ON THE THEORY OF ROTATING MAGNETIC STARS

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All observations of magnetic stars necessarily yield information only about their surface features. We are ignorant of the nature of the fields in the interiors of such stars, and equally we cannot be sure of the non-existence of interior fields in stars which are superficially non-magnetic. In fact, if we assume the truth of the 'fossil' theory – that the magnetic flux of an Ap star is a relic of the flux initially present in the gas cloud from which the star condensed – then it is surprising that magnetic stars are not observed to be much more common, since magnetic fields appear to be ubiquitous in interstellar gas clouds. For those stars with strong surface convection zones, we might expect that a fossil field of low energy would be expelled by the turbulence and would possibly be trapped in the interior. However, the majority of early-type stars with radiative envelopes also do not exhibit any observable magnetic field.

The work described in this paper was motivated by the possibility that the appearance or non-appearance of magnetic fields might depend on whether there are mass-motions within the star, which would tend to drag the field-lines beneath the surface.

Such motions are likely to be much more effective in rapidly rotating early-type stars (Mestel, 1965, 1967). If this idea is correct, then the non-appearance of magnetic fields in most rapidly-rotating stars is not just an aspect effect, due for example to Doppler broadening. There would then be a two-way interaction between a primeval magnetic field and stellar rotation. A strongly magnetic star which spends long enough in a pre-main sequence convective phase could suffer sufficient braking by a magnetically-controlled stellar wind for its rotation to be abnormally low, even after subsequent contraction to the main sequence. On the other hand a weakly magnetic star, one which has only a short-lived stellar wind phase, could reach the main sequence with a rotation rapid enough to cause the surface field to disappear well within the star's lifetime.

In the radiative envelope of a uniformly rotating star, the divergence of the radiative flux **F** is not zero, but is a prescribed function of position (Von Zeipel, 1924). The consequent buoyancy forces drive a circulation that is slow over the bulk of the star, but can become considerably faster in low-density surface regions (Eddington, 1929; Sweet, 1950; Opik, 1951; Baker and Kippenhahn, 1959; Mestel 1966). This circulation tends to convect both angular momentum and magnetic flux; however, it is possible to construct approximate self-consistent models in which the magnetic field is quite strong enough to keep the rotation uniform in spite of the circulation, but is too weak to affect sensibly the thermal-gravitational field over the bulk of the star (Roxburgh, 1963; Mestel, 1965). In particular, we expect such an inexorable rotationally-driven circulation steadily to distort a weak primeval field and to reduce the net flux emanating from the surface.

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In this paper we invert the problem and look for magnetic fields that not only keep the star rotating uniformly, but are strong enough to suppress the Eddington-Sweet circulation. In a linear perturbation theory centrifugal and magnetic forces cause independent disturbances to the pressure-density-temperature field, and so also to the flux of energy. We demand that (in an obvious notation)

$$(\nabla \cdot \mathbf{F})_\Omega + (\nabla \cdot \mathbf{F})_H = 0$$

over the whole star. This imposes a constraint on the magnetic field structure through the star. In these models there are no motions, so that field-lines emanating from the surface do not suffer a progressive distortion.

The models described in this paper are all assumed to possess axisymmetric magnetic fields which limits their observational applications, since most magnetic stars appear to have large angles of obliquity. The assumption of uniform rotation is more than is strictly required by Ferraro's Law of Isorotation, which could permit relative shearing of individual field-lines. Any departure from the axisymmetric assumption would however tend to maintain uniform rotation, so we select this case in order to best approximate the non-axisymmetric situation. It was further assumed that the magnetic field permeated the convective core of the star, i.e. that the turbulence was not strong enough to expel it. The calculations were in fact repeated with the condition that the field should not enter the core and, although the field-structures were different, the integrated results, which form the main conclusions of this paper, were not affected.

