divergences. We saw, at least by example, that softly broken SUSY theories have the virtue that they do not suffer from these: the masses of the superpartners set the scale for radiative corrections to the Higgs boson mass, and hence the weak scale. This is the *raison d'être* for weak scale supersymmetry. We thus *require* that the SUSY breaking parameters as well as μ are in the range of the weak scale, or at least not larger than a few TeV. This is our most important assumption.

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Next, we note that the scalar matter sector of the MSSM generically would have large flavor violation in both the squark and slepton sectors if the off-diagonal terms in the corresponding mass matrices or the **a** or **c** matrices are comparable to the diagonal terms. Moreover, we saw that the scalar sector of the MSSM has many physical phases that serve as novel sources of CP violation. Even at low energies (well below the SUSY threshold) SM particles would "feel" these sources of flavor and CP violation through SUSY particles in loop diagrams. The magnitude of these effects, of course, depends on the sparticle masses. There are experimental bounds on lepton flavor violation and on CP violation that stringently restrict the size of some of the off-diagonal terms as well as phases referred to in Section 8.1.1. For instance, large off-diagonal contributions to slepton mass matrices would lead to large decay rates for $\mu \to e\gamma$. Large off-diagonal terms in the squark mass matrices are greatly restricted by K^0–$\overline{K}^0$, D–$\overline{D}$, and B–$\overline{B}$ mixing, and by processes such as $b \to s\gamma$, $b \to s\ell\bar{\ell}$ or $K^0 \to \mu^+\mu^-$ decays. Large off-diagonal terms in the trilinear **a** and **c** matrices are similarly restricted. There are also strong constraints on CP-violating parameters from measurements of the electron and neutron electric dipole moments.

In the following, we will for simplicity set all SUSY sources of CP violation to zero. In addition, we will also assume that squark and slepton matrices as well as the **a** matrices are diagonal in the same basis that the fermion Yukawa couplings are diagonal. Following common practice, we will set the **c** terms to zero. This is because these terms are frequently small in many models. These simplifying assumptions may well prove to be incorrect. It could turn out that experiments may show that nature requires sources of flavor or CP violation beyond those present in the SM. While there is scant evidence for this at the present time, things could be different in the future. We should also stress that our predictions for even the simplest properties (such as mass) of SUSY particles are sensitive to these assumptions. In the interest of pedagogy, however, we will continue to work within the simplified framework, and leave it to the reader to make the appropriate modifications in more complicated frameworks.

Finally, since our main focus is on SUSY particles, we will keep track of only the third generation Yukawa couplings, and neglect Yukawa interactions of the first two generations. This is obviously unrealistic, but has little effect for most things that we will study. In other words, we will approximate the Yukawa matrices in the superpotential by,

$$\mathbf{f}_e \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & f_\tau \end{pmatrix}, \quad \mathbf{f}_u \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & f_t \end{pmatrix}, \quad \mathbf{f}_d \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & f_b \end{pmatrix}.$$

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Frequently, the matrices $\mathbf{a}_{ij}$ are written as $A_{ij}\mathbf{f}_{ij}$. Within our approximation, these will then take the form,

$$\mathbf{a}_e \sim \begin{pmatrix} 0\,0 & 0 \\ 0\,0 & 0 \\ 0\,0 & f_\tau A_\tau \end{pmatrix}, \quad \mathbf{a}_u \sim \begin{pmatrix} 0\,0 & 0 \\ 0\,0 & 0 \\ 0\,0 & f_t A_t \end{pmatrix}, \quad \mathbf{a}_d \sim \begin{pmatrix} 0\,0 & 0 \\ 0\,0 & 0 \\ 0\,0 & f_b A_b \end{pmatrix},$$

with A_τ, A_t, and A_b being real parameters (the A-terms). The bilinear b term is, likewise, written as $b = B\mu$, where B is taken to be real. The parametrization of the **a** and b terms in terms of the corresponding superpotential interactions is motivated by gravity-mediated models (to be discussed later). Indeed, within the MSSM framework, the soft breaking scalar parameters are completely unrelated to the parameters in the superpotential, i.e. **a** may be non-zero even if the Yukawa couplings vanish and, further, **c** terms need not be small.

From this point onwards, unless explicitly stated, we will assume that we are working within the simplified parameter space.

8.2 Electroweak symmetry breaking

The theory we have written down so far respects the gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$. Our next task is to ensure that the gauge symmetry of the MSSM can be successfully broken down to observed $SU(3)_C \times U(1)_{em}$, so that W and Z bosons and fermions may receive mass as they do in the SM.

To investigate electroweak symmetry breaking, we must examine the minima of the scalar potential in the MSSM. The tree-level scalar potential consists of three parts

$$V_{\text{MSSM}} = V_F + V_D + V_{\text{soft}}, \tag{8.13}$$

where

$$V_F = \sum_i \left| \frac{\partial \hat{f}}{\partial \hat{\mathcal{S}}_i} \right|^2_{\hat{\mathcal{S}}=\mathcal{S}}, \tag{8.14a}$$

$$V_D = \frac{1}{2} \sum_A \left[\sum_i \mathcal{S}_i^\dagger g t_A \mathcal{S}_i \right]^2 \quad \text{and} \tag{8.14b}$$

$$V_{\text{soft}} = \sum_i m_{\phi_i}^2 |\phi_i|^2 - B\mu\,(H_d H_u + \quad \text{h.c.}) + \text{a-terms}. \tag{8.14c}$$

The sum over i is over all scalar fields in the model. Each real component of each scalar field may be regarded as a separate direction in "field space". Thus, the scalar "field space" of the MSSM, with 14 real matter scalars per generation, plus four complex Higgs scalars, is a 50-dimensional space. We look for parameter

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regions where this scalar potential develops a minimum along "directions" of the Higgs scalars. If a deeper minimum develops along other scalar field directions, then the ground state of the theory could develop such that electric charge, color or lepton number symmetry is broken. In fact, these considerations can be used to put constraints on the parameters of the theory. We will assume here that such non-standard minima do not develop.

We can then restrict our attention to the scalar potential involving only the Higgs scalar fields. We may use the $SU(2)_L$ gauge symmetry freedom to rotate the VEV of H_u to its lower component which we have *defined* to be neutral. Minimization of the potential with respect to the other component of H_u then requires that $\langle h_d^- \rangle = 0$ as demonstrated in the following exercise. The MSSM Higgs potential, therefore, allows only charge-conserving vacua.[2]

Exercise *Verify that for the Higgs fields, V_D can be written as,*

$$V_D^{\text{Higgs}} = \frac{g^2 + g'^2}{8}(\mathcal{A}_u^2 + \mathcal{A}_d^2) + \frac{g^2 - g'^2}{4}\mathcal{A}_u\mathcal{A}_d - \frac{g^2}{2}|\mathcal{A}_{ud}|^2,$$

where $\mathcal{A}_u = |H_u|^2$, $\mathcal{A}_d = |H_d|^2$, and $\mathcal{A}_{ud} = H_u H_d$. The tree-level Higgs potential to be minimized is,

$$V^{\text{Higgs}} = (m_{H_u}^2 + \mu^2)\mathcal{A}_u + (m_{H_d}^2 + \mu^2)\mathcal{A}_d - B\mu(\mathcal{A}_{ud} + \mathcal{A}_{ud}^\dagger) + V_D^{\text{Higgs}}.$$

We see that for fixed magnitudes of H_u and H_d, i.e. fixed values of $\mathcal{A}_u$ and $\mathcal{A}_d$, the minimum of V^{Higgs} is obtained by making $|\mathcal{A}_{ud}|$ as large as possible. This means, of course, that H_d and H_u are aligned, so that $\langle h_d^- \rangle = 0$. Moreover, for real values of $B\mu$ the second last term in V^{Higgs} is minimized when $\mathcal{A}_{ud}$ is real and positive (negative) if $B\mu$ is positive (negative). Thus as long as the parameters of the Higgs potential are real, no CP-violating phases are induced by the interactions of Higgs bosons.

Notice that there is no loss of generality if we choose the VEVs of both fields to have the same sign as long as the sign of $B\mu$ can always be appropriately chosen.

We then only have to minimize the scalar potential for the "neutral fields" which now reads,

$$\begin{aligned} V_{\text{scalar}} = {} & (m_{H_u}^2 + \mu^2)|h_u^0|^2 + (m_{H_d}^2 + \mu^2)|h_d^0|^2 \\ & - B\mu(h_u^0 h_d^0 + \text{h.c.}) + \frac{1}{8}(g^2 + g'^2)\left(|h_u^0|^2 - |h_d^0|^2\right)^2. \end{aligned} \tag{8.15}$$

[2] Of course, we still have to assume that the matter scalars do not develop VEVs.

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To find the minimum of the scalar potential, we set the first derivatives of this potential with respect to the fields as well as to their conjugates to zero:

$$\frac{\partial V}{\partial h_u^{0*}} = (m_{H_u}^2 + \mu^2)h_u^0 - B\mu h_d^{0*} + \frac{1}{4}(g^2 + g'^2)h_u^0(|h_u^0|^2 - |h_d^0|^2) = 0, \tag{8.16a}$$

$$\frac{\partial V}{\partial h_d^{0*}} = (m_{H_d}^2 + \mu^2)h_d^0 - B\mu h_u^{0*} - \frac{1}{4}(g^2 + g'^2)h_d^0(|h_u^0|^2 - |h_d^0|^2) = 0. \tag{8.16b}$$

The point(s) in field space where these equations are satisfied is an extremum of the (tree-level) potential. One possible solution is $\langle h_u^0 \rangle = \langle h_d^0 \rangle = 0$, i.e. no electroweak symmetry breaking. To ensure that this does not occur, the origin must be a local maximum of the potential. In other words, the determinant of the matrix of second derivatives should be negative at the origin. Since we are interested in the evaluation of the second derivatives at the origin of field space just the bilinear terms contribute, and we must have,

$$(B\mu)^2 > (m_{H_u}^2 + \mu^2)(m_{H_d}^2 + \mu^2). \tag{8.17a}$$

We must also check that the scalar potential indeed has a stable minimum, and is not unbounded from below. For most field values this is not an issue because the positive definite quartic term dominates the scalar potential for large field values. However, in the direction of field space where $|h_u^0| = |h_d^0|$, the quartic term vanishes. This is a D-flat direction in field space, and in this direction we must require the scalar potential to be positive. This leads to

$$m_{H_u}^2 + m_{H_d}^2 + 2\mu^2 > 2|B\mu|. \tag{8.17b}$$

If these conditions are met, then the scalar potential should develop a well-defined local minimum in which electroweak symmetry is spontaneously broken. We write $\langle h_u^0 \rangle \equiv v_u$ and $\langle h_d^0 \rangle \equiv v_d$ with the VEVs as real numbers, and define a parameter,

$$\tan\beta \equiv \frac{v_u}{v_d} \tag{8.18}$$

that will play an important role in phenomenological studies of the MSSM. It is simple to see that the potential minimization conditions can be written as:

$$B\mu = \frac{(m_{H_u}^2 + m_{H_d}^2 + 2\mu^2)\sin 2\beta}{2} \quad \text{and} \tag{8.19a}$$

$$\mu^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{(\tan^2\beta - 1)} - \frac{M_Z^2}{2}. \tag{8.19b}$$

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To obtain (8.19b), we have used the relation (to be derived shortly), $M_Z^2 = \frac{(g^2+g'^2)}{2}(v_u^2 + v_d^2)$. The first of these equations allows one to trade the parameter $B\mu$ for the more commonly used parameter $\tan\beta$. Given the soft SUSY breaking Higgs masses $m_{H_u}^2$ and $m_{H_d}^2$, we use the second to fix the magnitude (but not the sign) of μ to reproduce the observed value of M_Z.

Up to now, we have focussed on the tree-level potential (8.15) for the electroweak symmetry breaking sector of the MSSM. A characteristic feature of this potential is that the quartic self-interactions of the Higgs fields are determined solely by the $SU(2) \times U(1)$ gauge couplings. This implies that the Higgs sector of the MSSM automatically satisfies perturbative unitarity constraints, in sharp contrast to the SM where the Higgs self-coupling constant is an independent parameter. This important feature of the MSSM can be traced to the fact that the μ term is the only possible superpotential term bilinear in the Higgs superfields. Indeed, as we will see, the structure of the self-couplings in the Higgs sector of the MSSM implies an *upper* limit of M_Z on the mass of the SM-like Higgs boson! This is a tree-level result, and radiative corrections modify it in an important way. We will, however, postpone any further discussion about this until we are ready to examine the spectrum of the relics of the electroweak symmetry breaking sector of the MSSM.

8.3 Particle masses in the MSSM

8.3.1 Gauge bosons

Once we are assured of the correct pattern of electroweak symmetry breaking, we can proceed to calculate the masses of the vector bosons. Since the vacuum does not spontaneously break the $U(1)_{\rm em}$ associated with electromagnetic gauge invariance, we expect that the photon will remain massless, while the $W^\pm$ and Z^0 will acquire a mass via the Higgs mechanism. As in the SM, these vector boson mass terms arise from the kinetic energy terms of the Higgs fields:

$$\mathcal{L} \ni |D_\mu H_u|^2 + |D_\mu H_d|^2, \tag{8.20}$$

where

$$D_\mu H_u = (\partial_\mu + {\rm i}g\frac{\tau_A}{2}W_{A\mu} + {\rm i}\frac{g'}{2}B_\mu)H_u \quad \text{and}$$
$$D_\mu H_d = (\partial_\mu + {\rm i}g(-\frac{\tau_A^*}{2})W_{A\mu} - {\rm i}\frac{g'}{2}B_\mu)H_d.$$

The vector boson masses are obtained by making the replacement,

$$\langle H_u \rangle \to \begin{pmatrix} 0 \\ v_u \end{pmatrix} \text{ and } \langle H_d \rangle \to \begin{pmatrix} 0 \\ v_d \end{pmatrix}. \tag{8.21}$$

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Identifying the charged fields,

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_{1\mu} \mp \mathrm{i}W_{2\mu}),$$

we find,

$$M_W^2 = \frac{g^2}{2}(v_u^2 + v_d^2). \tag{8.22a}$$

As in the SM, the neutral fields $W_{3\mu}$ and B_μ mix, and the neutral mass matrix has to be diagonalized. Diagonalizing this mass matrix yields the fields,

$$A_\mu = \frac{(g' W_{3\mu} + g B_\mu)}{\sqrt{g^2 + g'^2}}$$
$$Z_\mu = \frac{(-g W_{3\mu} + g' B_\mu)}{\sqrt{g^2 + g'^2}}.$$

A_μ is massless and identified as the photon field. The other field has a mass,

$$M_Z^2 = \frac{g^2 + g'^2}{2}(v_u^2 + v_d^2). \tag{8.22b}$$

Defining the weak mixing angle by $\tan\theta_W \equiv g'/g$, we recover the SM relation $M_W = M_Z \cos\theta_\mathrm{W}$.

Instead of working with H_u and H_d, we could have equally well worked with the linear combinations,

$$\phi = \sin\beta H_u + \cos\beta H_d^*,$$
$$\phi' = \cos\beta H_u - \sin\beta H_d^*.$$

The doublet ϕ acquires a VEV $v \equiv \sqrt{v_u^2 + v_d^2} \simeq 174\,\mathrm{GeV}$ for its neutral component and can be identified with the SM Higgs doublet. The field ϕ' does not acquire a VEV and is just an additional scalar field that has nothing to do with symmetry breaking.

8.3.2 Matter fermions

Matter fermions acquire masses via Yukawa interactions in the superpotential. Specifically, these masses arise from the terms

$$\mathcal{L} \ni -\frac{1}{2}\sum_{i,j}\bar{\psi}_i\left[\left(\frac{\partial^2 \hat{f}}{\partial\hat{\mathcal{S}}_i\partial\hat{\mathcal{S}}_j}\right)_{\hat{\mathcal{S}}=\mathcal{S}}\frac{1-\gamma_5}{2} + \left(\frac{\partial^2 \hat{f}}{\partial\hat{\mathcal{S}}_i\partial\hat{\mathcal{S}}_j}\right)^\dagger_{\hat{\mathcal{S}}=\mathcal{S}}\frac{1+\gamma_5}{2}\right]\psi_j$$

in our master formula. We will focus on the mass of the electron; the calculation of other SM fermion masses follows along identical lines.

We first note that, since the superpotential contains the term $\hat{f} \ni f_e \hat{e} \hat{h}_d^0 \hat{E}^c$, we find

$$\left.\frac{\partial^2 \hat{f}}{\partial \hat{e} \partial \hat{E}^c}\right|_{\hat{\mathcal{S}}=\mathcal{S}} = f_e \hat{h}_d^0|_{\hat{h}_d^0=h_d^0} = f_e h_d^0,$$

so that

$$\begin{aligned}\mathcal{L} \ni & -\frac{1}{2}\bar{\psi}_e \left[f_e h_d^0 \frac{1-\gamma_5}{2} + f_e h_d^{0*} \frac{1+\gamma_5}{2} \right] \psi_{E^c} \\ & -\frac{1}{2}\bar{\psi}_{E^c} \left[f_e h_d^0 \frac{1-\gamma_5}{2} + f_e h_d^{0*} \frac{1+\gamma_5}{2} \right] \psi_e \\ = & -\left[\bar{\psi}_{E^c} f_e h_d^0 \frac{1-\gamma_5}{2} \psi_e + \bar{\psi}_e f_e h_d^{0*} \frac{1+\gamma_5}{2} \psi_{E^c} \right],\end{aligned}$$

where in the last step we have used the Majorana bilinear relations to combine terms. Using the definition (8.3) of the Dirac electron field, and replacing the field h_d^0 by its VEV, the reader can easily check that these terms reduce to a mass term for the Dirac electron. Specifically,

$$\mathcal{L} \ni -f_e v_d \bar{e} e = -m_e \bar{e} e, \tag{8.23}$$

with $m_e \equiv f_e v_d$. Thus, as in the SM, the electron acquires a mass via its coupling to the Higgs field. This justifies our calling the superpotential coupling f_e as the electron Yukawa coupling. Note that in the MSSM the electron mass comes from $\langle h_d^0 \rangle$. The same is true for the other charged leptons and down-type quarks that couple just to the doublet H_d via superpotential interactions. A similar calculation for the masses of $T_3 = +1/2$ fermions of the SM finds their masses proportional to v_u. The neutrino, of course, remains massless just as in the SM, since we have not introduced a Yukawa coupling for it.

Exercise *Verify that the fermion Yukawa couplings can be written as,*

$$f_i = \frac{g m_i}{\sqrt{2} M_W} 1/\sin\beta, \text{ if } T_{3f} = \frac{1}{2}, \tag{8.24a}$$

and

$$f_i = \frac{g m_i}{\sqrt{2} M_W} 1/\cos\beta, \text{ if } T_{3f} = -\frac{1}{2}. \tag{8.24b}$$

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Notice that these expressions for the MSSM Yukawa couplings f_i in terms of the fermion masses are different from the corresponding expressions for the SM Yukawa couplings λ_i.

We remark that since the mass of the fermions arises from the superpotential, it must be supersymmetric, i.e. the scalar superpartners will get an identical contribution to the mass from the superpotential Yukawa couplings. This should not be surprising since we have already seen that we cannot have soft SUSY breaking masses for chiral fermions.

8.3.3 Higgs bosons

Before turning to the masses of the superpartners, let us examine the spectrum of physical particles from the electroweak symmetry breaking sector. Within the SM with just one complex doublet, we know that a single neutral spin zero particle – the Higgs boson – is left in the spectrum as a relic of the spontaneous breakdown of $SU(2)_{\mathrm{L}} \times U(1)_{\mathrm{Y}} \to U(1)_{\mathrm{em}}$. This is because the charged component of the doublet and one of the neutral components are the three would-be Goldstone bosons that become the longitudinal components of the $W^{\pm}$ and Z^0 after the Higgs mechanism. Since the symmetry breaking pattern of the MSSM is the same as that of the SM, we expect the same set of would-be Goldstone bosons: however, since we now start with two sets of complex doublets, one charged and three neutral spin zero bosons remain in the physical spectrum of the MSSM.

In order to identify these states and compute their masses , we must examine the Higgs potential:

$$\begin{aligned}
V^{\mathrm{Higgs}} = & (m_{H_u}^2 + \mu^2)(|h_u^0|^2 + |h_u^+|^2) + (m_{H_d}^2 + \mu^2)(|h_d^0|^2 + |h_d^-|^2) \\
& - B\mu(h_u^+ h_d^- + h_u^0 h_d^0 + \quad \text{h.c.}) \\
& + \frac{g^2}{8}\left\{(|h_u^+|^2 - |h_u^0|^2 + |h_d^0|^2 - |h_d^-|^2)^2 + 4|h_u^+|^2|h_u^0|^2 + 4|h_d^0|^2|h_d^-|^2\right. \\
& \left. - 4\,(h_u^{+*}h_d^{-*}h_u^0 h_d^0 + h_u^{0*}h_d^{0*}h_u^+ h_d^-)\right\} \\
& + \frac{g'^2}{8}\left[|h_u^+|^2 + |h_u^0|^2 - |h_d^0|^2 - |h_d^-|^2\right]^2 .
\end{aligned} \tag{8.25}$$

The neutral fields may be broken up into real and imaginary components $h_u^0 = \frac{h_{u\mathrm{R}}^0 + \mathrm{i}h_{u\mathrm{I}}^0}{\sqrt{2}}$ and $h_d^0 = \frac{h_{d\mathrm{R}}^0 + \mathrm{i}h_{d\mathrm{I}}^0}{\sqrt{2}}$, so that the scalar potential can be regarded as a function $V(h_{u\mathrm{R}}^0, h_{u\mathrm{I}}^0, h_{d\mathrm{R}}^0, h_{d\mathrm{I}}^0, h_u^+, h_u^{+*}, h_d^-, h_d^{-*})$ of eight independent fields. Since we are interested in excitations of the vacuum, we expand the Higgs potential about its

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minimum as,

$$V^{\text{Higgs}} = V_{\min} + \sum_{h_i} \left.\frac{\partial V}{\partial h_i}\right|_{h_i=\langle h_i\rangle} (h_i - \langle h_i\rangle) + \frac{1}{2}\sum_{h_i,h_j} \left.\frac{\partial^2 V}{\partial h_i \partial h_j}\right|_{h_{i,j}=\langle h_{i,j}\rangle} (h_i - \langle h_i\rangle)(h_j - \langle h_j\rangle) + \cdots , \quad (8.26)$$

where the h_i are the eight arguments of V as listed above, and the only non-vanishing VEVs are $\langle h^0_{d\text{R}}\rangle = \sqrt{2}v_d$ and $\langle h^0_{u\text{R}}\rangle = \sqrt{2}v_u$. The coefficients of the linear terms should all vanish, since the derivatives are evaluated at the minimum of the potential; the quadratic terms will then be Higgs boson mass terms, and since in general there will be mixing, these will form mass matrices. The conservation of electric charge means that there can be no mixing between charged and neutral Higgs fields, so that there is one mass matrix for the charged sector and a different one in the neutral sector. Moreover, because of the (assumed) CP invariance of the Higgs sector, the real and imaginary components of the neutral Higgs bosons do not mix either, so that the 4×4 mass matrix in the neutral sector decomposes into two 2×2 blocks.

First, let us construct the mass matrices that contain the would-be Goldstone bosons. These reside in the charged sector and in the CP-odd sector (i.e. the imaginary components) of the neutral fields. The states orthogonal to the Goldstone boson will automatically be the physical states in these sectors.

We begin with the charged fields. The Lagrangian will have the form

$$\mathcal{L} \ni \left(h_u^{+*}\; h_d^-\right) \mathcal{M}^2_{h^\pm} \begin{pmatrix} h_u^+ \\ h_d^{-*} \end{pmatrix}, \quad (8.27)$$

where

$$\mathcal{M}^2_{h^\pm} = \begin{pmatrix} \left.\frac{\partial^2 V}{\partial h_u^+ \partial h_u^{+*}}\right|_{h_i\to v_i} & \left.\frac{\partial^2 V}{\partial h_u^{+*} \partial h_d^{-*}}\right|_{h_i\to v_i} \\ \left.\frac{\partial^2 V}{\partial h_u^+ \partial h_d^-}\right|_{h_i\to v_i} & \left.\frac{\partial^2 V}{\partial h_d^- \partial h_d^{-*}}\right|_{h_i\to v_i} \end{pmatrix}.$$

The derivatives are simple to compute if we remember that we want to evaluate these at the VEV of the Higgs fields; then we can drop terms that are proportional to h_u^+, h_d^-, $h^0_{u\text{I}}$ or $h^0_{d\text{I}}$ (after the derivatives are taken) as these fields vanish in the vacuum. For instance,

$$\begin{aligned} \left.\frac{\partial^2 V}{\partial h_u^+ \partial h_u^{+*}}\right|_{h_i\to v_i} &= (m^2_{H_u} + \mu^2) + \frac{g^2}{4}\left(v_u^2 + v_d^2\right) + \frac{g'^2}{4}(v_u^2 - v_d^2) \\ &= B\mu \cot\beta + \frac{g^2}{2} v_d^2, \end{aligned}$$

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where in the last step we have used the first of the minimization conditions (8.16a) to eliminate $m_{H_u}^2 + \mu^2$ in favor of $B\mu$. The mass squared matrix in the charged sector is found to be,

$$\mathcal{M}_{h^\pm}^2 = \begin{pmatrix} B\mu\cot\beta + \frac{g^2}{2}v_d^2 & -B\mu - \frac{g^2}{2}v_u v_d \\ -B\mu - \frac{g^2}{2}v_u v_d & B\mu\tan\beta + \frac{g^2}{2}v_u^2 \end{pmatrix}, \tag{8.28}$$

where we have used (8.16b) to eliminate $m_{H_d}^2 + \mu^2$ from the lower right entry of the matrix. Its eigenvalues are given by

$$m_{G^\pm} = 0 \quad \text{and} \quad m_{H^\pm}^2 = B\mu(\cot\beta + \tan\beta) + M_W^2. \tag{8.29}$$

The zero eigenvalue merely confirms that, but for the Higgs mechanism, $G^\pm$ would have been the Goldstone boson. In the unitarity gauge, these do not appear in the Lagrangian with massive W bosons. The other state, $H^\pm$, remains in the spectrum. The mixing matrix takes the form,

$$\begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} h_d^{-*} \\ h_u^+ \end{pmatrix}. \tag{8.30}$$

Let us now turn to the neutral sector, focussing for the moment on the mass terms for the imaginary components of the neutral fields. These may be written as,

$$\mathcal{L} \ni \frac{1}{2}\begin{pmatrix} h_{u\mathrm{I}}^0 & h_{d\mathrm{I}}^0 \end{pmatrix} \mathcal{M}_{h_{i\mathrm{I}}^0}^2 \begin{pmatrix} h_{u\mathrm{I}}^0 \\ h_{d\mathrm{I}}^0 \end{pmatrix}, \tag{8.31}$$

with

$$\mathcal{M}_{h_{i\mathrm{I}}^0}^2 = \begin{pmatrix} \left.\frac{\partial^2 V}{\partial h_{u\mathrm{I}}^{02}}\right|_{h_i \to v_i} & \left.\frac{\partial^2 V}{\partial h_{u\mathrm{I}}^0 \partial h_{d\mathrm{I}}^0}\right|_{h_i \to v_i} \\ \left.\frac{\partial^2 V}{\partial h_{u\mathrm{I}}^0 \partial h_{d\mathrm{I}}^0}\right|_{h_i \to v_i} & \left.\frac{\partial^2 V}{\partial h_{d\mathrm{I}}^{02}}\right|_{h_i \to v_i} \end{pmatrix}.$$

A computation similar to that for the charged sector gives,

$$\mathcal{M}_{h_{i\mathrm{I}}^0}^2 = \begin{pmatrix} B\mu\cot\beta & B\mu \\ B\mu & B\mu\tan\beta \end{pmatrix}. \tag{8.32}$$

The eigenvalues are,

$$m_{G^0} = 0 \quad \text{and} \quad m_A^2 = B\mu(\cot\beta + \tan\beta). \tag{8.33}$$

From the eigenvalue corresponding to $m_{H^\pm}^2$, we see that

$$m_{H^\pm}^2 = m_A^2 + M_W^2, \tag{8.34}$$

so that, at least at tree level, $m_{H^\pm} \geq M_W$ and $m_{H^\pm} \geq m_A$.

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Exercise *We have already argued that the real and imaginary components of the neutral fields cannot mix. Explicitly verify that this is indeed the case. Also, verify the mass matrix is indeed given by (8.32). To obtain the diagonal entries, you will once again have to use the minimization conditions.*

Again, after a gauge transformation to the unitarity gauge, G^0 disappears from the Lagrangian which now includes a mass for the Z^0 boson. The massive A particle remains as a *pseudoscalar* Higgs boson, as will be seen when we calculate its couplings to matter fermions.[3] The mixing matrix for G^0 and A is

$$\begin{pmatrix} G^0 \\ A \end{pmatrix} = \begin{pmatrix} \sin\beta & -\cos\beta \\ \cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} h^0_{u\mathrm{I}} \\ h^0_{d\mathrm{I}} \end{pmatrix} . \tag{8.35}$$

Finally, let us turn to the mass matrix for the remaining neutral scalars involving the $h^0_{u\mathrm{R}}$ and $h^0_{d\mathrm{R}}$. The mass squared matrix of the real components of the neutral Higgs scalars occurs in the Lagrangian as,

$$\mathcal{L} \ni \frac{1}{2} \left(h^0_{u\mathrm{R}} \; h^0_{d\mathrm{R}} \right) \mathcal{M}^2_{h^0_{i\mathrm{R}}} \begin{pmatrix} h^0_{u\mathrm{R}} \\ h^0_{d\mathrm{R}} \end{pmatrix} , \tag{8.36}$$

with

$$\mathcal{M}^2_{h^0_{i\mathrm{R}}} = \begin{pmatrix} \left.\frac{\partial^2 V}{\partial h^{02}_{u\mathrm{R}}}\right|_{h_i \to v_i} & \left.\frac{\partial^2 V}{\partial h^0_{u\mathrm{R}} \partial h^0_{d\mathrm{R}}}\right|_{h_i \to v_i} \\ \left.\frac{\partial^2 V}{\partial h^0_{u\mathrm{R}} \partial h^0_{d\mathrm{R}}}\right|_{h_i \to v_i} & \left.\frac{\partial^2 V}{\partial h^{02}_{d\mathrm{R}}}\right|_{h_i \to v_i} \end{pmatrix}$$

$$= \begin{pmatrix} m_A^2 \cos^2\beta + M_Z^2 \sin^2\beta & -(m_A^2 + M_Z^2) \sin\beta\cos\beta \\ -(m_A^2 + M_Z^2) \sin\beta\cos\beta & m_A^2 \sin^2\beta + M_Z^2 \cos^2\beta \end{pmatrix} , \tag{8.37}$$

where to obtain the last step we have used manipulations very similar to those used to obtain the other mass matrices above. The eigenvalues of this mass matrix are

$$m^2_{h,H} = \frac{1}{2} \left[(m_A^2 + M_Z^2) \mp \sqrt{(m_A^2 + M_Z^2)^2 - 4 m_A^2 M_Z^2 \cos^2 2\beta} \right] , \tag{8.38}$$

where h and H denote the lighter and heavier of the neutral scalar mass eigenstates.

Exercise *The masses of h and H respect several important bounds. To see this, recall that the expectation value of the matrix (8.37) – for any vector* $(\cos\theta, \sin\theta)^T$ *– must lie between the eigenvalues m_h^2 and m_H^2. Verify that setting $\theta = \beta$ yields,*

$$m_h \le m_A |\cos 2\beta| \le m_H , \tag{8.39a}$$

[3] Because parity is not conserved in weak interactions, the attentive reader may wonder whether A remains an eigenstate beyond tree level. However, CP is conserved and the CP-odd A is precluded from mixing with the CP-even scalar Higgs bosons that we consider shortly.

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while setting $\theta = \pi/2 - \beta$ yields,

$$m_h \leq M_Z |\cos 2\beta| \leq m_H. \tag{8.39b}$$

Notice that this implies that $m_h = 0$ if $\tan\beta = 1$.

Note that these bounds hold only at tree level. Radiative corrections that we alluded to earlier allow h to be significantly heavier than M_Z. This is fortunate since otherwise the non-observation of h in experiments at LEP2 would have excluded the MSSM!

Finally, we may write the physical Higgs scalars in terms of $h^0_{u\mathrm{R}}$ and $h^0_{d\mathrm{R}}$ as

$$\begin{pmatrix} h \\ H \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h^0_{u\mathrm{R}} \\ h^0_{d\mathrm{R}} \end{pmatrix}, \tag{8.40a}$$

with α the Higgs scalar mixing angle being given by

$$\tan\alpha = \frac{(m_A^2 - M_Z^2)\cos 2\beta + \sqrt{(m_A^2 + M_Z^2)^2 - 4m_A^2 M_Z^2 \cos^2 2\beta}}{(m_A^2 + M_Z^2)\sin 2\beta}. \tag{8.40b}$$

Let us now turn to the mass spectrum of the superpartners. We first discuss masses of gauge and Higgs fermions, and then turn to the partners of the matter fermions.

8.3.4 Gluinos

The gluino $\tilde{g}$, the gaugino partner of the gluon, is the only color octet fermion. Since $SU(3)_\mathrm{C}$ is not broken, the gluino cannot mix with any other fermion, and must be a mass eigenstate. Its mass term then arises just from the soft supersymmetry breaking gaugino mass term,[4]

$$\mathcal{L} \ni -\frac{1}{2} M_3 \bar{\tilde{g}} \tilde{g} \tag{8.41}$$

so that its mass at tree level is simply $m_{\tilde{g}} = |M_3|$. If the real parameter M_3 is negative, following the discussion in the Technical Aside of Chapter 7, we can always redefine the gluino field $\tilde{g} \to -\mathrm{i}\gamma_5 \tilde{g}$. The new gluino field then has positive mass and retains its Majorana character. For later convenience, we will write this redefinition as $\tilde{g} \to (-\mathrm{i}\gamma_5)^{\theta_{\tilde{g}}} \tilde{g}$, where $\theta_{\tilde{g}} = 0\ (1)$ for $M_3 > 0\ (M_3 < 0)$.

Exercise *Show that the transformation $\psi \to \psi' = -\mathrm{i}\gamma_5 \psi$ changes the sign of the mass term in the Lagrangian for a free Majorana fermion, but not the kinetic energy term. Show also that if ψ is Majorana, then so is ψ'.*

---

[4] Recall that we have already discussed how the CP-violating mass M_3' can be removed by field redefinition.

8.3.5 Charginos and neutralinos

Spontaneous breakdown of $SU(2)_{\rm L} \times U(1)_{\rm Y}$ implies that states with the same electric charge, color, and spin will mix. This means that gauginos and higgsinos cannot be the physical particles with definite mass. Rather, the neutral fermion fields $\psi_{h_u^0}$, $\psi_{h_d^0}$, λ_3 and λ_0 mix to form neutral fermion mass eigenstates, the *neutralinos*, while the negatively charged fields $\psi_{h_u^+ {\rm R}}$, $\psi_{h_d^- {\rm L}}$, and the linear combination $\frac{\lambda_1 + i\lambda_2}{\sqrt{2}}$ (this is just the superpartner of the field W_μ^- defined earlier) mix to form the negative *charginos*.[5]

We first work out the form of the chargino and neutralino mass matrices, and then diagonalize them to identify the physical charginos and neutralinos. These mass matrices receive a supersymmetric contribution from the superpotential higgsino mass term μ, a SUSY breaking one from gaugino masses, and finally a contribution from electroweak symmetry breaking. This last contribution is also SUSY breaking unless $v_u = v_d$ because D-term contributions to the potential from the Higgs field do not vanish in the vacuum.

The supersymmetric contribution, which arises from the superpotential terms,

$$\hat{f} \ni \mu \left(\hat{h}_u^0 \hat{h}_d^0 + \hat{h}_u^+ \hat{h}_d^-\right) \tag{8.42}$$

gives rise to fermion bilinear terms,

$$\mathcal{L} \ni -\frac{1}{2} \sum_{i,j} \bar{\psi}_i \left(\frac{\partial^2 \hat{f}}{\partial \hat{\mathcal{S}}_i \partial \hat{\mathcal{S}}_j}\right)_{\hat{\mathcal{S}}=\mathcal{S}} P_{\rm L} \psi_j + \quad \text{h.c.},$$

which take the form,

$$\begin{aligned}\mathcal{L}_{\rm mass} \ni & -\frac{\mu}{2}\left[\bar{\psi}_{h_u^0}\psi_{h_d^0} + \bar{\psi}_{h_d^0}\psi_{h_u^0}\right] \\ & -\frac{\mu}{2}\left[\bar{\psi}_{h_u^+}\psi_{h_d^-} + \bar{\psi}_{h_d^-}\psi_{h_u^+}\right].\end{aligned} \tag{8.43}$$

Gaugino–higgsino bilinear terms coming from electroweak breaking arise from,

$$\mathcal{L} \ni -\sqrt{2} \sum_i g \mathcal{S}_i^\dagger t_A \bar{\lambda}_A P_{\rm L} \psi_i + \quad \text{h.c.}, \tag{8.44}$$

[5] Recall that $\psi_{h_u^+}$ is a Majorana spinor whose left-chiral component is positively charged while the right-chiral component is negatively charged.

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when $\mathcal{S}_i$ are the Higgs fields that develop VEVs. These contributions can be written as,

$$\begin{aligned}\mathcal{L} \ni &-\sqrt{2}\left(h_u^{+\dagger}, h_u^{0\dagger}\right)\frac{1}{2}\begin{bmatrix} g\bar{\lambda}_3 + g'\bar{\lambda}_0 & g\bar{\lambda}_1 - \mathrm{i}g\bar{\lambda}_2 \\ g\bar{\lambda}_1 + \mathrm{i}g\bar{\lambda}_2 & -g\bar{\lambda}_3 + g'\bar{\lambda}_0 \end{bmatrix} P_\mathrm{L}\begin{pmatrix}\psi_{h_u^+} \\ \psi_{h_u^0}\end{pmatrix} \\ &-\sqrt{2}\left(h_d^{-\dagger}, \ h_d^{0\dagger}\right)\frac{1}{2}\begin{bmatrix} -g\bar{\lambda}_3 - g'\bar{\lambda}_0 & -g\bar{\lambda}_1 - \mathrm{i}g\bar{\lambda}_2 \\ -g\bar{\lambda}_1 + \mathrm{i}g\bar{\lambda}_2 & g\bar{\lambda}_3 - g'\bar{\lambda}_0 \end{bmatrix} P_\mathrm{L}\begin{pmatrix}\psi_{h_d^-} \\ \psi_{h_d^0}\end{pmatrix} \\ &+ \quad \text{h.c.}\end{aligned} \tag{8.45}$$

Electroweak symmetry breaking contributions to gaugino–higgsino masses arise when the Higgs boson fields develop VEVs. The corresponding terms in (8.45) involving charged higgsinos are,

$$\begin{aligned}&-\frac{gv_u}{\sqrt{2}}\bar{\psi}_{h_u^+}P_\mathrm{R}(\lambda_1 - \mathrm{i}\lambda_2) - \frac{gv_d}{\sqrt{2}}(-\bar{\lambda}_1 + \mathrm{i}\bar{\lambda}_2)P_\mathrm{L}\psi_{h_d^-} + \quad \text{h.c.} \\ &\quad = -\frac{gv_u}{\sqrt{2}}(\bar{\lambda}_1 - \mathrm{i}\bar{\lambda}_2)P_\mathrm{R}\psi_{h_u^+} + \frac{gv_d}{\sqrt{2}}(\bar{\lambda}_1 - \mathrm{i}\bar{\lambda}_2)P_\mathrm{L}\psi_{h_d^-} + \quad \text{h.c.},\end{aligned}$$

where the first term in the first line comes from the Hermitian conjugate part of (8.45), and in the second step we have used the Majorana bilinear identities to swap the order of the spinors. This then leads us to define Dirac fields for the negatively charged gaugino,

$$\lambda = \frac{\lambda_1 + \mathrm{i}\lambda_2}{\sqrt{2}} \tag{8.46a}$$

and a negatively charged higgsino,

$$\tilde{\chi} = P_\mathrm{L}\psi_{h_d^-} - P_\mathrm{R}\psi_{h_u^+} \tag{8.46b}$$

in terms of which the charged and neutral gaugino–higgsino mass terms in (8.45) can then be written as,

$$\begin{aligned}\mathcal{L}_\text{mass} = &\, gv_u\bar{\lambda}\frac{1+\gamma_5}{2}\tilde{\chi} + gv_d\bar{\lambda}\frac{1-\gamma_5}{2}\tilde{\chi} + \quad \text{h.c.} \\ &+ \frac{gv_u}{\sqrt{2}}\bar{\lambda}_3\psi_{h_u^0} - \frac{g'v_u}{\sqrt{2}}\bar{\lambda}_0\psi_{h_u^0} - \frac{gv_d}{\sqrt{2}}\bar{\lambda}_3\psi_{h_d^0} + \frac{g'v_d}{\sqrt{2}}\bar{\lambda}_0\psi_{h_d^0}.\end{aligned} \tag{8.47}$$

Exercise *Verify that the charged higgsino mass term in the Lagrangian (8.43) simply becomes* $+\mu\bar{\tilde{\chi}}\tilde{\chi}$.

Finally, the Lagrangian contribution from the soft SUSY breaking gaugino masses is,

$$\mathcal{L}_\text{mass} = -\frac{1}{2}M_1\bar{\lambda}_0\lambda_0 - M_2\frac{1}{2}\bar{\lambda}_3\lambda_3 - M_2\bar{\lambda}\lambda. \tag{8.48}$$

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The gaugino–higgsino mass terms (8.43), (8.47), and (8.48) can be written as

$$\mathcal{L}_{\text{neutralino}} = -\frac{1}{2}\left(\bar{\psi}_{h_u^0},\, \bar{\psi}_{h_d^0},\, \bar{\lambda}_3,\, \bar{\lambda}_0\right) \mathcal{M}_{\text{neutral}} \begin{pmatrix} \psi_{h_u^0} \\ \psi_{h_d^0} \\ \lambda_3 \\ \lambda_0 \end{pmatrix},$$

with

$$\mathcal{M}_{\text{neutral}} = \begin{pmatrix} 0 & \mu & -\frac{g v_u}{\sqrt{2}} & \frac{g' v_u}{\sqrt{2}} \\ \mu & 0 & \frac{g v_d}{\sqrt{2}} & -\frac{g' v_d}{\sqrt{2}} \\ -\frac{g v_u}{\sqrt{2}} & \frac{g v_d}{\sqrt{2}} & M_2 & 0 \\ \frac{g' v_u}{\sqrt{2}} & -\frac{g' v_d}{\sqrt{2}} & 0 & M_1 \end{pmatrix} \tag{8.49a}$$

and

$$\mathcal{L}_{\text{chargino}} = -\left(\bar{\lambda},\, \bar{\tilde{\chi}}\right)\left(\mathcal{M}_{\text{charge}} P_{\text{L}} + \mathcal{M}^T_{\text{charge}} P_{\text{R}}\right)\begin{pmatrix} \lambda \\ \tilde{\chi} \end{pmatrix},$$

with

$$\mathcal{M}_{\text{charge}} = \begin{pmatrix} M_2 & -g v_d \\ -g v_u & -\mu \end{pmatrix}. \tag{8.49b}$$

The physical charginos and neutralinos are eigenstates of these mass matrices. The neutralino mass matrix is real and Hermitian, and so can be diagonalized by an orthogonal transformation as usual. The chargino mass matrix is not symmetric, so that the chargino mass terms are "γ_5-dependent". The diagonalization of charginos is performed as described in the Technical Note of Chapter 7.