We write the radial and transverse components of the field in terms of a stream function ψ:

$$H_r = -\frac{1}{r^2}\frac{\partial\psi}{\partial\mu} \quad \text{and} \quad H_\theta = -\frac{1}{r\sin\theta}\frac{\partial\psi}{\partial r},$$

where $\mu = \cos\theta$. It is found that if ψ is taken to have the basically dipolar form

$$\psi = (1 - \mu^2)\, B(r)$$

then the angular dependence in the basic equations of stellar structure can be eliminated, simplifying the problem considerably. This simplification is made solely on mathematical grounds and we have to assume that it will yield results qualitatively similar to those which would be given by a full analysis of the evolution and decay of the field. The method of formulating the problem by means of separating the radial and $P_2(\mu)$ components and comparing the coefficients of P_2 has been described by Roxburgh (in Lüst, 1965) and Monaghan (1966) in the case of zero rotation and only a slight extension is required to include the effects of uniform rotation (Wright, 1969). There results a set of 5 non-linear ordinary differential equations which have now been solved both by ourselves and independently by Davies (1968).

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In order to describe these results we define two integrals over the equatorial plane:

$$F_t = \left| \int_0^{r_0} 2\pi r H_\theta \, \mathrm{d}r \right|$$

(where r_0 is the radius of the neutral point) is the total flux of the field, and

$$F_s = \left| \int_0^{R} 2\pi r H_\theta \, \mathrm{d}r \right|$$

is the flux which emerges from the surface. In all models it was found that the field-lines were concentrated towards the centre of the star and that if F_t was regarded as fixed and the angular velocity, Ω, gradually increased, then F_s tended to zero for a *finite* value of Ω.

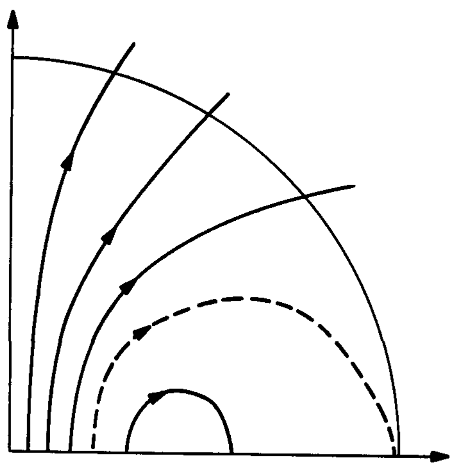

Fig. 1. Field-lines for zero angular velocity.

The *structure* of the models is fixed by the value of $\Omega/\bar{H}$, where $\bar{H}$ is the surface polar field. For $\Omega=0$, the structure is as in Figure 1; and Figure 2 represents the structure when Ω and $\bar{H}$ have values corresponding to a period of $4\frac{1}{2}$ days with a surface field of 10^3 gauss. In the first case the central field strength is about 40 times the surface field, and in the second case about 1600 times. The flux inside the dotted field-lines never emerges above the surface of the star. Figure 3 shows F_t plotted against Ω for differing constant values of $\bar{H}$. For a given value of Ω there corresponds a unique minimum total flux, which has $\bar{H}=0$.

These models have all been computed under the idealisation of infinite conductivity. In fact, the value of F_t for any particular star must monotonically decrease as field-lines slowly contract into the neutral point. If radiative equilibrium is to be maintained,

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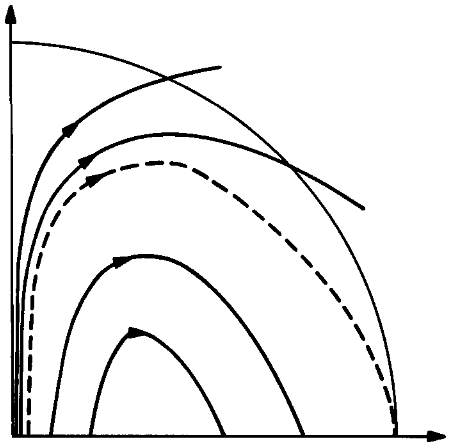

Fig. 2. Field-lines for $\bar{H} = 10^3$ gauss and a period of $4\frac{1}{2}$ days.