Diagonalization of neutralinos

The neutralino mass matrix $\mathcal{M}_{\text{neutral}}$ is guaranteed to have real eigenvalues since it is Hermitian. It can be diagonalized by a unitary (in fact, real orthogonal) matrix V_n such that,

$$V_n^\dagger \mathcal{M}_{\text{neutral}} V_n = \mathcal{M}_D$$

where $\mathcal{M}_D$ is the diagonal matrix of eigenvalues which, though real, are not necessarily positive. The matrix V_n is the matrix whose columns are the eigenvectors of $\mathcal{M}_{\text{neutral}}$. The neutral higgsino and gaugino fields are related to the mass eigenstate fields by,

$$\begin{pmatrix} \psi_{h_u^0} \\ \psi_{h_d^0} \\ \lambda_3 \\ \lambda_0 \end{pmatrix} = \begin{pmatrix} v_1^{(1)} & v_1^{(2)} & v_1^{(3)} & v_1^{(4)} \\ v_2^{(1)} & v_2^{(2)} & v_2^{(3)} & v_2^{(4)} \\ v_3^{(1)} & v_3^{(2)} & v_3^{(3)} & v_3^{(4)} \\ v_4^{(1)} & v_4^{(2)} & v_4^{(3)} & v_4^{(4)} \end{pmatrix} \begin{pmatrix} \widetilde{Z}'_1 \\ \widetilde{Z}'_2 \\ \widetilde{Z}'_3 \\ \widetilde{Z}'_4 \end{pmatrix}. \tag{8.50}$$

It is customary to define mass eigenstate fields with positive eigenvalues. We thus define mass eigenstates such that

$$\widetilde{Z}_i = (-\mathrm{i}\gamma_5)^{\theta_i}\widetilde{Z}'_i, \tag{8.51}$$

with θ_i equals 0 (1) if the eigenvalue corresponding to $\widetilde{Z}'_i$ is positive (negative). The neutralinos are labeled according to increasing mass, with $\widetilde{Z}_1$ being the lightest neutralino and $\widetilde{Z}_4$ the heaviest.

The neutralino mass matrix can be diagonalized analytically, but the resulting formulae are lengthy and not particularly illuminating. Usually, the eigenvalues and eigenvectors are calculated numerically.

Diagonalization of charginos

The chargino mass terms are γ_5-dependent and, as discussed in the Technical Note of Chapter 7, can be diagonalized by different unitary transformations of the left- and right-handed components of the fields. We can write

$$P_{\mathrm{L}}\begin{pmatrix} \lambda \\ \tilde{\chi} \end{pmatrix} = U P_{\mathrm{L}}\begin{pmatrix} \widetilde{W}_2 \\ \widetilde{W}_1 \end{pmatrix} \;\; ; \;\; P_{\mathrm{R}}\begin{pmatrix} \lambda \\ \tilde{\chi} \end{pmatrix} = V P_{\mathrm{R}}\begin{pmatrix} \widetilde{W}_2 \\ \widetilde{W}_1 \end{pmatrix}, \tag{8.52}$$

with U and V being 2×2 unitary matrices. Then,

$$\begin{aligned} \mathcal{L} \ni &-\left(\overline{\widetilde{W}}_2\; \overline{\widetilde{W}}_1\right) V^\dagger \mathcal{M}_{\text{charge}} U P_{\mathrm{L}}\begin{pmatrix} \widetilde{W}_2 \\ \widetilde{W}_1 \end{pmatrix} \\ &-\left(\overline{\widetilde{W}}_2,\; \overline{\widetilde{W}}_1\right) U^\dagger \mathcal{M}^T_{\text{charge}} V P_{\mathrm{R}}\begin{pmatrix} \widetilde{W}_2 \\ \widetilde{W}_1 \end{pmatrix}. \end{aligned}$$

We construct matrices U and V so that these mass terms are diagonal, i.e.

$$\begin{aligned} V^\dagger \mathcal{M}_{\text{charge}} U &= \begin{pmatrix} m_{\widetilde{W}_2} & 0 \\ 0 & m_{\widetilde{W}_1} \end{pmatrix} \equiv \mathcal{M}_D \quad \text{and} \\ U^\dagger \mathcal{M}^T_{\text{charge}} V &= \begin{pmatrix} m_{\widetilde{W}_2} & 0 \\ 0 & m_{\widetilde{W}_1} \end{pmatrix} \equiv \mathcal{M}_D^\dagger, \end{aligned} \tag{8.53}$$

with $m_{\widetilde{W}_1}$ and $m_{\widetilde{W}_2}$ as real (but not necessarily positive) numbers. U is simply the unitary matrix that diagonalizes the Hermitian matrix $\mathcal{M}^T_{\text{charge}}\mathcal{M}_{\text{charge}}$, while V is the corresponding matrix that diagonalizes $\mathcal{M}_{\text{charge}}\mathcal{M}^T_{\text{charge}}$. The eigenvalues of the

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matrix $\mathcal{M}_{\text{charge}}^T \mathcal{M}_{\text{charge}}$ (which are the same as those of the matrix $\mathcal{M}_{\text{charge}} \mathcal{M}_{\text{charge}}^T$) are of course real and positive. Since

$$\mathcal{M}_D^\dagger \mathcal{M}_D = U^\dagger \left(\mathcal{M}_{\text{charge}}^T \mathcal{M}_{\text{charge}} \right) U,$$

these eigenvalues are just $m^2_{\widetilde{W}_{2,1}}$, and are given by

$$m^2_{\widetilde{W}_{1,2}} = \frac{1}{2} \left[(\mu^2 + M_2^2 + 2M_W^2) \mp \zeta \right], \tag{8.54}$$

with

$$\zeta^2 = (\mu^2 - M_2^2)^2 + 4M_W^2 \left[M_W^2 \cos^2 2\beta + \mu^2 + M_2^2 - 2\mu M_2 \sin 2\beta \right].$$

We define $\widetilde{W}_1$ to be the lighter chargino mass eigenstate, and $\widetilde{W}_2$ the heavier one.

It is easy to see that U, the matrix of eigenvectors of $\mathcal{M}_{\text{charge}}^T \mathcal{M}_{\text{charge}}$, is

$$U = \begin{pmatrix} \frac{1}{\sqrt{1+x_2^2}} & \frac{1}{\sqrt{1+x_1^2}} \\ \frac{x_2}{\sqrt{1+x_2^2}} & \frac{x_1}{\sqrt{1+x_1^2}} \end{pmatrix},$$

where

$$x_{2/1} = \frac{\mu^2 - M_2^2 + 2M_W^2 \cos 2\beta \pm \zeta}{2\sqrt{2} M_W (-M_2 \cos\beta + \mu \sin\beta)}. \tag{8.55}$$

Likewise, the matrix V, constructed from the eigenvectors of $\mathcal{M}_{\text{charge}} \mathcal{M}_{\text{charge}}^T$, is given by

$$V = \begin{pmatrix} \frac{1}{\sqrt{1+y_2^2}} & \frac{1}{\sqrt{1+y_1^2}} \\ \frac{y_2}{\sqrt{1+y_2^2}} & \frac{y_1}{\sqrt{1+y_1^2}} \end{pmatrix},$$

with

$$y_{2/1} = \frac{\mu^2 - M_2^2 - 2M_W^2 \cos 2\beta \pm \zeta}{2\sqrt{2} M_W (-M_2 \sin\beta + \mu \cos\beta)}. \tag{8.56}$$

It is straightforward to check that $x_1 x_2 = y_1 y_2 = -1$, as expected from the orthogonality of the eigenvectors. Using this to eliminate x_2 and y_2, the U and V matrices can be recast as,

$$U = \begin{pmatrix} \theta_{x_1} \cos\gamma_{\text{L}} & \sin\gamma_{\text{L}} \\ -\theta_{x_1} \sin\gamma_{\text{L}} & \cos\gamma_{\text{L}} \end{pmatrix} \tag{8.57a}$$

and

$$V = \begin{pmatrix} \theta_y \cos\gamma_{\text{R}} & \sin\gamma_{\text{R}} \\ -\theta_y \sin\gamma_{\text{R}} & \cos\gamma_{\text{R}} \end{pmatrix}, \tag{8.57b}$$

with $\theta_x = \text{sign}(x_1)$ and $\theta_y = \text{sign}(y_1)$. The mixing angles γ_L and γ_R lie in the range $0 \le \gamma_L, \gamma_R \le 180^\circ$, and are given by,

$$\tan \gamma_L = 1/x_1 \quad \text{and} \quad \tan \gamma_R = 1/y_1. \tag{8.58}$$

From Eq. (8.53) we see that *unsquared* chargino masses are given by,

$$m_{\widetilde{W}_1} = \sin \gamma_R \left(M_2 \sin \gamma_L - \sqrt{2} M_W \cos \beta \cos \gamma_L \right) - \cos \gamma_R \left(\sqrt{2} M_W \sin \beta \sin \gamma_L + \mu \cos \gamma_L \right) \tag{8.59a}$$

and

$$m_{\widetilde{W}_2} = \theta_x \theta_y \left[\cos \gamma_R \left(M_2 \cos \gamma_L + \sqrt{2} M_W \cos \beta \sin \gamma_L \right) + \sin \gamma_R \left(\sqrt{2} M_W \sin \beta \cos \gamma_L - \mu \sin \gamma_L \right) \right]. \tag{8.59b}$$

If either of the $m_{\widetilde{W}_i}$ is negative, we replace $\widetilde{W}_i \to \gamma_5 \widetilde{W}_i$, and work with fields with positive mass eigenvalues.

In general, the chargino and neutralino mixing patterns are complex, and depend on the parameters, μ, M_1, M_2, and $\tan \beta$. However, if $|\mu| \gg |M_{1,2}|, M_W$, then $\widetilde{W}_2$ and $\widetilde{Z}_{3,4}$ are approximately higgsinos with squared masses of about μ^2, while the lighter chargino and the two lighter neutralinos are gaugino-like. If $|M_{1,2}| \gg |\mu|, M_W$, the situation is reversed, and the heavier chargino, and the two heavy neutralinos are gaugino-like, while the lighter chargino and the lighter neutralinos are approximately higgsino-like. These properties will be useful in understanding sparticle decay patterns discussed in Chapter 13.

Exercise *From the "squared mass" matrices of charginos and neutralinos, show that,*

$$m^2_{\widetilde{W}_1} + m^2_{\widetilde{W}_2} - 2M_W^2 = \mu^2 + M_2^2,$$

and

$$m^2_{\widetilde{Z}_1} + m^2_{\widetilde{Z}_2} + m^2_{\widetilde{Z}_3} + m^2_{\widetilde{Z}_4} - 2M_Z^2 = 2\mu^2 + M_1^2 + M_2^2.$$

These are, of course, tree-level relations.

Exercise *If soft SUSY breaking gaugino masses are zero, show that the lightest neutralino is a massless photino,* $\tilde{\gamma} \equiv \sin \theta_W \lambda_3 + \cos \theta_W \lambda_0$. *In this case, show that* $\widetilde{W}_1$ *and* $\widetilde{Z}_2$ *are lighter than* M_W *and* M_Z*, respectively.*

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Although we note this in the context of the MSSM, this result is much more general, in the sense that it does not depend on the details of the electroweak symmetry breaking sector.

Incidently, note also that if $M_1 = M_2 = M$, the photino, defined above, is an eigenstate of the neutralino mass matrix with mass M.

Exercise *We have just seen that the lightest neutralino is a massless photino if gaugino masses are zero. Show that it is a massless higgsino* $\cos\beta\bar{\psi}_{h_u^0} + \sin\beta\bar{\psi}_{h_d^0}$ *if, instead, μ vanishes.*

Show that a massless neutralino can also occur if

$$\mu + M_W^2 \sin 2\beta \left(\frac{1}{M_2} + \frac{\tan^2\theta_W}{M_1} \right) = 0.$$

Find the appropriate eigenvector in this case.

8.3.6 Squarks and sleptons

Now we turn to squark and slepton masses. Unlike matter fermions whose masses only arise from superpotential Yukawa interactions, squarks and sleptons (collectively referred to as sfermions) have four distinct sources for these mass terms. For definiteness, we will write these terms for top squarks, but it will be obvious how to write the corresponding terms for other squarks as well as sleptons.

Superpotential terms

We expect that sfermions must get a mass contribution equal to the corresponding fermion mass. The relevant part of the superpotential is,

$$\hat{f} \ni \mu\hat{h}_u^0\hat{h}_d^0 + f_t\hat{t}\hat{h}_u^0\hat{T}^c.$$

Since $\mathcal{L} \ni -\sum_i \left|\partial\hat{f}/\partial\hat{\mathcal{S}}_i\right|^2_{\hat{\mathcal{S}}=\mathcal{S}}$, we see that the squares of $\partial\hat{f}/\partial\hat{t} = f_t\hat{h}_u^0\hat{T}^c$ and of $\partial\hat{f}/\partial\hat{T}^c = f_t\hat{t}\hat{h}_u^0$, upon the replacement $h_u^0 \to v_u$, give the anticipated terms,

$$\mathcal{L} \ni -m_t^2\tilde{t}_L^\dagger\tilde{t}_L - m_t^2\tilde{t}_R^\dagger\tilde{t}_R. \tag{8.60a}$$

This is, however, not the only t-squark bilinear that can come from the superpotential because the cross terms from $|\partial\hat{f}/\partial\hat{h}_u^0|^2$, upon the replacement $h_d^0 \to v_d$, yield an intra-generational mixing contribution to the $\tilde{t}$ mass,

$$\mathcal{L} \ni -(\mu m_t \cot\beta)\left(\tilde{t}_L^\dagger\tilde{t}_R + \tilde{t}_R^\dagger\tilde{t}_L\right). \tag{8.60b}$$

Notice that both these contributions will vanish if the corresponding *quark* Yukawa coupling is zero.

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Soft SUSY breaking scalar masses

These terms arise from

$$\begin{aligned}\mathcal{L} &\ni -\tilde{Q}_i^\dagger \mathbf{m}_{Qij}^2 \tilde{Q}_j - \tilde{u}_{\mathrm{R}i}^\dagger \mathbf{m}_{Uij}^2 \tilde{u}_{\mathrm{R}j} \\ &\ni -m_{\tilde{t}_\mathrm{L}}^2 \tilde{t}_\mathrm{L}^\dagger \tilde{t}_\mathrm{L} - m_{\tilde{t}_\mathrm{R}}^2 \tilde{t}_\mathrm{R}^\dagger \tilde{t}_\mathrm{R}.\end{aligned} \tag{8.61}$$

Remember that there is just one soft SUSY breaking squark (slepton) mass for each generation of left-squarks (left-sleptons); i.e.

$$m_{\tilde{t}_\mathrm{L}} = m_{\tilde{b}_\mathrm{L}} = m_{Q3},\ m_{\tilde{e}_\mathrm{L}} = m_{\tilde{\nu}_e} = m_{\mathrm{L}1}, \quad \text{etc.}$$

Clearly, these terms come from SUSY breaking and are present regardless of whether or not electroweak symmetry is spontaneously broken.

Soft SUSY breaking trilinear terms

Soft SUSY breaking interactions of squarks with neutral Higgs bosons,

$$\mathcal{L} \ni A_t f_t \tilde{t}_\mathrm{L} h_u^0 \tilde{t}_\mathrm{R}^\dagger + \quad \text{h.c.},$$

give rise to intra-generational squark mixing terms

$$\mathcal{L} \ni -(-A_t m_t)(\tilde{t}_\mathrm{L}^\dagger \tilde{t}_\mathrm{R} + \tilde{t}_\mathrm{R}^\dagger \tilde{t}_\mathrm{L}), \tag{8.62}$$

when the Higgs field is replaced by its VEV. That these terms appear proportional to m_t is an artifact of writing a_t as $A_t f_t$. Nevertheless, like the superpotential terms, these terms are absent if the electroweak symmetry is unbroken.

D-term contributions

We write these terms which come from

$$\begin{aligned}\mathcal{L} &\ni -\frac{1}{2}\sum_A |\sum_i \mathcal{S}_i^\dagger g_\alpha t_{\alpha A} \mathcal{S}_i|^2 \\ &\ni -\frac{1}{2} g^2 |\tilde{Q}^\dagger T_{3Q} \tilde{Q} + H_u^\dagger \frac{\tau_3}{2} H_u + H_d^\dagger (-\frac{\tau_3}{2}) H_d|^2 \\ &\quad - \left(\frac{g'}{2}\right)^2 |H_u^\dagger Y_{H_u} H_u + H_d^\dagger Y_{H_d} H_d + \tilde{Q}^\dagger Y_Q \tilde{Q} + \tilde{u}_{\mathrm{R}i}^\dagger Y_{U^c} \tilde{u}_{\mathrm{R}i} + \tilde{d}_{\mathrm{R}i}^\dagger Y_{D^c} \tilde{d}_{\mathrm{R}i}|^2,\end{aligned}$$

for both top and bottom squarks. Squark mass contributions arise from cross terms between squark and Higgs boson fields. The $SU(2)$ D-term gives,

$$\begin{aligned}\mathcal{L} &\ni -\frac{1}{2}\left[2(\frac{g}{2})^2 (v_d^2 - v_u^2)(\tilde{t}_\mathrm{L}^\dagger \tilde{t}_\mathrm{L} - \tilde{b}_\mathrm{L}^\dagger \tilde{b}_\mathrm{L})\right] \\ &= -M_W^2 \cos 2\beta T_{3Q_i} \tilde{Q}_{\mathrm{L}i}^\dagger \tilde{Q}_{\mathrm{L}i},\end{aligned} \tag{8.63a}$$

while the hypercharge D-term gives,

$$\mathcal{L} \ni \sin^2\theta_{\mathrm{W}} \cos 2\beta M_Z^2 \left(\tilde{t}_{\mathrm{L}}^\dagger \frac{Y_Q}{2} \tilde{t}_{\mathrm{L}} + \tilde{b}_{\mathrm{L}}^\dagger \frac{Y_Q}{2} \tilde{b}_{\mathrm{L}} + \tilde{t}_{\mathrm{R}}^\dagger \frac{Y_{U^c}}{2} \tilde{t}_{\mathrm{R}} + \tilde{b}_{\mathrm{R}}^\dagger \frac{Y_{D^c}}{2} \tilde{b}_{\mathrm{R}} \right). \quad (8.63b)$$

Note that the hypercharges that appear in the terms involving right-handed fields are those for the corresponding *left*-handed antiquark fields that appear in Table 8.1. Eliminating the hypercharge in favor of the electric charge, the D-term contribution to any MSSM sfermion squared mass can be written as,

$$m_{D\text{-term}}^2 = M_Z^2 \cos 2\beta \left(T_3 - Q \sin^2\theta_{\mathrm{W}} \right). \quad (8.64)$$

We can now assemble the mass squared matrices for the sfermions. For top squarks, we have

$$\mathcal{L} \ni - \left(\tilde{t}_{\mathrm{L}}^\dagger, \tilde{t}_{\mathrm{R}}^\dagger \right) \mathcal{M}_{\tilde{t}}^2 \begin{pmatrix} \tilde{t}_{\mathrm{L}} \\ \tilde{t}_{\mathrm{R}} \end{pmatrix},$$

where the matrix $\mathcal{M}_{\tilde{t}}^2$ is given by

$$\begin{pmatrix} m_{\tilde{t}_{\mathrm{L}}}^2 + m_t^2 + D(\tilde{t}_{\mathrm{L}}) & m_t(-A_t + \mu \cot\beta) \\ m_t(-A_t + \mu\cot\beta) & m_{\tilde{t}_{\mathrm{R}}}^2 + m_t^2 + D(\tilde{t}_{\mathrm{R}}) \end{pmatrix}, \quad (8.65a)$$

and

$$D(\tilde{t}_{\mathrm{L}}) = M_Z^2 \cos 2\beta (\frac{1}{2} - \frac{2}{3}\sin^2\theta_{\mathrm{W}}),$$
$$D(\tilde{t}_{\mathrm{R}}) = M_Z^2 \cos 2\beta (+\frac{2}{3}\sin^2\theta_{\mathrm{W}}),$$

are the hypercharge D-term contributions (8.64) to the squared masses of $\tilde{t}_{\mathrm{L}}$ and $\tilde{t}_{\mathrm{R}}$. The eigenvalues of this matrix are,

$$m_{\tilde{t}_{1,2}}^2 = \frac{1}{2}\left(m_{\tilde{t}_{\mathrm{L}}}^2 + m_{\tilde{t}_{\mathrm{R}}}^2 \right) + \frac{1}{4} M_Z^2 \cos 2\beta + m_t^2$$

$$\mp \left\{ \left[\frac{1}{2}(m_{\tilde{t}_{\mathrm{L}}}^2 - m_{\tilde{t}_{\mathrm{R}}}^2) + M_Z^2 \cos 2\beta (\frac{1}{4} - \frac{2}{3} x_W) \right]^2 + m_t^2(\mu\cot\beta - A_t)^2 \right\}^{\frac{1}{2}}, \quad (8.65b)$$

with $\tilde{t}_1$ the lighter top squark mass eigenstate, and $\tilde{t}_2$ the heavier one, and $x_W \equiv \sin^2\theta_{\mathrm{W}}$. The top squark mixing matrix is defined by

$$\begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_t & -\sin\theta_t \\ \sin\theta_t & \cos\theta_t \end{pmatrix} \begin{pmatrix} \tilde{t}_{\mathrm{L}} \\ \tilde{t}_{\mathrm{R}} \end{pmatrix}, \quad (8.65c)$$

with the top squark mixing angle θ_t given by,

$$\tan\theta_t = \frac{m_{\tilde{t}_L}^2 + m_t^2 + M_Z^2\cos 2\beta\left(\frac{1}{2} - \frac{2}{3}x_W\right) - m_{\tilde{t}_1}^2}{m_t\left(-A_t + \mu\cot\beta\right)}. \tag{8.65d}$$

For bottom squarks, we find the mass matrix $\mathcal{M}_{\tilde{b}}^2$ to be

$$\begin{pmatrix} m_{\tilde{b}_L}^2 + m_b^2 + D(\tilde{b}_L) & m_b(-A_b + \mu\tan\beta) \\ m_b(-A_b + \mu\tan\beta) & m_{\tilde{b}_R}^2 + m_b^2 + D(\tilde{b}_R) \end{pmatrix}, \tag{8.66a}$$

with $m_{\tilde{b}_L} = m_{\tilde{t}_L}$ by $SU(2)$ symmetry, and

$$D(\tilde{b}_L) = M_Z^2\cos 2\beta(-\frac{1}{2} + \frac{1}{3}\sin^2\theta_W),$$
$$D(\tilde{b}_R) = M_Z^2\cos 2\beta(-\frac{1}{3}\sin^2\theta_W).$$

The corresponding eigenvalues are,

$$m_{\tilde{b}_{1,2}}^2 = \frac{1}{2}\left(m_{\tilde{b}_L}^2 + m_{\tilde{b}_R}^2\right) - \frac{1}{4}M_Z^2\cos 2\beta + m_b^2$$

$$\mp\left\{\left[\frac{1}{2}(m_{\tilde{b}_L}^2 - m_{\tilde{b}_R}^2) - M_Z^2\cos 2\beta(\frac{1}{4} - \frac{1}{3}x_W)\right]^2 + m_b^2(\mu\tan\beta - A_b)^2\right\}^{\frac{1}{2}}, \tag{8.66b}$$

and the bottom squark mixing angle (defined the same way as in Eq. (8.65c)) is

$$\tan\theta_b = \frac{m_{\tilde{b}_L}^2 + m_b^2 + M_Z^2\cos 2\beta\left(-\frac{1}{2} + \frac{1}{3}x_W\right) - m_{\tilde{b}_1}^2}{m_b\left(-A_b + \mu\tan\beta\right)}. \tag{8.66c}$$

For tau sleptons we have,

$$\begin{pmatrix} m_{\tilde{\tau}_L}^2 + m_\tau^2 + D(\tilde{\tau}_L) & m_\tau(-A_\tau + \mu\tan\beta) \\ m_\tau(-A_\tau + \mu\tan\beta) & m_{\tilde{\tau}_R}^2 + m_\tau^2 + D(\tilde{\tau}_R) \end{pmatrix}, \tag{8.67a}$$

with

$$D(\tilde{\tau}_L) = M_Z^2\cos 2\beta(-\frac{1}{2} + \sin^2\theta_W),$$
$$D(\tilde{\tau}_R) = M_Z^2\cos 2\beta\left(-\sin^2\theta_W\right),$$

and

$$m_{\tilde{\tau}_{1,2}}^2 = \frac{1}{2}\left(m_{\tilde{\tau}_L}^2 + m_{\tilde{\tau}_R}^2\right) - \frac{1}{4}M_Z^2\cos 2\beta + m_\tau^2$$

$$\mp\left\{\left[\frac{1}{2}(m^2_{\tilde{\tau}_L}-m^2_{\tilde{\tau}_R})-M_Z^2\cos 2\beta(\frac{1}{4}-x_W)\right]^2+m_\tau^2(\mu\tan\beta-A_\tau)^2\right\}^{\frac{1}{2}}, \tag{8.67b}$$

and

$$\tan\theta_\tau=\frac{m^2_{\tilde{\tau}_L}+m_\tau^2+M_Z^2\cos 2\beta\left(-\frac{1}{2}+x_W\right)-m^2_{\tilde{\tau}_1}}{m_\tau\left(-A_\tau+\mu\tan\beta\right)}. \tag{8.67c}$$

Since we have ignored neutrino masses, the MSSM only contains the scalar partner for the left-handed neutrino, one for each flavor. Also, because lepton flavor has been assumed to be conserved, the three sneutrinos cannot mix with one another, and hence, must be mass eigenstates. For the third generation, we thus have

$$m^2_{\tilde{\nu}_\tau}=m^2_{L3}+\frac{1}{2}M_Z^2\cos 2\beta, \tag{8.68}$$

where the first term is the soft SUSY breaking mass for the third generation scalar lepton doublet, and the second term comes from the D-term contribution to the sneutrino mass. Since there are only superpartners of left-handed neutrinos in the MSSM, we will henceforth drop the subscript L on the sneutrinos.

The masses of the first and second generation squarks and sleptons can be obtained in exactly the same fashion. However, since first and second generation quark and lepton masses are small compared to the soft SUSY breaking masses, intra-generation mixing effects can be neglected so that $\tilde{f}_L$ and $\tilde{f}_R$ are essentially mass eigenstates. To a very good approximation, the masses of the first generation of sfermions are given by

$$m^2_{\tilde{u}_L}=m^2_{Q_1}+m_u^2+M_Z^2\cos 2\beta(\frac{1}{2}-\frac{2}{3}\sin^2\theta_W) \tag{8.69a}$$

$$m^2_{\tilde{d}_L}=m^2_{Q_1}+m_d^2+M_Z^2\cos 2\beta(-\frac{1}{2}+\frac{1}{3}\sin^2\theta_W) \tag{8.69b}$$

$$m^2_{\tilde{u}_R}=m^2_{U_1}+m_u^2+M_Z^2\cos 2\beta(\frac{2}{3}\sin^2\theta_W) \tag{8.69c}$$

$$m^2_{\tilde{d}_R}=m^2_{D_1}+m_d^2+M_Z^2\cos 2\beta(-\frac{1}{3}\sin^2\theta_W) \tag{8.69d}$$

$$m^2_{\tilde{e}_L}=m^2_{L_1}+m_e^2+M_Z^2\cos 2\beta(-\frac{1}{2}+\sin^2\theta_W) \tag{8.69e}$$

$$m^2_{\tilde{\nu}_e}=m^2_{L_1}+M_Z^2\cos 2\beta(\frac{1}{2}) \tag{8.69f}$$

$$m^2_{\tilde{e}_R}=m^2_{E_1}+m_e^2+M_Z^2\cos 2\beta(-\sin^2\theta_W), \tag{8.69g}$$

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where the first terms on the right-hand side of these expressions are the soft SUSY breaking masses for the first generation of sfermions. There are analogous expressions for second generation masses. Notice that we are abusing notation here in that sometimes we use $m_{\tilde{t}_L}$ to denote the entire entry in the squark mass matrix (as implied by these equations), while at other times we use it to denote just the corresponding soft SUSY breaking mass. We trust that the meaning will be clear from the context.

We remind the reader that in deriving these MSSM mass spectra, we have ignored the possibility of **c**-terms. If such terms are present, like a-terms, they will contribute to intra-generation sfermion mixing, and possibly also to flavor physics.

Finally, we stress that (8.68) follows only from $SU(2)_L$ gauge symmetry, so that its analogue for the first two generations of sleptons and squarks (whose Yukawa couplings are negligible) gives a *model independent* relation between the physical masses of the up and down components of the slepton/squark doublets. Most importantly, it tells us that the mass gap between $\tilde{\ell}_L$ and the corresponding sneutrino ($\ell = e, \mu$), and likewise for the left-squarks, can never be too large. This is clearly relevant for collider searches for SUSY.

Exercise *The alert reader may wonder why the sfermion masses do not equal the corresponding fermion mass even if we take the "SUSY limit" in the sfermion mass squared matrix, and set the soft-masses and A-parameters to zero, and take* $\tan\beta = 1$ *so that the Higgs field D-terms vanish in the vacuum. The point is that within the MSSM, electroweak symmetry is unbroken unless we introduce soft SUSY breaking masses for the Higgs fields. Then, fermion and sfermion masses become equal as both vanish!*

There is, however, an interesting extension of the MSSM that leads to a SUSY limit in which electroweak symmetry is spontaneously broken. We need to introduce a SM "singlet Higgs" superfield $\hat{N}$*, and choose the superpotential as,*

$$\hat{f} = \hat{f}_{\text{MSSM}} + \lambda \hat{H}_u \hat{H}_d \hat{N} - K\hat{N} \tag{8.70a}$$

where the parameter $K > 0$ *has dimensions of mass squared, and appropriate group contractions are implied. Show that the scalar potential is given by,*

$$V = |\lambda h_d^0 N + \mu h_d^0|^2 + |\lambda h_u^0 N + \mu h_u^0|^2 + |\lambda h_u^0 h_d^0 - K|^2 + \cdots, \tag{8.70b}$$

where the ellipsis refers to terms involving charged Higgs boson or squark and slepton fields. Again assuming that these do not develop any VEV, show that this potential can have a minimum with $v_u = v_d \neq 0$ *and* $\langle N\rangle \neq 0$ *with* $\lambda\langle N\rangle + \mu = 0$*. Notice that because* N *condenses, there is effectively an additional contribution to* μ *equal to* $\lambda\langle N\rangle$*. In other words, the total "effective* μ *term" vanishes!*

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Work out the top squark mass squared matrix for this model. Show that the off-diagonal terms vanish, while the diagonal terms are just m_t^2. In the sfermion sector, we thus have what looks like a SUSY limit of the MSSM but with non-vanishing masses for the fermions and sfermions. This model with the extra gauge singlet superfield is referred to as the Next to Minimal Supersymmetric Standard Model, or the NMSSM.

8.4 Interactions in the MSSM

In order to work out the phenomenological implications of the MSSM, we must first evaluate the interactions of the various superpartners, i.e. the mass eigenstates, with SM particles. This is done in two steps. First, we write down the interactions of the primitive fields of the MSSM (the fields with definite $SU(3)_C \times SU(2)_L \times U(1)_Y$ quantum numbers) using our master formula, and then transform these to the interactions of the mass eigenstates by performing the "rotations" (and, in the case of the Higgs sector, also a shift) of these fields discussed in the last section.

As in any gauge theory, before proceeding further we must fix a gauge. Since our attention will be mainly on tree level processes, we will write these in the unitarity gauge, where only physical fields are present. For many loop calculations, it is more convenient to work in the renormalizable R_ξ gauges, in which the propagator has better high energy behavior. Then, additional couplings involving Dewitt–Fadeev–Popov ghosts and unphysical Goldstone bosons must be included. We do not work these couplings out in this book.

In the following, we first evaluate the interactions in supersymmetric QCD. Next, we work out the interactions between matter fermions, sfermions, electroweak gauge bosons and the charginos and neutralinos. We then list the couplings of the MSSM Higgs bosons to other particles and sparticles. Finally, we list some "hybrid" interactions of matter sfermions.

8.4.1 QCD interactions in the MSSM

We begin by showing that we can recover the SM QCD Lagrangian written in Chapter 1 using our master formula. Clearly, the gluon field kinetic energy term $\mathcal{L} = -\frac{1}{4}F_{\mu\nu A}F^{\mu\nu}_A$ in the master formula has the usual form, and leads to the three and four gluon interactions listed in Eq. (1.7).

The kinetic energies and gauge couplings of quarks are contained in the terms,

$$\mathcal{L} \ni \frac{\mathrm{i}}{2}\sum_i \bar{\psi}_i \not{D}\psi_i,$$

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of the master formula, where the ψ_i are the fermion components of the quark superfields $\hat{q}$ and $\hat{Q}^c$. Just as for the electron field in Eq. (8.4), manipulation of the kinetic energy terms for ψ_q and ψ_{Q^c} leads to canonically normalized kinetic energy terms for the Dirac quark field q defined by,

$$q = P_{\rm L}\psi_q + P_{\rm R}\psi_{Q^c}.$$

To obtain the coupling between quarks and gluons, we must examine the interaction terms. Using (3.8c) and (3.8d) it is easy to see that

$$\bar{\psi}_q t_A^* \not{G}_A P_{\rm R}\psi_q = -\bar{\psi}_q t_A \not{G}_A P_{\rm L}\psi_q,$$

so that

$$\begin{aligned} \mathcal{L}_{gq\bar{q}} &\ni -g_s\bar{\psi}_q t_A \not{G}_A P_{\rm L}\psi_q - g_s\bar{\psi}_{Q^c} t_A \not{G}_A P_{\rm R}\psi_{Q^c} \\ &= -g_s\bar{q}\gamma_\mu\frac{\lambda_A}{2}G_{A\mu}q, \end{aligned} \tag{8.71}$$

which is just the interaction in Eq. (1.7).

The gauge invariant kinetic energy term for any flavor of left- or right-type squark field is,

$$\begin{aligned} \mathcal{L} &\ni (D^\mu\tilde{q})^\dagger(D_\mu\tilde{q}) \\ &= (\partial^\mu\tilde{q}^\dagger - {\rm i}g_s\tilde{q}^\dagger t_A G_A^\mu)(\partial_\mu\tilde{q} + {\rm i}g_s t_A G_{A\mu}\tilde{q}). \end{aligned}$$

The cross terms lead to

$$\mathcal{L}_{g\tilde{q}\bar{\tilde{q}}} = -{\rm i}g_s\left(\tilde{q}^\dagger\frac{\lambda_A}{2}\partial_\mu\tilde{q} - \partial_\mu\tilde{q}^\dagger\frac{\lambda_A}{2}\tilde{q}\right)G_A^\mu, \tag{8.72}$$

while the remaining interaction term yields,

$$\mathcal{L}_{gg\tilde{q}\bar{\tilde{q}}} = g_s^2\tilde{q}^\dagger\frac{\lambda_A}{2}\frac{\lambda_B}{2}\tilde{q}G_{A\mu}G_B^\mu, \tag{8.73}$$

where matrix multiplication is implied.

Exercise *We have obtained the interactions of gluons with $\tilde{q}_{\rm L}$ and $\tilde{q}_{\rm R}$. Show that the interactions of the squark mass eigenstates $\tilde{q}_1$ and $\tilde{q}_2$ with gluons have the same forms as in (8.72) and (8.73). This is just the familiar GIM (Glashow–Iliopoulos–Maiani) mechanism in a different setting.*

The gluino–quark–squark interaction comes from the Lagrangian term

$$\mathcal{L} \ni -\sqrt{2}\sum_{i,A}\mathcal{S}_i^\dagger g t_A\bar{\lambda}_A\frac{1-\gamma_5}{2}\psi_i + \quad {\rm h.c.},$$

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with $(\mathcal{S}_i, \psi_i) = (\tilde{q}_L, \psi_q)$ or $(\tilde{q}_R^\dagger, \psi_{Q^c})$. For the contribution from the superfield $\hat{Q}^c$, we write the term involving the right projector from the Hermitian conjugate part, and then use the Majorana bilinear identities to get the interaction,

$$\mathcal{L} \ni -\sqrt{2}\tilde{q}_{La}^\dagger \left(\frac{g\lambda_A}{2}\right)_{ab} \bar{\lambda}_A \frac{1-\gamma_5}{2}\psi_{qb} - \sqrt{2}\tilde{q}_{Ra}^\dagger \left(-\frac{g\lambda_A}{2}\right)_{ab} \bar{\lambda}_A \frac{1+\gamma_5}{2}\psi_{Q^c b},$$

where $\tilde{q}_R^\dagger$ is the field that *annihilates* the scalar partner of the weak singlet antiquark, or *creates* the scalar partner of the right-handed quark. To obtain this form, we must remember that the superfields $\hat{q}$ and $\hat{Q}^c$ belong to the **3** and **3*** representations, respectively, and write the generator t_A accordingly. We can allow for the possibly negative value of M_3 by replacing the gaugino λ_A by $(+i\gamma_5)^{\theta_{\tilde{g}}}\tilde{g}_A$ (rather than just $\tilde{g}_A$). Making the additional replacements of $P_L\psi_q = P_L q$ and $P_R\psi_{Q^c} = P_R q$ to write the interaction in terms of the Dirac quark field q leads to

$$\mathcal{L}_{\tilde{g}q\tilde{q}} = -\sqrt{2}g_s(-i)^{\theta_{\tilde{g}}}\tilde{q}_L^\dagger \bar{\tilde{g}}_A \frac{\lambda_A}{2} P_L q + \sqrt{2}g_s(i)^{\theta_{\tilde{g}}}\tilde{q}_R^\dagger \bar{\tilde{g}}_A \frac{\lambda_A}{2} P_R q + \quad \text{h.c.} \tag{8.74}$$

We can take into account intra-generation squark mixing by writing $\tilde{q}_L$ and $\tilde{q}_R$ in terms of the squark mass eigenstates $\tilde{q}_1$ and $\tilde{q}_2$ defined as in (8.65c). The quark–squark–gluino interaction then depends on the squark mixing angle, and we have

$$\begin{aligned}\mathcal{L}_{\tilde{g}q\tilde{q}_i} = &- \sqrt{2}g_s\tilde{q}_1^\dagger \bar{\tilde{g}}_A \frac{\lambda_A}{2}\left[(-i)^{\theta_{\tilde{g}}}\cos\theta_q P_L + (i)^{\theta_{\tilde{g}}}\sin\theta_q P_R\right] q \\ &- \sqrt{2}g_s\tilde{q}_2^\dagger \bar{\tilde{g}}_A \frac{\lambda_A}{2}\left[(-i)^{\theta_{\tilde{g}}}\sin\theta_q P_L - (i)^{\theta_{\tilde{g}}}\cos\theta_q P_R\right] q + \quad \text{h.c.}\end{aligned} \tag{8.75}$$

Although we have written this for generic squarks, in practice, mixing angle effects are usually only important for the third generation.

We have a gluon–gluino–gluino interaction arising from the minimal coupling of the color octet gluino,

$$\mathcal{L} \ni \frac{i}{2}\bar{\lambda}_A \not{D}\lambda_A \ni -\frac{1}{2}g_s\bar{\tilde{g}}_A(t_B^{\text{adj}}\not{G}_B)_{AC}\tilde{g}_C,$$

which leads to

$$\mathcal{L}_{g\tilde{g}\tilde{g}} = i\frac{g_s}{2}f_{ABC}\bar{\tilde{g}}_A\gamma_\mu\tilde{g}_B G_C^\mu. \tag{8.76}$$

Notice that this interaction is not altered by the transformation, $\tilde{g}_A \to (-i\gamma_5)^{\theta_g}\tilde{g}_A$.

Finally, supersymmetry necessarily implies the existence of four squark interactions. These arise from the D-terms on the third line of our master formula, and

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take the form,

$$\mathcal{L}_{4\tilde{q}} = -\frac{g_s^2}{8}\sum_A \left(\sum_i \tilde{q}_{Li}^\dagger \lambda_A \tilde{q}_{Li} - \sum_i \tilde{q}_{Ri}^\dagger \lambda_A \tilde{q}_{Ri}\right)^2, \tag{8.77}$$

where i denotes the flavor of the squark. Notice that the cross terms in the sum over flavors and types imply vertices such as $\tilde{u}_{\rm R}^\dagger \tilde{u}_{\rm R} \tilde{b}_{\rm L}^\dagger \tilde{b}_{\rm L}$, where the squark pairs could have different flavors and/or types. Moreover, for the same reason as in the last exercise, we see that writing this in terms of mass eigenstates ($\tilde{q}_1$ and $\tilde{q}_2$) does not lead to "cross terms" (such as $\tilde{q}_1^\dagger \tilde{q}_2$) in this coupling.