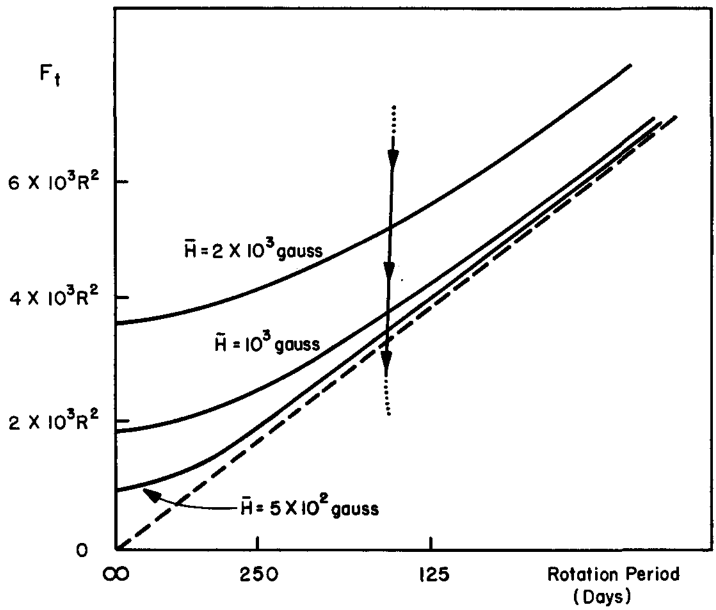

Fig. 3. Total field flux plotted against rotation period for differing constant values of the surface polar field.

with a basically P_1-field and with no change in Ω, then we must assume that the small circulation currents which arise as F_t decays will restore the field-lines to the new equilibrium configuration. As this evolution proceeds, the value of the surface field is gradually reduced to zero, an effect represented by the downward vertical line in Figure 3. Once the flux is reduced below this critical value, then either radiative equilibrium is maintained by fields of a more complex structure than the simple P_1-form assumed so far, or slow circulation currents are started which will steadily distort the field. In either case it is unlikely that strong surface fields will re-appear above the surface.

When compared with observation, assuming that these computed structures are to some degree a good approximation to the real state of affairs inside magnetic stars, most stars appear to have values of $\bar{H}$ and Ω which would indicate that their values of F_t are no more that 10% above the critical value for vanishing surface fields, and that their ratios of magnetic to rotational energies are of the order of 0.07. A decay of about 10% would lead ultimately to the disappearance of all the surface flux. Further, these values of F_t are much below the maximum permitted by the virial theorem. If the stellar flux is primeval, this raises the question as to why such a small fraction remained. One possibility is that during star formation there was substantial motion across as well as along the field (Mestel and Spitzer, 1956). The difficulty is to find a reason why this should yield an upper limit, as required by comparison of our models with observation. Another possibility is that a star with too much flux would be thermally unstable, leading to motions of flux-tubes and accelerated Ohmic decay. Fricke (1969) has shown that uniformly rotating stars with poloidal magnetic fields are secularly stable to axisymmetric perturbations, but this work has yet to be extended to non-axisymmetric perturbations and the inclusion of finite conductivity. If these hypothetical instabilities slow up as flux is lost, it may be possible to explain the upper limit (however it may turn out that no magnetic star is sufficiently stable unless there is a negative gradient of mean molecular weight in at least part of the star).

This concludes the analysis and discussion of the results so far computed. Work is now proceeding on the investigation of the secular stability of these models, and on the production of plausible 'quasi-steady' solutions representing stars with radiative equilibrium in the surface regions but with a slow circulation within. In addition we are analysing the models of this paper to see how the perturbations to the luminosity and surface temperature, caused by the mixture of rotation and magnetism, affect their position on the H-R diagram.

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