8.4.2 Electroweak interactions in the MSSM

Standard Model interactions

The triple and quartic vector boson gauge self-couplings arise from the squared field strength term in the master formula Eq. (6.44) and so are exactly as given by (1.19a) and (1.19b). Next, we turn to the SM electroweak interactions of quarks and leptons from the master formula. We will first evaluate the couplings of the up and down quarks to the gauge bosons $W^\pm$, Z^0 and γ. The starting point in the master formula is the term,

$$\mathcal{L} \ni \frac{\rm i}{2}\sum_i \bar{\psi}_i \not{D}\psi_i$$

where $D_\mu = \partial_\mu + {\rm i}g(t\cdot V_\mu)P_{\rm L} - {\rm i}g(t^*\cdot V_\mu)P_{\rm R}$ and $i = \hat{Q}$, $\hat{U}^c$, and $\hat{D}^c$. We will leave it to the reader to verify that the second and third terms of the covariant derivative yield identical contributions to the Lagrangian. The $SU(2)_{\rm L}$ and $U(1)_{\rm Y}$ gauge boson interactions take the form,

$$\begin{aligned}\mathcal{L} \ni &-\frac{g}{2}\left(\bar{\psi}_{u_{\rm L}}\ \bar{\psi}_{d_{\rm L}}\right)\begin{pmatrix} \not{W}_3 & \not{W}_1 - {\rm i}\not{W}_2 \\ \not{W}_1 + {\rm i}\not{W}_2 & -\not{W}_3\end{pmatrix}\begin{pmatrix}\psi_{u_{\rm L}}\\ \psi_{d_{\rm L}}\end{pmatrix}\\ &-\frac{1}{3}\frac{g'}{2}\bar{\psi}_u \not{B} P_{\rm L}\psi_u - \frac{1}{3}\frac{g'}{2}\bar{\psi}_d \not{B} P_{\rm L}\psi_d - \frac{4}{3}\frac{g'}{2}\bar{\psi}_{U^c}\not{B}P_{\rm R}\psi_{U^c}\\ &+\frac{2}{3}\frac{g'}{2}\bar{\psi}_{D^c}\not{B}P_{\rm R}\psi_{D^c}.\end{aligned}$$

To write these in terms of the Dirac quark fields u and d, we substitute $P_{\rm L}\psi_u = P_{\rm L}u$, $P_{\rm L}\psi_d = P_{\rm L}d$, $P_{\rm R}\psi_{U^c} = P_{\rm R}u$, and $P_{\rm R}\psi_{D^c} = P_{\rm R}d$ and, finally, we eliminate the fields W_i and B in favor of the gauge boson mass eigenstates. The resulting Lagrangian is,

$$\mathcal{L}_{W\bar{u}d} = -\frac{g}{\sqrt{2}}\bar{u}\gamma^\mu P_{\rm L} d W_\mu^+ + \quad \text{h.c.} \tag{8.78}$$

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for the charged gauge bosons, and

$$\mathcal{L} = -e(+\frac{2}{3})\bar{u}\gamma_\mu u A^\mu + e\bar{u}\gamma_\mu \left[(-\frac{5}{12}t + \frac{1}{4}c) + (-\frac{1}{4}c - \frac{1}{4}t)\gamma_5\right] u Z^{0\mu} \tag{8.79}$$

for the electromagnetic and Z-boson interactions with u-quarks. Aside from inter-generational mixing between the quarks, these results are in accord with the SM interactions that we obtained in Chapter 1. The gauge interactions of other quarks and leptons can be obtained in the same fashion. These interactions have all been listed in Eq. (1.16a) and Eq. (1.16b), with coupling constants defined in Table 1.2.

Gauge boson couplings to matter scalars

The interactions of gauge bosons with sfermions originate in the gauge invariant kinetic terms,

$$\mathcal{L} \ni \left(D_\mu \mathcal{S}_i\right)^\dagger (D^\mu \mathcal{S}_i)\,,$$

for the scalars. Notice that in addition to the coupling of a vector boson to a sfermion pair, these terms also include a two-gauge boson–two-sfermion interaction.

Three-point couplings: $W^\pm$ bosons do not couple to the $SU(2)$ singlet sfermions $\tilde{f}_\mathrm{R}$. The coupling of $W^\pm$ to doublet sfermions of the first generation takes the form,

$$\mathcal{L} \ni -\frac{\mathrm{i}g}{\sqrt{2}}\left(\tilde{u}_\mathrm{L}^\dagger \partial_\mu \tilde{d}_\mathrm{L} - \tilde{d}_\mathrm{L}\partial_\mu \tilde{u}_\mathrm{L}^\dagger\right) W^{+\mu} - \frac{\mathrm{i}g}{\sqrt{2}}\left(\tilde{\nu}_e^\dagger \partial_\mu \tilde{e}_\mathrm{L} - \tilde{e}_\mathrm{L}\partial_\mu \tilde{\nu}_e^\dagger\right) W^{+\mu} + \quad \text{h.c.} \tag{8.80}$$

Except for intrageneration sfermion mixing, other sfermion generations couple to W in exactly the same way. For third generation squarks and sleptons, mixing effects can be important. These couplings can be readily obtained from Eq. (8.80) via the replacement,

$$\tilde{f}_\mathrm{L} = \cos\theta_f \tilde{f}_1 + \sin\theta_f \tilde{f}_2,$$

where $f = t, b$ or τ. In addition to these three-point couplings the kinetic energy term for sfermions also includes a two-gauge boson–two-sfermion interaction. We will list these couplings shortly.

The interaction of a photon with a sfermion pair is given by,

$$\mathcal{L} \ni -\mathrm{i}eq_f\left(\tilde{f}_i^\dagger \partial_\mu \tilde{f}_i - \tilde{f}_i \partial_\mu \tilde{f}_i^\dagger\right) A^\mu, \tag{8.81}$$

where $\tilde{f}$ is any squark or slepton, q_f is the electric charge of the sfermion (which is, of course, the same as the charge of the corresponding fermion), and $i = \mathrm{L}$ or R. Notice that the photon couples just to left- or to right-sfermion pairs, i.e. there is no $\tilde{f}_\mathrm{L}\tilde{f}_\mathrm{R}\gamma$ interaction. Intra-generational (or for that matter, inter-generational) mixing does not alter the form of (8.81).

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Exercise *Show that the conservation of electric current for the coupling,*

$$\mathcal{L} = J_\mu A^\mu,$$

implies that the photon cannot couple two sfermions with different masses. This explains why there is no $\tilde{f}_1 \tilde{f}_2 \gamma$ interaction.

The interactions of sfermions with a Z^0 boson are given by,

$$\mathcal{L} \ni \mathrm{i}e\left[\left(\alpha_f - \beta_f\right) \tilde{f}_\mathrm{L}^\dagger \partial_\mu \tilde{f}_\mathrm{L} + \left(\alpha_f + \beta_f\right) \tilde{f}_\mathrm{R}^\dagger \partial_\mu \tilde{f}_\mathrm{R}\right] Z^\mu + \quad \text{h.c.}, \tag{8.82}$$

where again $\tilde{f}_i$ is any squark or slepton of type i and α_f and β_f, which also determine the couplings of Z^0 to matter *fermions*, are given in Table 1.2. Like the photon, Z^0 interactions do not couple left- and right-type sfermions to each other. This should not be surprising since gauge bosons do not couple left-handed and right-handed *fermions* to each other. Supersymmetry then implies that they cannot couple the respective superpartners to one another either.

Exercise *In the presence of intra-generational mixing show that the couplings of Z^0 to sfermions are modified to,*

$$\begin{aligned}\mathcal{L} \ni \mathrm{i}e\Big[&\left(\alpha_f - \beta_f \cos 2\theta_f\right) \tilde{f}_1^\dagger \partial_\mu \tilde{f}_1 + \left(\alpha_f + \beta_f \cos 2\theta_f\right) \tilde{f}_2^\dagger \partial_\mu \tilde{f}_2 \\ &- \beta_f \sin 2\theta_f \left(\tilde{f}_1^\dagger \partial_\mu \tilde{f}_2 + \tilde{f}_2^\dagger \partial_\mu \tilde{f}_1\right)\Big] Z^\mu + \quad \text{h.c.}\end{aligned} \tag{8.83}$$

Notice that unlike the photon, Z^0 does couple sfermions of different masses together.

Four-point Couplings: We now work out the two-vector boson–two-sfermion couplings that are also contained in the gauge invariant kinetic energy terms. The covariant derivative for squark fields can be written as,

$$\begin{aligned}
D_\mu \tilde{u}_\mathrm{L} &= \partial_\mu \tilde{u}_\mathrm{L} + \mathrm{i}\left(eq_u A_\mu - e(\alpha_u - \beta_u) Z_\mu + g_s \frac{\lambda_A}{2} G_{A\mu}\right) \tilde{u}_\mathrm{L} + \frac{\mathrm{i}g}{\sqrt{2}} W_\mu^+ \tilde{d}_\mathrm{L}, \\
D_\mu \tilde{d}_\mathrm{L} &= \partial_\mu \tilde{d}_\mathrm{L} + \mathrm{i}\left(eq_d A_\mu - e(\alpha_d - \beta_d) Z_\mu + g_s \frac{\lambda_A}{2} G_{A\mu}\right) \tilde{d}_\mathrm{L} + \frac{\mathrm{i}g}{\sqrt{2}} W_\mu^+ \tilde{u}_\mathrm{L}, \\
D_\mu \tilde{u}_\mathrm{R} &= \partial_\mu \tilde{u}_\mathrm{R} + \mathrm{i}\left(eq_u A_\mu - e(\alpha_u + \beta_u) Z_\mu + g_s \frac{\lambda_A}{2} G_{A\mu}\right) \tilde{u}_\mathrm{R}, \\
D_\mu \tilde{d}_\mathrm{R} &= \partial_\mu \tilde{d}_\mathrm{R} + \mathrm{i}\left(eq_d A_\mu - e(\alpha_d + \beta_d) Z_\mu + g_s \frac{\lambda_A}{2} G_{A\mu}\right) \tilde{d}_\mathrm{R},
\end{aligned}$$

where q_f, α_f, and β_f are defined in Table 1.2. Here, $\tilde{u}$ and $\tilde{d}$ denote any up- or down-type squark. Except for obvious replacements and the absence of the gluon

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field, $\tilde{\ell}_{\rm L}$, $\tilde{\ell}_{\rm R}$, and sneutrino covariant derivatives are identical to those for $\tilde{d}_{\rm L}$, $\tilde{d}_{\rm R}$ and $\tilde{u}_{\rm L}$, respectively.[6]

The quartic interactions that we mentioned are now easy to work out. The interactions with photons, Z^0, and gluons can be written as

$$\mathcal{L}_{VV\tilde{f}\tilde{f}} = \tilde{f}^{\dagger}_{\rm L/R}\left(eq_f A_\mu - e(\alpha_f \mp \beta_f)Z_\mu + \xi_f g_s \frac{\lambda_A}{2} G_{A\mu}\right) \times \left(eq_f A^\mu - e(\alpha_f \mp \beta_f)Z^\mu + \xi_f g_s \frac{\lambda_B}{2} G^\mu_B\right)\tilde{f}_{\rm L/R}, \quad (8.84a)$$

where the minus sign in the terms involving Z^0 is for $\tilde{f}_{\rm L}$ and the plus sign for $\tilde{f}_{\rm R}$ and $\xi_f = 1$ for squarks and $\xi_f = 0$ for charged sleptons and sneutrinos. Notice that in addition to just electroweak interactions, squarks also have QCD–electroweak hybrid interactions. Quartic interactions involving $W^\pm$ bosons can be written as

$$\mathcal{L}_{WW\tilde{f}\tilde{f}} = \frac{1}{2}g^2 \tilde{f}^{\dagger}_{\rm L}\tilde{f}_{\rm L} W^{\pm}_{\mu} W^{\mp\mu} \quad (8.84b)$$

where $\tilde{f}_{\rm L} = \tilde{u}_{\rm L}, \tilde{d}_{\rm L}, \tilde{\ell}_{\rm L}$ or $\tilde{\nu}$. Finally, the interactions involving both neutral and charged gauge bosons are,

$$\mathcal{L}_{VW\tilde{u}\tilde{d}} = \frac{g}{\sqrt{2}}\tilde{u}^{\dagger}_{\rm L}\left(e(q_u+q_d)A_\mu - e(\alpha_u+\alpha_d)Z_\mu + g_s\frac{\lambda_A}{2}G_{A\mu}\right)W^{+\mu}\tilde{d}_{\rm L} + \frac{g}{\sqrt{2}}\tilde{\nu}^{\dagger}_{\rm L}\left(eq_\ell A_\mu - e(\alpha_\ell+\alpha_\nu)Z_\mu\right)W^{+\mu}\tilde{\ell}_{\rm L} + \quad \text{h.c.} \quad (8.84c)$$

Left-type squark pairs have a contact interaction with the W-boson gluon pair.

In writing Eq. (8.84a)–(8.84c) we have ignored intragenerational mixing of sfermions. This can be easily included by writing $\tilde{f}_{\rm L}$ and $\tilde{f}_{\rm R}$ in terms of the mass eigenstates. Clearly, the four-point interactions involving just gluons and photons will couple just $\tilde{f}_1\tilde{f}_1$ and $\tilde{f}_2\tilde{f}_2$ pairs, while the others will couple $\tilde{f}_1\tilde{f}_2$ pairs as well.

Chargino and neutralino couplings to matter

Because these are dimension four interactions, these interactions are unaffected by the soft SUSY breaking terms. There are just two sources of these couplings. First, the gaugino components of charginos and neutralinos couple to fermions and sfermions via the term

$$\mathcal{L} \ni -\sqrt{2}\sum_{i,A} g\mathcal{S}^{\dagger}_i t_A \bar{\lambda}_A \frac{1-\gamma_5}{2}\psi_i + \quad \text{h.c.},$$

[6] These covariant derivatives give an alternative way to write down the coupling of any gauge boson to a sfermion pair.

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in Eq. (6.44). These couplings are completely determined by gauge interactions and various sparticle mixing matrices. The higgsino components of the charginos and neutralinos also contribute to these couplings via superpotential Yukawa interactions contained in

$$\mathcal{L} \ni -\frac{1}{2}\bar{\psi}_i \left(\frac{\partial^2 \hat{f}}{\partial \hat{\mathcal{S}}_i \partial \hat{\mathcal{S}}_j}\right)_{\hat{\mathcal{S}}=\mathcal{S}} P_L \psi_j + \quad \text{h.c.}$$

For most purposes, these couplings are only important for the third generation.

We begin by evaluating the neutralino–quark–squark couplings arising from gaugino interactions. The relevant terms are contained in

$$\mathcal{L} \ni -\frac{1}{\sqrt{2}}\left\{\begin{pmatrix}\tilde{u}_L^\dagger & \tilde{d}_L^\dagger\end{pmatrix}\begin{pmatrix} g\bar{\lambda}_3 + \frac{g'}{3}\bar{\lambda}_0 & g(\bar{\lambda}_1 - i\bar{\lambda}_2) \\ g(\bar{\lambda}_1 + i\bar{\lambda}_2) & -g\bar{\lambda}_3 + \frac{g'}{3}\bar{\lambda}_0 \end{pmatrix} P_L \begin{pmatrix}\psi_u \\ \psi_d\end{pmatrix} \right.$$
$$\left. + \tilde{u}_R^\dagger g'\left(-\frac{4}{3}\right)\bar{\lambda}_0 P_R \psi_{U^c} + \tilde{d}_R^\dagger g'\left(+\frac{2}{3}\right)\bar{\lambda}_0 P_R \psi_{D^c}\right\} + \quad \text{h.c.}, \tag{8.85}$$

where, for convenience, we have written the Hermitian conjugate of the terms involving the $SU(2)$ singlet antiquarks. The interactions of charged sleptons and sneutrinos can be obtained by replacing $\tilde{u}_L \to \tilde{\nu}$, $\tilde{d}_L \to \tilde{\ell}$, $\tilde{d}_R \to \tilde{\ell}_R$, dropping the term involving $\tilde{u}_R$, and replacing 2/3, the weak hypercharge of the $SU(2)$ singlet $\bar{d}$, by 2, the hypercharge of the antilepton. We also need to replace the quark hypercharges that multiply $\bar{\lambda}_0$ by corresponding lepton or neutrino hypercharges, and also appropriately replace the quark Majorana spinors by those of the lepton/neutrino. We proceed, however, to extract the quark–squark–neutral gaugino interactions, and eliminate the Majorana fields in favor of the Dirac quark fields using $P_L\psi_u = P_L u$, $P_L\psi_d = P_L d$, $P_R\psi_{U^c} = P_R u$ and $P_R\psi_{D^c} = P_R d$. Finally, using Eq. (8.50) and (8.51), we substitute $\lambda_3 = \sum_i v_3^{(i)}(i\gamma_5)^{\theta_i}\widetilde{Z}_i$, and $\lambda_0 = \sum_i v_4^{(i)}(i\gamma_5)^{\theta_i}\widetilde{Z}_i$ to write,

$$\mathcal{L}_{\tilde{f}f\widetilde{Z}_i} = \sum_{f=u,d,\ell,\nu}\left[iA^f_{\widetilde{Z}_i}\tilde{f}_L^\dagger\overline{\widetilde{Z}}_i P_L f + iB^f_{\widetilde{Z}_i}\tilde{f}_R^\dagger\overline{\widetilde{Z}}_i P_R f + \quad \text{h.c.}\right], \tag{8.86}$$

where

$$A^u_{\widetilde{Z}_i} = \frac{(-i)^{\theta_i - 1}}{\sqrt{2}}\left[g v_3^{(i)} + \frac{g'}{3}v_4^{(i)}\right], \tag{8.87a}$$

$$A^d_{\widetilde{Z}_i} = \frac{(-i)^{\theta_i - 1}}{\sqrt{2}}\left[-g v_3^{(i)} + \frac{g'}{3}v_4^{(i)}\right], \tag{8.87b}$$

$$B^u_{\widetilde{Z}_i} = \frac{4}{3\sqrt{2}}g'(i)^{\theta_i - 1}v_4^{(i)} \quad \text{and} \tag{8.87c}$$

$$B^d_{\widetilde{Z}_i} = -\frac{2}{3\sqrt{2}}g'(i)^{\theta_i - 1}v_4^{(i)}. \tag{8.87d}$$

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The couplings of leptons and sleptons to neutralinos have the same form as in (8.86) above, but with couplings given by

$$A^{\ell}_{\widetilde{Z}_i} = -\frac{(-\mathrm{i})^{\theta_i-1}}{\sqrt{2}}\left[g v_3^{(i)} + g' v_4^{(i)}\right], \tag{8.88a}$$

$$A^{\nu}_{\widetilde{Z}_i} = \frac{(-\mathrm{i})^{\theta_i-1}}{\sqrt{2}}\left[g v_3^{(i)} - g' v_4^{(i)}\right], \tag{8.88b}$$

$$B^{\ell}_{\widetilde{Z}_i} = -(\mathrm{i})^{\theta_i-1}\sqrt{2}g' v_4^{(i)} \quad \text{and} \tag{8.88c}$$

$$B^{\nu}_{\widetilde{Z}_i} = 0. \tag{8.88d}$$

Next, we turn to the contribution to fermion–sfermion–neutralino interactions that arise from the superpotential terms,

$$\mathcal{L} \ni -\frac{1}{2}\bar{\psi}_i \left(\frac{\partial^2 \hat{f}}{\partial \hat{\mathcal{S}}_i \partial \hat{\mathcal{S}}_j}\right)_{\hat{\mathcal{S}}=\mathcal{S}} P_{\mathrm{L}}\psi_j + \quad \text{h.c.},$$

with

$$\hat{f} \ni f_u \hat{u}\hat{h}_u^0 \hat{U}^c + f_d \hat{d}\hat{h}_d^0 \hat{D}^c + f_e \hat{e}\hat{h}_d^0 \hat{E}^c + \cdots,$$

where the ellipsis denotes Yukawa couplings for the second and third generations. For up- (down-)type (s)fermions, we have contributions when one of ψ_i, ψ_j is $\psi_{h_u^0}$ ($\psi_{h_d^0}$), with the other one being ψ_f or ψ_{F^c}. It is straightforward to check that these contributions can be written as,

$$\mathcal{L} \ni -f_f v_a^{(i)}(-\mathrm{i})^{\theta_i} \tilde{f}_{\mathrm{R}}^{\dagger}\overline{\widetilde{Z}}_i P_{\mathrm{L}} f - f_f v_a^{(i)}(\mathrm{i})^{\theta_i} \tilde{f}_{\mathrm{L}}^{\dagger}\overline{\widetilde{Z}}_i P_{\mathrm{R}} f,$$

with $a = 1$ for up-type (s)fermions, and $a = 2$ for down-type ones. Combining this with the contributions (8.86) from the gaugino components of neutralinos, we have,

$$\begin{aligned}\mathcal{L}_{\widetilde{Z}_i f\tilde{f}} \ni\; & \tilde{f}_{\mathrm{L}}^{\dagger}\overline{\widetilde{Z}}_i\left(\mathrm{i}A^{f}_{\widetilde{Z}_i}P_{\mathrm{L}} - (\mathrm{i})^{\theta_i} f_f v_a^{(i)} P_{\mathrm{R}}\right) f \\ & + \tilde{f}_{\mathrm{R}}^{\dagger}\overline{\widetilde{Z}}_i\left(\mathrm{i}B^{f}_{\widetilde{Z}_i}P_{\mathrm{R}} - (-\mathrm{i})^{\theta_i} f_f v_a^{(i)} P_{\mathrm{L}}\right) f + \quad \text{h.c.}\end{aligned} \tag{8.89}$$

Finally, eliminating $\tilde{f}_{\mathrm{L}}$ and $\tilde{f}_{\mathrm{R}}$ in favor of the sfermion mass eigenstates $\tilde{f}_1$ and $\tilde{f}_2$, we arrive at

$$\mathcal{L}_{\widetilde{Z}_i f\tilde{f}} = \tilde{f}_j^{\dagger}\overline{\widetilde{Z}}_i\left[\alpha^{\tilde{f}_j}_{\widetilde{Z}_i}P_{\mathrm{L}} + \beta^{\tilde{f}_j}_{\widetilde{Z}_i}P_{\mathrm{R}}\right] f + \quad \text{h.c.}, \tag{8.90}$$

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with

$$\alpha_{\widetilde{Z}_i}^{\tilde{f}_1} = \mathrm{i}A_{\widetilde{Z}_i}^{f} \cos\theta_f + (-\mathrm{i})^{\theta_i} f_f v_a^{(i)} \sin\theta_f, \tag{8.91a}$$

$$\beta_{\widetilde{Z}_i}^{\tilde{f}_1} = -\mathrm{i}B_{\widetilde{Z}_i}^{f} \sin\theta_f - (\mathrm{i})^{\theta_i} f_f v_a^{(i)} \cos\theta_f, \tag{8.91b}$$

$$\alpha_{\widetilde{Z}_i}^{\tilde{f}_2} = \mathrm{i}A_{\widetilde{Z}_i}^{f} \sin\theta_f - (-\mathrm{i})^{\theta_i} f_f v_a^{(i)} \cos\theta_f, \tag{8.91c}$$

$$\beta_{\widetilde{Z}_i}^{\tilde{f}_2} = \mathrm{i}B_{\widetilde{Z}_i}^{f} \cos\theta_f - (\mathrm{i})^{\theta_i} f_f v_a^{(i)} \sin\theta_f. \tag{8.91d}$$

Again, $a = 1$ if f is an up-type quark, and $a = 2$ if it is a down-type quark or a charged lepton. Since we do not have a right-handed neutrino superfield, the neutrino–sneutrino–neutralino coupling is given by (8.86).

The interactions of charginos with either squarks and quarks or sleptons and leptons can be calculated in a similar fashion. For chargino–quark–squark interactions, using (8.85) we find that

$$\mathcal{L} \ni -g\tilde{u}_{\mathrm{L}}^{\dagger}\bar{\lambda}P_{\mathrm{L}}d \; - g\tilde{d}_{\mathrm{L}}^{\dagger}\overline{\lambda^c}P_{\mathrm{L}}u + \quad \text{h.c.}$$

Here, λ^c is the charge conjugate of the charged Dirac gaugino λ. Eliminating λ and λ^c in favor of the chargino mass eigenstates, we find

$$\mathcal{L} \ni \mathrm{i}A_{\widetilde{W}_i}^{d}\tilde{u}_{\mathrm{L}}^{\dagger}\overline{\widetilde{W}}_i P_{\mathrm{L}}d + \mathrm{i}A_{\widetilde{W}_i}^{u}\tilde{d}_{\mathrm{L}}^{\dagger}\overline{\widetilde{W}_i^c}P_{\mathrm{L}}u + \quad \text{h.c.}, \tag{8.92}$$

where

$$A_{\widetilde{W}_1}^{d} = \mathrm{i}(-1)^{\theta_{\widetilde{W}_1}} g \sin\gamma_{\mathrm{R}}, \tag{8.93a}$$

$$A_{\widetilde{W}_2}^{d} = \mathrm{i}(-1)^{\theta_{\widetilde{W}_2}} \theta_y g \cos\gamma_{\mathrm{R}}, \tag{8.93b}$$

$$A_{\widetilde{W}_1}^{u} = \mathrm{i}g \sin\gamma_{\mathrm{L}}, \tag{8.93c}$$

$$A_{\widetilde{W}_2}^{u} = \mathrm{i}\theta_x g \cos\gamma_{\mathrm{L}}. \tag{8.93d}$$

These couplings, which originate in the gauge interactions, are generation independent; i.e. u and d ($\tilde{u}_{\mathrm{L}}$ and $\tilde{d}_{\mathrm{L}}$) respectively refer to any up- and down-type quark (squark). Moreover, the coupling of charginos to leptons and sleptons is identical, with the identification $u \to \nu$ and $d \to \ell$.

There are also superpotential contributions to these chargino interactions that can be worked out in the same way as for neutralinos. We will leave it to the reader to work out that including these leads to the couplings,

$$\begin{aligned}\mathcal{L}_{\tilde{u}d\widetilde{W}_i} = {} & \tilde{u}_1^{\dagger}\overline{\widetilde{W}}_i\left[(\mathrm{i}A_{\widetilde{W}_i}^{d}\cos\theta_u - B_{\widetilde{W}_i}\sin\theta_u)P_{\mathrm{L}} + B'_{\widetilde{W}_i}\cos\theta_u P_{\mathrm{R}}\right]d \\ & + \tilde{u}_2^{\dagger}\overline{\widetilde{W}}_i\left[(\mathrm{i}A_{\widetilde{W}_i}^{d}\sin\theta_u + B_{\widetilde{W}_i}\cos\theta_u)P_{\mathrm{L}} + B'_{\widetilde{W}_i}\sin\theta_u P_{\mathrm{R}}\right]d + \quad \text{h.c.},\end{aligned} \tag{8.94}$$

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where

$$B_{\widetilde{W}_1} = -(-1)^{\theta_{\widetilde{W}_1}} f_u \cos\gamma_{\mathrm{R}}, \tag{8.95a}$$

$$B_{\widetilde{W}_2} = (-1)^{\theta_{\widetilde{W}_2}} \theta_y f_u \sin\gamma_{\mathrm{R}}, \tag{8.95b}$$

$$B'_{\widetilde{W}_1} = -f_d \cos\gamma_{\mathrm{L}}, \tag{8.95c}$$

$$B'_{\widetilde{W}_2} = f_d \theta_x \sin\gamma_{\mathrm{L}}. \tag{8.95d}$$

For chargino–sbottom–top interactions, we have

$$\mathcal{L}_{\tilde{d}u\widetilde{W}_i} = \tilde{d}_1^\dagger \overline{\widetilde{W}_i^c} \left[(\mathrm{i}A^u_{\widetilde{W}_i} \cos\theta_d - B'_{\widetilde{W}_i} \sin\theta_d) P_{\mathrm{L}} + B_{\widetilde{W}_i} \cos\theta_d P_{\mathrm{R}} \right] u + \tilde{d}_2^\dagger \overline{\widetilde{W}_i^c} \left[(\mathrm{i}A^u_{\widetilde{W}_i} \sin\theta_d + B'_{\widetilde{W}_i} \cos\theta_d) P_{\mathrm{L}} + B_{\widetilde{W}_i} \sin\theta_d P_{\mathrm{R}} \right] u + \text{h.c.} \tag{8.96}$$

Finally, the chargino–slepton–neutrino and chargino–sneutrino–lepton interactions can be obtained by replacing $u \to \nu$ and $d \to \ell$ everywhere *including* in the definitions of the couplings in (8.95a)–(8.95d). We then have,

$$\mathcal{L}_{\tilde{\tau}\nu_\tau\widetilde{W}_i} = \tilde{\tau}_1^\dagger \overline{\widetilde{W}_i^c} \left[(\mathrm{i}A^\nu_{\widetilde{W}_i} \cos\theta_\tau - B''_{\widetilde{W}_i} \sin\theta_\tau) P_{\mathrm{L}} \nu_\tau \right] + \tilde{\tau}_2^\dagger \overline{\widetilde{W}_i^c} \left[(\mathrm{i}A^\nu_{\widetilde{W}_i} \sin\theta_\tau + B''_{\widetilde{W}_i} \cos\theta_\tau) P_{\mathrm{L}} \right] \nu_\tau + \tilde{\nu}_\tau^\dagger \overline{\widetilde{W}}_i \left[\mathrm{i}A^\tau_{\widetilde{W}_i} P_{\mathrm{L}} + B''_{\widetilde{W}_i} P_{\mathrm{R}} \right] \tau + \text{h.c.}, \tag{8.97}$$

with

$$A^\nu_{\widetilde{W}_i} = A^u_{\widetilde{W}_i}, \tag{8.98a}$$

$$A^\tau_{\widetilde{W}_i} = A^d_{\widetilde{W}_i}, \tag{8.98b}$$

$$B''_{\widetilde{W}_1} = -f_\tau \cos\gamma_{\mathrm{L}}, \tag{8.98c}$$

$$B''_{\widetilde{W}_2} = f_\tau \theta_x \sin\gamma_{\mathrm{L}}. \tag{8.98d}$$

Gauge boson interactions with charginos and neutralinos

These interactions arise from two sources, both of which are supersymmetric. First, there is the contribution from gaugino kinetic energy terms,

$$\mathcal{L} \ni \frac{\mathrm{i}}{2} \bar{\lambda} \not{D} \lambda,$$

in the master formula, with the covariant derivative involving gauge group generators in the adjoint representation: $(\not{D}\lambda)_A = \not{\partial}\lambda_A + \mathrm{i}g(t_B^{\mathrm{adj}} \not{V}_B)_{AC}\lambda_C$, with

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$[t_B^{\mathrm{adj}}]_{AC} = -\mathrm{i}\epsilon_{ACB}$. The $SU(2)_\mathrm{L}$ gauginos thus have a coupling of the form

$$\begin{aligned}\mathcal{L} \ni \frac{-\mathrm{i}g}{2}&\big(-\bar{\lambda}_1 \not{W}_3\lambda_2 + \bar{\lambda}_1 \not{W}_2\lambda_3 + \bar{\lambda}_2 \not{W}_3\lambda_1 - \bar{\lambda}_2 \not{W}_1\lambda_3 \\ &-\bar{\lambda}_3 \not{W}_2\lambda_1 + \bar{\lambda}_3 \not{W}_1\lambda_2\big), \\ &= g\left[\bar{\lambda} \not{W}_3\lambda - (\bar{\lambda} \not{W}^-\lambda_3 + \quad \text{h.c.})\right],\end{aligned}$$

while there is no coupling to the hypercharge gaugino. To obtain the last step, we have used $\bar{\lambda}_3\gamma_\mu\lambda^c = -\bar{\lambda}\gamma_\mu\lambda_3$, as the reader can readily verify.

There are also higgsino contributions

$$\begin{aligned}\mathcal{L} \ni \frac{\mathrm{i}}{2}\Bigg[&\left(\bar{\psi}_{h_u^+}\ \bar{\psi}_{h_u^0}\right)\frac{\mathrm{i}}{2}\begin{bmatrix} g \not{W}_3 + g' \not{B} & g \not{W}_1 - \mathrm{i}g \not{W}_2 \\ g \not{W}_1 + \mathrm{i}g \not{W}_2 & -g \not{W}_3 + g' \not{B}\end{bmatrix} P_\mathrm{L}\begin{pmatrix}\psi_{h_u^+} \\ \psi_{h_u^0}\end{pmatrix} \\ +&\left(\bar{\psi}_{h_d^-}\ \bar{\psi}_{h_d^0}\right)\frac{\mathrm{i}}{2}\begin{bmatrix} -g \not{W}_3 - g' \not{B} & -g \not{W}_1 - \mathrm{i}g \not{W}_2 \\ -g \not{W}_1 + \mathrm{i}g \not{W}_2 & g \not{W}_3 - g' \not{B}\end{bmatrix} P_\mathrm{L}\begin{pmatrix}\psi_{h_d^-} \\ \psi_{h_d^0}\end{pmatrix}\Bigg] + \quad \text{h.c.}\end{aligned}$$

Exercise *Verify that we can write the gaugino and higgsino contributions as:*

$$\begin{aligned}\mathcal{L} \ni g&\left\{\bar{\lambda} \not{W}_3\lambda - (\bar{\lambda} \not{W}^-\lambda_3 + \quad \text{h.c.})\right\} \\ &+ \frac{1}{2}\bar{\tilde{\chi}}\left(g \not{W}_3 + g' \not{B}\right)\tilde{\chi} \\ &+ \frac{1}{4}\sqrt{g^2 + g'^2}\left(\bar{\psi}_{h_u^0}\gamma_\mu\gamma_5\psi_{h_u^0} - \bar{\psi}_{h_d^0}\gamma_\mu\gamma_5\psi_{h_d^0}\right) Z^\mu \\ &- \frac{g}{\sqrt{2}}(\bar{\tilde{\chi}} \not{W}^- P_\mathrm{R}\psi_{h_u^0} - \bar{\tilde{\chi}} \not{W}^- P_\mathrm{L}\psi_{h_d^0} + \quad \text{h.c.}).\end{aligned}$$

Here, the first line clearly comes from the couplings of the gauginos to gauge bosons, while the rest comes from the gauge interactions of higgsinos.

We can now write these in terms of the chargino and neutralino mass eigenstates to obtain the following couplings to the photon and Z^0 boson:

$$\begin{aligned}\mathcal{L} = e&\left(\overline{\widetilde{W}}_1\gamma_\mu\widetilde{W}_1 + \overline{\widetilde{W}}_2\gamma_\mu\widetilde{W}_2\right)A^\mu \\ &- e\cot\theta_\mathrm{W}\overline{\widetilde{W}}_1\gamma_\mu(x_c - y_c\gamma_5)\widetilde{W}_1 Z^\mu - e\cot\theta_\mathrm{W}\overline{\widetilde{W}}_2\gamma_\mu(x_s - y_s\gamma_5)\widetilde{W}_2 Z^\mu \\ &+ (-1)^{(\theta_{\widetilde{W}_1} + \theta_{\widetilde{W}_2})}\frac{e}{2}(\cot\theta_\mathrm{W} + \tan\theta_\mathrm{W}) \\ &\times\left[\overline{\widetilde{W}}_1\gamma_\mu(x\gamma_5 - y)(\gamma_5)^{(\theta_{\widetilde{W}_1} + \theta_{\widetilde{W}_2})}\widetilde{W}_2 Z^\mu + \quad \text{h.c.}\right],\end{aligned} \tag{8.99}$$

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where

$$x_c = 1 - \frac{1}{4}\sec^2\theta_W(\cos^2\gamma_L + \cos^2\gamma_R), \tag{8.100a}$$

$$y_c = \frac{1}{4}\sec^2\theta_W(\cos^2\gamma_R - \cos^2\gamma_L), \tag{8.100b}$$

$$x_s = 1 - \frac{1}{4}\sec^2\theta_W(\sin^2\gamma_L + \sin^2\gamma_R), \tag{8.100c}$$

$$y_s = \frac{1}{4}\sec^2\theta_W(\sin^2\gamma_R - \sin^2\gamma_L), \tag{8.100d}$$

$$x = \frac{1}{2}(\theta_x \sin\gamma_L\cos\gamma_L - \theta_y \sin\gamma_R\cos\gamma_R), \quad \text{and} \tag{8.100e}$$

$$y = \frac{1}{2}(\theta_x \sin\gamma_L\cos\gamma_L + \theta_y \sin\gamma_R\cos\gamma_R). \tag{8.100f}$$

Notice that the photon does not couple to the $\widetilde{W}_1^+\widetilde{W}_2^-$ pair, as may be expected from the conservation of electromagnetic current.

The couplings of Z^0 with the neutralinos arise only via their higgsino components, and are given by,

$$\begin{aligned}\mathcal{L} &= \frac{1}{4}\sqrt{g^2+g'^2}\sum_{i,j}(-\mathrm{i})^{\theta_i}(\mathrm{i})^{\theta_j}(v_1^{(i)}v_1^{(j)} - v_2^{(i)}v_2^{(j)})\,\overline{\widetilde{Z}}_i\gamma_\mu(\gamma_5)^{\theta_i+\theta_j+1}\widetilde{Z}_j Z^\mu \\ &\equiv \sum_{ij} W_{ij}\overline{\widetilde{Z}}_i\gamma_\mu(\gamma_5)^{\theta_i+\theta_j+1}\widetilde{Z}_j Z^\mu. \end{aligned}\tag{8.101}$$

In models where $|\mu| \gg (\ll)|M_{1,2}|$, the neutralinos $\widetilde{Z}_1$ and $\widetilde{Z}_2$ ($\widetilde{Z}_3$ and $\widetilde{Z}_4$) are mainly gaugino-like so that their couplings to Z^0 are strongly suppressed by mixing angles. The couplings of neutralino pairs to gauge bosons are, therefore, very sensitive to model parameters. This is not the case for charginos. Their couplings to the photon are fixed by their electric charge. Moreover, chargino pairs couple to Z^0 via both their gaugino as well as their higgsino components, so that their couplings to vector bosons are much more robust.

Exercise *If* $\tan\beta = 1$ *show that the higgsino* $\frac{1}{\sqrt{2}}(\psi_{h_u^0} + \psi_{h_d^0})$ *has mass* $|\mu|$ *but that* Z^0 *does not couple to a pair of these higgsinos.*

Finally for charged vector bosons, substituting in terms of the mass eigenstates, we obtain,

$$\mathcal{L} = -g(-\mathrm{i})^{\theta_j}\sum_{i,j}\overline{\widetilde{W}}_i\left(X_i^j + Y_i^j\gamma_5\right)\gamma_\mu\widetilde{Z}_j W^\mu + \quad \text{h.c.}, \tag{8.102}$$

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with

$$X_1^j = \frac{1}{2}\left[(-1)^{\theta_{\widetilde{W}_1}+\theta_j}\left(\frac{\cos\gamma_R}{\sqrt{2}}v_1^{(j)} + \sin\gamma_R v_3^{(j)}\right)\right.$$
$$\left. -\frac{\cos\gamma_L}{\sqrt{2}}v_2^{(j)} + \sin\gamma_L v_3^{(j)}\right], \tag{8.103a}$$
$$X_2^j = \frac{1}{2}\left[(-1)^{\theta_{\widetilde{W}_2}+\theta_j}\theta_y\left(\frac{-\sin\gamma_R}{\sqrt{2}}v_1^{(j)} + \cos\gamma_R v_3^{(j)}\right)\right.$$
$$\left. +\theta_x\left(\frac{\sin\gamma_L}{\sqrt{2}}v_2^{(j)} + \cos\gamma_L v_3^{(j)}\right)\right]. \tag{8.103b}$$

The $Y_{1,2}^j$ can be obtained from the $X_{1,2}^j$ by changing the sign of just the first term inside the square brackets. We see that W bosons couple to the chargino–neutralino system via both gaugino and higgsino components. In this sense, $W\widetilde{W}_i\widetilde{Z}_j$ couplings should, like the couplings of Z^0 to charginos, also be quite robust. Only if $|M_1| \ll |M_2|$ and $|\mu|$ (in which case the neutralino is dominantly a hypercharge gaugino) is this coupling dynamically suppressed.

8.4.3 Interactions of MSSM Higgs bosons

Higgs boson couplings to SM fermions

The interactions of Higgs bosons with SM fermions arise directly from the terms,

$$\mathcal{L} \ni -\frac{1}{2}\sum_{i,j}\bar{\psi}_i \left.\frac{\partial^2 \hat{f}}{\partial\hat{\mathcal{S}}_i\partial\hat{\mathcal{S}}_j}\right|_{\hat{\mathcal{S}}=\mathcal{S}} P_L\psi_j + \quad \text{h.c.},$$

in our master formula. We have already examined a portion of these terms when we discussed masses for the SM fermions. Our present discussion proceeds along the same lines. The superpotential contains

$$\hat{f} \ni f_u(\hat{u}\hat{h}_u^0 - \hat{d}\hat{h}_u^+)\hat{U}^c + f_d(\hat{u}\hat{h}_d^- + \hat{d}\hat{h}_d^0)\hat{D}^c + f_e(\hat{\nu}_\tau\hat{h}_d^- + \hat{e}\hat{h}_d^0)\hat{E}^c + \cdots$$

We can easily work out the coupling of Dirac fermions to the scalar components in $\hat{h}_u$ to obtain,

$$\mathcal{L} \ni -f_u\bar{u}P_L u h_u^0 - f_u\bar{u}P_R u h_u^{0\dagger}$$

We can now eliminate h_u^0 in favor of the Higgs mass eigenstates using (8.35) and (8.40a). Recalling that $f_u = gm_u/\sqrt{2}M_W\sin\beta$ we find the required Lagrangian density,

$$\mathcal{L} \ni -\frac{gm_u}{2M_W\sin\beta}\left[\cos\alpha\bar{u}uh - \sin\alpha\bar{u}uH - \mathrm{i}\cos\beta\bar{u}\gamma_5 uA\right]. \tag{8.104}$$

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A similar calculation for the down-type quark and charged lepton Yukawa interactions yields,

$$\mathcal{L} \ni -\frac{g m_d}{2M_W \cos\beta}\left[\sin\alpha \bar{d} d h + \cos\alpha \bar{d} d H - \mathrm{i}\sin\beta \bar{d}\gamma_5 d A\right]$$
$$-\frac{g m_e}{2M_W \cos\beta}\left[\sin\alpha \bar{e} e h + \cos\alpha \bar{e} e H - \mathrm{i}\sin\beta \bar{e}\gamma_5 e A\right]. \qquad (8.105)$$

The interactions with charged Higgs bosons can be similarly obtained by eliminating h_u^+ and h_d^- using (8.30):

$$\mathcal{L} \ni \frac{g}{2\sqrt{2}M_W} H^+ [(m_u \cot\beta + m_d \tan\beta)\bar{u}d + (m_d \tan\beta - m_u \cot\beta)\bar{u}\gamma_5 d$$
$$+ m_e \tan\beta \bar{\nu}_e(1+\gamma_5)e] + \quad \text{h.c.} \qquad (8.106)$$

Higgs boson couplings to vector bosons

As in any Yang–Mills theory, the coupling of vector bosons to Higgs boson pairs is fixed by the minimal coupling prescription; i.e. these arise from cross terms in the scalar field kinetic energy terms

$$\mathcal{L} \ni (D_\mu \mathcal{S}_i)^\dagger (D^\mu \mathcal{S}_i),$$

where $\mathcal{S}_i = H_u$ and H_d. Expanding these terms and substituting for the physical vector boson and Higgs fields yields the expected photon coupling to the charged Higgs boson pair,

$$\mathcal{L} \ni \mathrm{i}e \left(H^+ \partial_\mu H^- - H^- \partial_\mu H^+\right) A^\mu. \qquad (8.107)$$

The Z^0 boson couples to both charged as well as neutral Higgs fields, with couplings given by,

$$\mathcal{L} \ni \frac{\mathrm{i}}{2}(g' \sin\theta_{\mathrm{W}} - g\cos\theta_{\mathrm{W}})\left(H^+ \partial_\mu H^- - H^- \partial_\mu H^+\right) Z^{0\mu}, \qquad (8.108)$$

and

$$\mathcal{L} \ni \frac{1}{2}(g' \sin\theta_{\mathrm{W}} + g\cos\theta_{\mathrm{W}})\left[\cos(\alpha+\beta)\left(h\partial_\mu A - A\partial_\mu h\right)\right.$$
$$\left. - \sin(\alpha+\beta)\left(H\partial_\mu A - A\partial_\mu H\right)\right] Z^{0\mu}. \qquad (8.109)$$

Notice that Z^0 only couples the pseudoscalar boson to a scalar boson. Couplings of Z^0 to hh, hH, and HH pairs are forbidden by the assumed CP invariance of the Higgs boson sector.

The couplings of W bosons to pairs of Higgs bosons are given by,

$$\mathcal{L} \ni \mathrm{i}\frac{g}{2}\left[\cos(\alpha+\beta)\left(h\partial_\mu H^- - H^-\partial_\mu h\right) - \sin(\alpha+\beta)\left(H\partial_\mu H^- - H^-\partial_\mu H\right) + \mathrm{i}\left(A\partial_\mu H^- - H^-\partial_\mu A\right)\right] W^{+\mu} + \quad \text{h.c.} \tag{8.110}$$

The gauge kinetic term for the Higgs fields also contains two-vector boson–two-Higgs boson couplings. These are given by,

$$\mathcal{L} \ni H^+H^-\left[\mathrm{e}^2 A^\mu A_\mu + \frac{1}{4}(g'\sin\theta_\mathrm{W} - g\cos\theta_\mathrm{W})^2 Z^{0\mu}Z^0_\mu + e(g'\sin\theta_\mathrm{W} - g\cos\theta_\mathrm{W})A^\mu Z^0_\mu + \frac{g^2}{2}W^{+\mu}W^-_\mu\right], \tag{8.111a}$$

$$\mathcal{L} \ni \left(\frac{g^2}{4}W^{+\mu}W^-_\mu + \frac{1}{8}(g\cos\theta_\mathrm{W} + g'\sin\theta_\mathrm{W})^2 Z^{0\mu}Z^0_\mu\right)\left[h^2 + H^2 + A^2\right], \tag{8.111b}$$

and

$$\mathcal{L} \ni \frac{1}{2}eg\left(A^\mu + \tan\theta_\mathrm{W} Z^{0\mu}\right)W^-_\mu H^+ \times \left[\cos(\alpha+\beta)h - \sin(\alpha+\beta)H - \mathrm{i}A\right] + \quad \text{h.c.} \tag{8.111c}$$

Finally, a vector boson–vector boson–Higgs boson coupling can also arise from the four-point interactions in the case when one of the neutral Higgs fields is replaced by its vacuum expectation value. Instead of starting over, we can get these couplings from the four-point couplings that we have just obtained in (8.111b) and (8.111c), and simply set one of the neutral fields to their VEV using Eq. (8.40a):

$$\begin{aligned}\langle h\rangle &= \sqrt{2}\,(\cos\alpha\, v_u + \sin\alpha\, v_d)\,,\\ \langle H\rangle &= \sqrt{2}\,(\cos\alpha\, v_d - \sin\alpha\, v_u)\,,\\ \langle A\rangle &= 0.\end{aligned}$$

The resulting interaction is,

$$\mathcal{L} \ni gM_W\left(W^{+\mu}W^-_\mu + \frac{Z^{0\mu}Z^0_\mu}{2\cos^2\theta_\mathrm{W}}\right)\left[\sin(\alpha+\beta)\,h + \cos(\alpha+\beta)\,H\right]. \tag{8.112}$$

Notice that there is no $Z^0W^-H^+$ coupling and, by electromagnetic gauge invariance, also no γW^-H^+ coupling.

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Higgs boson self-couplings

We have already remarked at the end of Section 8.2 that in the MSSM, quartic interactions of Higgs fields arise only from D-terms, and so are completely determined by gauge couplings. Since we have already worked out the complete potential in (8.25), it is straightforward to write the quartic couplings in terms of mass eigenstates. We find,

$$\begin{aligned}\mathcal{L} \ni -\frac{1}{8}\big\{&2g^2 H^+H^-\big[\cos^2(\beta-\alpha)h^2+\sin^2(\beta-\alpha)H^2\\&+\sin 2(\beta-\alpha)hH+\cos^2 2\beta A^2\big]\\&+(g^2+g'^2)\cos^2 2\beta(H^+H^-)^2+\frac{1}{4}(g^2+g'^2)\\&\times\big[\cos 2\alpha\,(h^2-H^2)-2\sin 2\alpha\, hH+\cos 2\beta\ A^2\big]^2\\&-(g^2-g'^2)\cos 2\beta\ H^+H^-\big[\cos 2\alpha(h^2-H^2)-2\sin 2\alpha hH+\cos 2\beta A^2\big]\big\}\,.\end{aligned} \tag{8.113}$$

We see that the Higgs quartic scalar self-couplings are all fixed by gauge interactions. This is the origin of the tree-level bounds on m_h in (8.39a) and (8.39b), respectively. That these bounds are special to the MSSM is exemplified by the following exercise.

Exercise *Show that if the Higgs sector of the MSSM is extended by the inclusion of an extra $SU(3)_C \times SU(2)_L \times U(1)_Y$ singlet (as in the exercise at the end of Section 8.3), the quartic self interactions of Higgs bosons are no longer determined by just the gauge couplings. Convince yourself that the tree-level bounds on m_h are not valid in this case.*

The D-terms also result in trilinear couplings amongst the Higgs fields. As before, we can obtain these by setting one of the neutral Higgs fields to their VEV. The result is,

$$\begin{aligned}\mathcal{L} \ni -\frac{1}{8}\big\{&H^+H^-[8gM_W(\sin(\alpha+\beta)h+\cos(\alpha+\beta)H)\\&+\frac{4gM_Z\cos 2\beta}{\cos\theta_W}(\sin(\beta-\alpha)\,h-\cos(\beta-\alpha)\,H)]\\&+\frac{2gM_Z}{\cos\theta_W}[\sin(\beta-\alpha)\,h-\cos(\beta-\alpha)\,H]\\&\times\big[\cos 2\alpha\, h^2-\cos 2\alpha\ H^2-2\sin 2\alpha\, hH+\cos 2\beta\ A^2\big]\big\}\,.\end{aligned} \tag{8.114}$$

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Notice that although these are all dimension 3 operators, there are no explicit soft SUSY breaking contributions to these interactions. This is because there is no gauge-invariant combination of three Higgs field doublets.

Higgs boson couplings to charginos and neutralinos

Supersymmetry dictates that Higgs bosons must interact with charginos and neutralinos. Since trilinear Higgs boson terms in the superpotential are forbidden by gauge invariance, these interactions can arise only from the couplings of Higgs bosons and higgsinos to $SU(2) \times U(1)$ gauginos. Letting $\mathcal{S}_i = H_u$ and H_d in the terms

$$\mathcal{L} \ni -\sqrt{2} \sum_{i,A} \mathcal{S}_i^\dagger g t_A \bar{\lambda}_A \frac{1-\gamma_5}{2} \psi_i + \quad \text{h.c.}$$

in the master formula, and eliminating the original fields in favor of the mass eigenstate fields leads to the required interactions. Since we have already done several similar calculations, we will simply present the final results.

The couplings of the light Higgs scalar to charginos and neutralinos are given by

$$\mathcal{L} = g\sqrt{2} S_1^h \overline{\widetilde{W}}_1 \widetilde{W}_1 h + g\sqrt{2} S_2^h \overline{\widetilde{W}}_2 \widetilde{W}_2 h + \left[\frac{g}{\sqrt{2}} \overline{\widetilde{W}}_1 (S^h + P^h \gamma_5) \widetilde{W}_2 h + \quad \text{h.c.} \right] + \sum_{i,j} X_{ij}^h \overline{\widetilde{Z}}_i (-i\gamma_5)^{\theta_i + \theta_j} \widetilde{Z}_j h, \tag{8.115}$$

where

$$S_1^h = \frac{1}{2}(-1)^{\theta_{\widetilde{W}_1}} [\sin\alpha \sin\gamma_R \cos\gamma_L + \cos\alpha \sin\gamma_L \cos\gamma_R], \tag{8.116a}$$

$$S_2^h = \frac{1}{2}(-1)^{\theta_{\widetilde{W}_2}+1} \theta_x \theta_y [\sin\alpha \cos\gamma_R \sin\gamma_L + \cos\alpha \cos\gamma_L \sin\gamma_R], \tag{8.116b}$$

$$S^h = \frac{1}{2} \left[-(-1)^{\theta_{\widetilde{W}_1}} \theta_x \sin\gamma_R \sin\gamma_L \sin\alpha + (-1)^{\theta_{\widetilde{W}_1}} \theta_x \cos\gamma_L \cos\gamma_R \cos\alpha - (-1)^{\theta_{\widetilde{W}_2}} \theta_y \sin\gamma_L \sin\gamma_R \cos\alpha + (-1)^{\theta_{\widetilde{W}_2}} \theta_y \cos\gamma_L \cos\gamma_R \sin\alpha \right], \tag{8.116c}$$

and P^h is the same as S^h except that the signs of the first two terms are reversed. Finally,

$$X_{ij}^h = -\frac{1}{2}(-1)^{\theta_i + \theta_j} \left(v_2^{(i)} \sin\alpha - v_1^{(i)} \cos\alpha \right) \left(g v_3^{(j)} - g' v_4^{(j)} \right). \tag{8.117}$$

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The couplings of the heavy scalar H can be obtained from those of h by replacing $\cos\alpha \to -\sin\alpha$ and $\sin\alpha \to \cos\alpha$.

The corresponding couplings of the pseudoscalar A are given by

$$\begin{aligned}\mathcal{L} \ni\ & \mathrm{i}g\sqrt{2}S_1^A\overline{\widetilde{W}}_1\gamma_5\widetilde{W}_1A + \mathrm{i}g\sqrt{2}S_2^A\overline{\widetilde{W}}_2\gamma_5\widetilde{W}_2A \\ & + \left[\frac{-\mathrm{i}g}{\sqrt{2}}\overline{\widetilde{W}}_1(S^A + P^A\gamma_5)\widetilde{W}_2A + \quad \text{h.c.}\right] \\ & + \sum_{i,j} X_{ij}^A\overline{\widetilde{Z}}_i(-\mathrm{i}\gamma_5)^{\theta_i+\theta_j+1}\widetilde{Z}_jA, \end{aligned} \tag{8.118}$$

where

$$S_1^A = \frac{1}{2}(-1)^{\theta_{\widetilde{W}_1}}[\sin\gamma_R\cos\gamma_L\sin\beta + \sin\gamma_L\cos\gamma_R\cos\beta], \tag{8.119a}$$

$$S_2^A = -\frac{1}{2}(-1)^{\theta_{\widetilde{W}_2}}\theta_x\theta_y[\cos\gamma_R\sin\gamma_L\sin\beta + \cos\gamma_L\sin\gamma_R\cos\beta], \tag{8.119b}$$

$$\begin{aligned} S^A = \frac{1}{2}\big[& -(-1)^{\theta_{\widetilde{W}_1}}\theta_x\sin\gamma_R\sin\gamma_L\sin\beta + (-1)^{\theta_{\widetilde{W}_1}}\theta_x\cos\gamma_L\cos\gamma_R\cos\beta \\ & + (-1)^{\theta_{\widetilde{W}_2}}\theta_y\sin\gamma_L\sin\gamma_R\cos\beta - (-1)^{\theta_{\widetilde{W}_2}}\theta_y\cos\gamma_L\cos\gamma_R\sin\beta\big], \end{aligned} \tag{8.119c}$$

and P^A is obtained by reversing the sign of the first two terms of the expression for S^A. The coupling of A to neutralinos is,

$$X_{ij}^A = \frac{1}{2}(-1)^{\theta_i+\theta_j}\left(v_2^{(i)}\sin\beta - v_1^{(i)}\cos\beta\right)\left(gv_3^{(j)} - g'v_4^{(j)}\right). \tag{8.120}$$

Note that h and H couple to the scalar combination of $\widetilde{W}_i\widetilde{W}_i$ or $\widetilde{Z}_i\widetilde{Z}_i$ while A couples to the pseudoscalar combination. It is for this reason that we refer to h and H as scalars, and to A as a pseudoscalar.

Finally, the interactions of the charged Higgs bosons are given by,

$$\begin{aligned}\mathcal{L} = \sum_k(\mathrm{i})^{\theta_k}\Big[& \cos\beta A_1^{(k)}\theta_y(-1)^{\theta_{\widetilde{W}_2}}\overline{\widetilde{Z}}_kP_R\widetilde{W}_2 + \cos\beta A_2^{(k)}(-1)^{\theta_{\widetilde{W}_1}}\overline{\widetilde{Z}}_kP_R\widetilde{W}_1 \\ & - \sin\beta A_3^{(k)}\theta_x(-1)^{\theta_k}\overline{\widetilde{Z}}_kP_L\widetilde{W}_2 - \sin\beta A_4^{(k)}(-1)^{\theta_k}\overline{\widetilde{Z}}_kP_L\widetilde{W}_1\Big]H^+ + \quad \text{h.c.} \end{aligned} \tag{8.121}$$

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with

$$A_1^{(k)} = -\frac{1}{\sqrt{2}}\left(gv_3^{(k)} + g'v_4^{(k)}\right)\sin\gamma_R - gv_1^{(k)}\cos\gamma_R, \tag{8.122a}$$

$$A_2^{(k)} = \frac{1}{\sqrt{2}}\left(gv_3^{(k)} + g'v_4^{(k)}\right)\cos\gamma_R - gv_1^{(k)}\sin\gamma_R, \tag{8.122b}$$

$$A_3^{(k)} = -\frac{1}{\sqrt{2}}\left(gv_3^{(k)} + g'v_4^{(k)}\right)\sin\gamma_L + gv_2^{(k)}\cos\gamma_L, \tag{8.122c}$$

$$A_4^{(k)} = \frac{1}{\sqrt{2}}\left(gv_3^{(k)} + g'v_4^{(k)}\right)\cos\gamma_L + gv_2^{(k)}\sin\gamma_L. \tag{8.122d}$$

Higgs boson couplings to squarks and sleptons

In addition to the couplings that we have listed, there are several four scalar interactions in the MSSM. Since these are dimension four operators, there are no explicit soft-SUSY breaking contributions to these.

The D-term contributions from the term

$$\mathcal{L} \ni -\frac{1}{2}\sum_A \left|\sum_i \mathcal{S}_i^\dagger g_\alpha t_{\alpha A}\mathcal{S}_i\right|^2$$

in the master formula can be written as,

$$\begin{aligned}\mathcal{L} \ni -\frac{1}{2}\Bigg\{&\frac{g^2}{4}\Big[(h_u^{+\dagger}h_u^0 + h_u^{0\dagger}h_u^+) - (h_d^{-\dagger}h_d^0 + h_d^{0\dagger}h_d^-)\\
&+ (\tilde{\nu}_e^\dagger\tilde{e}_L + \tilde{e}_L^\dagger\tilde{\nu}_e) + (\tilde{u}_L^\dagger\tilde{d}_L + \tilde{d}_L^\dagger\tilde{u}_L) + \cdots\Big]^2\\
&- \frac{g^2}{4}\Big[(h_u^{+\dagger}h_u^0 - h_u^{0\dagger}h_u^+) + (h_d^{-\dagger}h_d^0 - h_d^{0\dagger}h_d^-)\\
&+ (\tilde{\nu}_e^\dagger\tilde{e}_L - \tilde{e}_L^\dagger\tilde{\nu}_e) + (\tilde{u}_L^\dagger\tilde{d}_L - \tilde{d}_L^\dagger\tilde{u}_L) + \cdots\Big]^2\\
&+ \frac{g^2}{4}\Big[(h_u^{+\dagger}h_u^+ - h_u^{0\dagger}h_u^0) - (h_d^{-\dagger}h_d^- - h_d^{0\dagger}h_d^0)\\
&+ (\tilde{\nu}_e^\dagger\tilde{\nu}_e - \tilde{e}_L^\dagger\tilde{e}_L) + (\tilde{u}_L^\dagger\tilde{u}_L - \tilde{d}_L^\dagger\tilde{d}_L) + \cdots\Big]^2\\
&+ g'^2\Bigg[\frac{H_u^\dagger H_u - H_d^\dagger H_d - \tilde{L}_e^\dagger\tilde{L}_e + \frac{1}{3}\tilde{Q}_1^\dagger\tilde{Q}_1 + \cdots}{2}\\
&+ \tilde{e}_R^\dagger\tilde{e}_R - \frac{2}{3}\tilde{u}_R^\dagger\tilde{u}_R + \frac{1}{3}\tilde{d}_R^\dagger\tilde{d}_R + \cdots\Bigg]^2\Bigg\}\\
&- \frac{g_s^2}{8}\sum_A\left(\sum_i \tilde{q}_{Li}^\dagger\lambda_A\tilde{q}_{Li} - \sum_i \tilde{q}_{Ri}^\dagger\lambda_A\tilde{q}_{Ri}\right)^2.\end{aligned} \tag{8.123}$$

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The ellipses denote sfermion terms from the second and third generations. In the first term of the last square parenthesis, an $SU(2)$ matrix product is implied; i.e. $h_u^\dagger h_u \equiv h_u^{+\dagger} h_u^+ + h_u^{0\dagger} h_u^0$, etc. and $\tilde{L}_e$ and $\tilde{Q}_1$ denote the first generation slepton and squark doublets, respectively. The last term is just the squark D-terms from supersymmetric QCD discussed previously.

We have already seen some of these terms before. For instance, terms involving the squares of bilinears in just the Higgs fields lead to the quartic self-interactions in (8.113). We see that there are several other quartic self-interactions that originate in these D-terms:

1. The cross terms between the Higgs and scalar matter bilinears lead to four-point vertices involving a pair of Higgs bosons and a pair of scalars (squarks or sleptons). These Higgs boson couplings are fixed by gauge interactions and, hence, *are generation-independent.* In the case where both the Higgs bosons are neutral, a quick examination shows that there is no $hA\tilde{q}\tilde{q}$ or $HA\tilde{q}\tilde{q}$ coupling or, for that matter, the corresponding slepton couplings.
2. The squares of the sfermion bilinears lead to several new quartic interactions amongst squarks and sleptons. These include four squark interactions, four slepton interactions, and also two squark two slepton contact interactions. All these couplings are again fixed by gauge interactions. Notice that the sfermions participating in these interactions may be of the same or different type (L or R) and of the same or different flavor. Note also that although some of the four squark couplings, for instance, the four $\tilde{u}_\mathrm{R}$ couplings from the hypercharge D-term, superficially resemble that from the QCD interaction, the color structure of these interactions is quite different.

Trilinear superpotential terms also yield four scalar interactions determined by the Yukawa couplings. Clearly there are many such terms – even for just one generation there are $7 + 4 = 11$ terms corresponding to taking the derivative of the superpotential with respect to any of the seven chiral matter fields ($\hat{u}$, $\hat{d}$, $\hat{e}$, $\hat{\nu}$, $\hat{U}^c$, $\hat{D}^c$, and $\hat{E}^c$) or the four Higgs fields. We will leave it to the interested reader to enumerate all the terms which are straightforward to list, but only illustrate the form of the result with just one term arising from the derivative with respect to $\hat{h}_d^-$. This yields the interactions,

$$\mathcal{L} = -\left| f_d \tilde{u}_\mathrm{L} \tilde{d}_\mathrm{R}^\dagger + f_e \tilde{\nu} \tilde{e}_\mathrm{R}^\dagger + \cdots \right|^2, \tag{8.124}$$

where the ellipsis denotes contributions from the second and third generations. The following features of the four-point interactions from D-terms and F-terms might be worth noting.

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- The superpotential F-terms do not contribute to Higgs potential.
- Both D- and F-terms yield four scalar interactions as well as two sfermion two Higgs boson couplings. However, unlike the generation-independent D-terms, the superpotential couplings are important only for the third generation. In particular, four scalar couplings from the superpotential that involve sfermions of different generations are small.
- The bilinears that enter the D-terms always involve matter sfermions of the same type (L or R). In contrast, the corresponding F-term bilinears always couple L and R sfermions together. The form of the couplings in (8.123) and in (8.124) is, therefore, quite different.

It is now straightforward to write the interactions in (8.123) and (8.124) in terms of the mass eigenstate fields. However, since we will not have any occasion to use these couplings in the remainder of this book, we have chosen not to list the lengthy and cumbersome formulae that result upon doing so.

The quartic interactions of Higgs and sfermion fields also lead to $H\tilde{f}\tilde{f}$ couplings if one of the Higgs fields acquires a VEV. In addition, soft SUSY breaking scalar trilinear couplings (A-terms) are an additional source of these interactions. The process of obtaining the couplings of the physical Higgs fields to the left- and right-squark fields is lengthy but straightforward. We present here the results for a single generation of squarks. Of course, $\tilde{q}_{\rm L}$ and $\tilde{q}_{\rm R}$ need to be replaced by the corresponding mass eigenstates to obtain the coupling to physical particles.

The couplings of squarks to charged Higgs bosons are given by,

$$\begin{aligned}\mathcal{L}_{H^+\tilde{q}\tilde{q}} \ni g&\left[-\frac{M_W}{\sqrt{2}}\sin 2\beta + \frac{m_d^2\tan\beta + m_u^2\cot\beta}{\sqrt{2}M_W}\right]\left(\tilde{u}_{\rm L}^\dagger\tilde{d}_{\rm L}H^+ + \tilde{d}_{\rm L}^\dagger\tilde{u}_{\rm L}H^-\right)\\ &+\left[\frac{gm_u m_d(\cot\beta+\tan\beta)}{\sqrt{2}M_W}\right]\left(\tilde{u}_{\rm R}^\dagger\tilde{d}_{\rm R}H^+ + \tilde{d}_{\rm R}^\dagger\tilde{u}_{\rm R}H^-\right)\\ &+\left[\frac{-gm_d}{\sqrt{2}M_W}(A_d\tan\beta+\mu)\right]\left(\tilde{u}_{\rm L}^\dagger\tilde{d}_{\rm R}H^+ + \tilde{d}_{\rm R}^\dagger\tilde{u}_{\rm L}H^-\right)\\ &+\left[\frac{-gm_u}{\sqrt{2}M_W}(A_u\cot\beta+\mu)\right]\left(\tilde{u}_{\rm R}^\dagger\tilde{d}_{\rm L}H^+ + \tilde{d}_{\rm L}^\dagger\tilde{u}_{\rm R}H^-\right). \qquad (8.125a)\end{aligned}$$

Here, and in the following, we have eliminated the Yukawa couplings in favor of the corresponding *quark* mass.

The couplings to the lighter scalar h are,

$$\begin{aligned}\mathcal{L}_{h\tilde{q}\tilde{q}} \ni g&\left[M_W(T_{3\hat{u}_{\rm L}} - \frac{1}{2}Y_{\hat{u}_{\rm L}}\tan^2\theta_{\rm W})\sin(\beta-\alpha) - \frac{m_u^2\cos\alpha}{M_W\sin\beta}\right]\tilde{u}_{\rm L}^\dagger\tilde{u}_{\rm L}h\\ &+g\left[M_W(T_{3\hat{d}_{\rm L}} - \frac{1}{2}Y_{\hat{d}_{\rm L}}\tan^2\theta_{\rm W})\sin(\beta-\alpha) - \frac{m_d^2\sin\alpha}{M_W\cos\beta}\right]\tilde{d}_{\rm L}^\dagger\tilde{d}_{\rm L}h\end{aligned}$$

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$$+ g\left[M_W(T_{3\hat{U}^c} - \frac{1}{2}Y_{\hat{U}^c}\tan^2\theta_W)\sin(\beta-\alpha) - \frac{m_u^2\cos\alpha}{M_W\sin\beta}\right]\tilde{u}_R^\dagger\tilde{u}_R h$$
$$+ g\left[M_W(T_{3\hat{D}^c} - \frac{1}{2}Y_{\hat{D}^c}\tan^2\theta_W)\sin(\beta-\alpha) - \frac{m_d^2\sin\alpha}{M_W\cos\beta}\right]\tilde{d}_R^\dagger\tilde{d}_R h$$
$$+ \frac{gm_d}{2M_W\cos\beta}(-\mu\cos\alpha + A_d\sin\alpha)\left(\tilde{d}_L^\dagger\tilde{d}_R + \tilde{d}_R^\dagger\tilde{d}_L\right)h$$
$$+ \frac{gm_u}{2M_W\sin\beta}(-\mu\sin\alpha + A_u\cos\alpha)\left(\tilde{u}_L^\dagger\tilde{u}_R + \tilde{u}_R^\dagger\tilde{u}_L\right)h, \qquad (8.125b)$$

while the corresponding couplings to H are given by,

$$\mathcal{L}_{H\tilde{q}\tilde{q}} \ni g\left[-M_W(T_{3\hat{u}_L} - \frac{1}{2}Y_{\hat{u}_L}\tan^2\theta_W)\cos(\beta-\alpha) + \frac{m_u^2\sin\alpha}{M_W\sin\beta}\right]\tilde{u}_L^\dagger\tilde{u}_L H$$
$$+ g\left[-M_W(T_{3\hat{d}_L} - \frac{1}{2}Y_{\hat{d}_L}\tan^2\theta_W)\cos(\beta-\alpha) - \frac{m_d^2\cos\alpha}{M_W\cos\beta}\right]\tilde{d}_L^\dagger\tilde{d}_L H$$
$$+ g\left[-M_W(T_{3\hat{U}^c} - \frac{1}{2}Y_{\hat{U}^c}\tan^2\theta_W)\cos(\beta-\alpha) + \frac{m_u^2\sin\alpha}{M_W\sin\beta}\right]\tilde{u}_R^\dagger\tilde{u}_R H$$
$$+ g\left[-M_W(T_{3\hat{D}^c} - \frac{1}{2}Y_{\hat{D}^c}\tan^2\theta_W)\cos(\beta-\alpha) - \frac{m_d^2\cos\alpha}{M_W\cos\beta}\right]\tilde{d}_R^\dagger\tilde{d}_R H$$
$$+ \frac{gm_d}{2M_W\cos\beta}(\mu\sin\alpha + A_d\cos\alpha)\left(\tilde{d}_L^\dagger\tilde{d}_R + \tilde{d}_R^\dagger\tilde{d}_L\right)H$$
$$+ \frac{gm_u}{2M_W\sin\beta}(-\mu\cos\alpha - A_u\sin\alpha)\left(\tilde{u}_L^\dagger\tilde{u}_R + \tilde{u}_R^\dagger\tilde{u}_L\right)H. \qquad (8.125c)$$

Note that the isospin and hypercharge values that appear in (8.125b) and (8.125c) refer to the corresponding quantities for the MSSM fields in Table 8.1.

Finally, the couplings to the pseudoscalar neutral Higgs field are given by,

$$\mathcal{L}_{A\tilde{q}\tilde{q}} \ni \mathrm{i}\frac{gm_d}{2M_W}(\mu + A_d\tan\beta)\left(\tilde{d}_R^\dagger\tilde{d}_L - \tilde{d}_L^\dagger\tilde{d}_R\right)A$$
$$+\mathrm{i}\frac{gm_u}{2M_W}(\mu + A_u\cot\beta)\left(\tilde{u}_R^\dagger\tilde{u}_L - \tilde{u}_L^\dagger\tilde{u}_R\right)A. \qquad (8.125d)$$

As already noted, especially for the third generation squarks and sleptons, mixing effects must be included by substituting for the appropriate mass eigenstates.

The corresponding couplings to sleptons can be obtained by substituting $m_d \to m_e, m_u \to 0, A_d \to A_e, A_u \to 0, \tilde{u}_L \to \tilde{\nu}_L, \tilde{d}_L \to \tilde{\ell}_L, \tilde{u}_R \to 0$, and $\tilde{d}_R \to \tilde{\ell}_R$, and by making appropriate weak isospin and hypercharge assignments.

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8.5 Radiative corrections

Up to now, we have focussed our attention on the tree-level masses and couplings of MSSM particles. Since MSSM couplings are all assumed to be in the perturbative regime, this should be a good approximation to the true masses and couplings. There are, however, some situations where radiative corrections are very important. The best known of these is in the Higgs boson sector where the tree-level bound (8.39b), if applicable, would already exclude the model! Clearly, such a correction cannot be neglected. In this section we briefly discuss the radiative corrections that cannot be neglected in phenomenological analyses of SUSY. This discussion is not meant to be complete, but is included as a caution, and to provide the reader a flavor of the issues involved. For a comprehensive discussion, we refer the reader to the original literature.

8.5.1 Higgs boson masses

We have already mentioned that radiative corrections to Higgs boson masses can be large, and are especially important for the lightest Higgs scalar h. The biggest corrections arise from the top (quark and squark) Yukawa coupling to Higgs field H_u. For large values of $\tan\beta$ b-Yukawa, and to a lesser degree τ-Yukawa, contributions are also significant. Smaller corrections also arise from gauge interactions of the Higgs bosons.

These radiative corrections can be included diagrammatically, by calculating the relevant Higgs boson self-energy graphs, and by identifying the location of the pole in the propagator. An alternative procedure involves analyzing the one-loop corrected effective potential. The form of the one-loop correction to the scalar potential can be written as

$$\Delta V = \sum_i \frac{(-1)^{2s_i}}{64\pi^2} Tr\left((\mathcal{M}_i\mathcal{M}_i^\dagger)^2\left[\log\frac{\mathcal{M}_i\mathcal{M}_i^\dagger}{Q^2} - \frac{3}{2}\right]\right), \qquad (8.126)$$

where the sum over i runs over all fields that couple to Higgs fields, $\mathcal{M}_i^2$ is the *Higgs field dependent* mass squared matrix (defined as the second derivative of the tree-level Lagrangian) of each of these fields, and the trace is over the internal as well as any spin indices. The function ΔV depends on the Higgs fields through $\mathcal{M}$, and must be added to the tree-level potential. It is this corrected effective potential that must be used to obtain the vacuum state as well as the masses and mixings of the physical particles in the Higgs sector. Here, we illustrate how to obtain the dominant corrections arising from top Yukawa couplings. To keep things simple, we also ignore intra-generational mixing.

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Exercise *Show that the neutral Higgs field dependent mass matrix for stops in the* $(\tilde{t}_{\rm L}, \tilde{t}_{\rm R})$ *basis is given by,*

$$\begin{pmatrix} m^2_{\tilde{t}_{\rm L}} + f_t^2|h_u^0|^2 & 0 \\ 0 & m^2_{\tilde{t}_{\rm R}} + f_t^2|h_u^0|^2 \end{pmatrix},$$

while the corresponding top quark mass is given by $f_t h_u^{0*}$ *where, for simplicity, we have ignored any* $\tilde{t}_{\rm L}$–$\tilde{t}_{\rm R}$ *mixing. Use these to show that the one-loop correction to the effective Higgs potential due to top Yukawa couplings is given by,*

$$\begin{aligned}\Delta V \simeq \frac{3}{32\pi^2}\Big[&(m^2_{\tilde{t}_{\rm L}} + f_t^2|h_u^0|^2)^2 \log(m^2_{\tilde{t}_{\rm L}} + f_t^2|h_u^0|^2) \\ &+ (m^2_{\tilde{t}_{\rm R}} + f_t^2|h_u^0|^2)^2 \log(m^2_{\tilde{t}_{\rm R}} + f_t^2|h_u^0|^2) \\ &- 2f_t^4|h_u^0|^4 \log(f_t^2|h_u^0|^2) - \frac{3}{2}\Big].\end{aligned} \tag{8.127}$$

To obtain this, we have to remember that in Eq. (8.126) the contribution from scalar loops is written for real scalar fields. Since $\tilde{t}_{\rm L}$ *and* $\tilde{t}_{\rm R}$ *are complex, their contribution needs to be doubled. The factor 3 is a color factor.*

Finally, we remark that to obtain the effective potential for the charged as well as neutral Higgs fields, we must allow both top and bottom quarks and squarks in the loops. Technically, this means that we have to construct a 4×4 *field-dependent mass matrix for the squarks, and* 2×2 *mass matrix for the fermions. Even for our simplified calculation, these matrices are no longer diagonal. The trace can be evaluated by evaluating the (field-dependent) eigenvalues of these squared mass matrices, inserting these in place of* $\mathcal{M}_i$ *in (8.126) and summing. Carry out these steps, and show that you obtain an* $SU(2) \times U(1)$ *invariant effective potential.*

We can use this effective potential to construct corrected Higgs boson mass matrices in the same way as before. We will now have additional contributions from the top quark Yukawa coupling f_t, and involving the top quark and top squark masses. The result for the scalar Higgs bosons is simple to write down in this approximation:

$$m^2_{h,H} = \frac{1}{2}\left[(m_A^2 + M_Z^2 + \delta) \mp \xi^{1/2}\right] \tag{8.128}$$

where

$$\xi = \left[(m_A^2 - M_Z^2)\cos 2\beta + \delta\right]^2 + \sin^2 2\beta(m_A^2 + M_Z^2)^2,$$

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and

$$\delta = \frac{3g^2 m_t^4}{16\pi^2 M_W^2 \sin^2\beta} \log\left[(1+\frac{m_{\tilde{t}_L}^2}{m_t^2})(1+\frac{m_{\tilde{t}_R}^2}{m_t^2})\right]. \quad (8.129)$$

In addition, the relation

$$m_{H^\pm}^2 = m_A^2 + M_W^2$$

is unaltered as long as bottom quark Yukawa couplings are neglected. Finally, the Higgs scalar mixing angle α is modified to

$$\tan\alpha = \frac{(m_A^2 - M_Z^2)\cos 2\beta + \delta + \xi^{1/2}}{\sin 2\beta(m_A^2 + M_Z^2)}. \quad (8.130)$$

In these formulae, we have eliminated f_t using (8.24a). Notice that the presence of δ in the expression for m_h allows h to exceed M_Z as seems to be required by the LEP data discussed earlier.

Although we have illustrated these corrections keeping only top quark Yukawa couplings and neglecting intra-generation stop mixing, many phenomenological analyses include mixing effects as well as the corrections due to b and τ Yukawa couplings (these are important if $\tan\beta$ is large), and also gauge couplings. Much effort has gone into making as precise predictions as possible for Higgs boson masses, especially m_h. At the present time, state-of-the-art calculations including dominant two-loop effects indicate that the value of m_h can be as high as about 130 GeV, well beyond the reach of the LEP2 e^+e^- collider at CERN, which ran at a maximum energy of $\sim$ 208 GeV, and even larger if $m_t > 175$ GeV.

Gluino mass

It has been noted by Martin and Vaughn that the tree-level gluino mass suffers large corrections – up to 25% – due to loop corrections.[7] In this case, one must compute the gluino self-energy diagrams, and look for the pole position in the gluino propagator. Including loop graphs with gluon exchange and quark–squark loops, they find

$$m_{\tilde{g}} = M_3(Q)\left(1 + \frac{\alpha_s}{4\pi}[15 + 6\log(Q/M_3) + \sum A_{\tilde{q}}]\right) \quad (8.131)$$

in the $\overline{DR}$ regularization scheme.[8] Here,

$$A_{\tilde{q}} = \int_0^1 \mathrm{d}x\; x \log[x m_{\tilde{q}}^2/M_3^2 + (1-x)m_q^2/M_3^2 - x(1-x)], \quad (8.132)$$

[7] S. Martin and M. Vaughn, *Phys. Lett.* **B318**, 331 (1993).
[8] The calculation is performed in the $\overline{DR}$ renormalization scheme: see Chapter 9.

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where the sum is over the 12 different quark–squark multiplets, and squark mixing has been neglected.

8.5.2 Squark mass

The dominant corrections to squark masses come from strong interactions, and so are the same for $\tilde{q}_L$ and $\tilde{q}_R$, and also independent of flavor. The radiatively corrected squark mass is given by,

$$\begin{aligned}\delta m_{\tilde{q}}^2 &= m_{\tilde{q}}^2 - m_{\tilde{q}}^2(Q)\\ &= \frac{2\alpha_s(Q)}{3\pi} m_{\tilde{q}}(Q)^2 \left\{1 + 3x + (x-1)^2 \ln|x-1| - x^2 \ln x + 2x \ln \frac{Q^2}{m_{\tilde{q}}^2}\right\}.\end{aligned} \tag{8.133}$$

If intra-generation squark mixing is not negligible, the form of the corrections is more complicated, and we refer the reader to Pierce *et al.* for the complete result.[9]

8.5.3 Chargino and neutralino masses

By and large the corrections to these masses are not very large, but there are regions of parameter space where they can be several percent. Nevertheless, there are important circumstances where inclusion of these corrections could be important. We will see later that the phenomenology is to a great extent determined by what the lightest supersymmetric particle (LSP) is. This is largely because (as long as R-parity is conserved) all sparticle decays terminate in the LSP. In many models, the LSP is the lightest neutralino or the lighter stau, depending on the values of model parameters. In the case where these sparticles are approximately degenerate at tree level, the radiative corrections might prove to be crucial in identifying the LSP.[10]

Another scenario where radiative corrections are crucial occurs when $|M_2|$ is much smaller than other soft SUSY breaking parameters so that the $SU(2)$ gauginos are the lightest of the sparticles. It is then the radiative corrections that break the degeneracy between the charged and neutral partners, making the latter slightly lighter. A realization of such a scenario occurs in the so-called anomaly-mediated SUSY breaking model discussed in Chapter 11.

[9] See, D. Pierce *et al.*, *Nucl. Phys.* **B491**, 3 (1997).

[10] We have oversimplified the discussion here. In gauge-mediated SUSY breaking models that we will discuss in Chapter 11 the LSP is a gravitino: since couplings of sparticles to the gravitino are extremely weak, all other sparticles cascade decay to the next lightest super particle (NLSP) which then decays to the gravitino. It is very important to correctly identify the NLSP to obtain the correct phenomenology.

These radiative corrections have been analyzed in the literature. Complete formulae can be found in Pierce *et al.*, where details are provided.

8.5.4 Yukawa couplings and SM fermion masses

At tree level, the Yukawa couplings that enter the superpotential are simply related to the corresponding SM fermion masses via (8.24a) and (8.24b). This is because $\hat{h}_u^0$ ($\hat{h}_d^0$) couple only to up-(down-)type fermions. At one-loop level, the field h_u^0 can also couple to down-type quarks via its couplings to up- and down-type squarks.

Exercise *Draw a Feynman diagram involving a gluino and down-type squark, or a chargino and an up-type squark, in a loop to show that h_u^0 can couple to the down quarks at the one-loop level.*

Thus, beyond tree level, down-type quarks can obtain contributions to their masses proportional to v_u. Although these contributions are loop-suppressed, they can be comparable to the tree-level contribution (proportional to v_d) if $\tan\beta \gg 1$. Clearly, then the relation between Yukawa couplings and the corresponding quark mass is considerably modified. We refer the interested reader to the paper by Pierce *et al.* for details regarding these corrections.

8.6 Should the goldstino be part of the MSSM?

The MSSM is the simplest viable supersymmetric extension of the SM. Within this framework, our ignorance of the underlying mechanism of SUSY breaking is reflected in the 178 parameters discussed in Section 8.1.2. We should regard the MSSM as an effective theory that will someday be obtained from a more fundamental theory, once we understand the principles behind the physics of SUSY breaking. Presumably, this will result in a dramatic reduction in the set of parameters that one will regard as fundamental, in the sense that most soft SUSY breaking parameters will be derived from more basic considerations.

Indeed despite the many suggestions for how SUSY breaking effects are felt by the superpartners of SM particles, no compelling theory has as yet emerged. There are two common themes to all models of SUSY breaking.

- First, it appears that the SUSY breaking occurs in a sector of the theory that differs from that containing the SM particles and their superpartners. We are forced into considering such theories because models where SUSY breaking occurs in the SM sector run into phenomenological troubles with the sum rules such as (7.35) that led to light scalars as discussed in Chapter 7. This then raises an

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additional question: even if we can dynamically break SUSY, how do we convey this information to the observable sector of SM particles and their superpartners? The answer to this question will be taken up in Chapter 11 where we discuss various models.

- Second, supersymmetry is broken spontaneously rather than explicitly. Clearly, this is the more appealing route, and also affords a rationale for why SUSY breaking is soft in the MSSM: since SUSY breaking operators are proportional to a VEV, dimensional analysis tells us that dimension four SUSY breaking interactions are forbidden at least in a renormalizable theory.

The attentive reader will, however, be disturbed by the fact that spontaneous breaking of SUSY should be accompanied by a massless Goldstone fermion in the low energy spectrum. This should then be the LSP. Yet, our discussion of the MSSM has made no mention of this. Indeed the MSSM (as we have formulated it with explicit SUSY breaking terms) does not contain a goldstino.

The naive reason that we can get away with doing so is that in most models we promote SUSY to a local supersymmetry. This, as we will discuss in Chapter 11, results in a theory that necessarily incorporates gravity, and requires the introduction of the (spin 2) graviton and its superpartner, a spin 3/2 fermion, the gravitino. Then, when SUSY is spontaneously broken, the would-be Goldstone fermion combines with the (originally massless) gravitino to form a massive gravitino and disappears from the physical spectrum, while the graviton (which is protected from acquiring a mass by the unbroken reparametrization invariance) remains massless. This phenomenon is analogous to what happens in spontaneously broken local gauge theories: the would-be Goldstone bosons combine to form the longitudinal components of a massive gauge field, and no massless spin zero excitations remain in the spectrum.

In principle, if the gravitino is light enough we ought to include it as part of the low energy theory. It is, generally speaking, not usual to do so for the same reason that we do not include the graviton: like the graviton, the gravitino typically couples too weakly to matter for particle physics.[11] Thus the MSSM is a parametrization of the effective low energy theory, but with some prejudices thrown in.

[11] We will discuss an exception to this in Chapter 11 when we discuss gauge-mediated SUSY breaking. If the SUSY breaking scale is low enough, we will see that the couplings of the longitudinal components of the gravitino (i.e. the goldstino) play an important role for collider signals.

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