BULL. AUSTRAL. MATH. SOC. 58F13, 58F14, 70K50
VOL. 42 (1990) [527–528]

Order and chaos in reversible dynamical systems

JOHN ANTHONY GERARD ROBERTS

A dynamical system is called reversible if there is an involution in phase space which reverses the direction of time. This thesis is a study of the dynamics of reversible systems without the usual constraint that the system is also conservative. This study takes the form of studying reversible mappings of the plane which are not necessarily measure preserving. Reversible mappings are mappings that can be written as the composition of two involutions. The application of either of these (reversing) involutions to an orbit of the mapping gives another orbit of the mapping when followed in the opposite sense, which is the essence of reversibility. We show that non measure preserving reversible mappings of the plane are in general hybrid dynamical systems that combine the features of conservative systems (for example, KAM tori) and dissipative systems (for example, attractors).

It has long been recognised that the study of mappings of the plane is one of the simplest, yet most useful, ways of becoming acquainted with the behaviour of large classes of dynamical systems. This is pointed out in the introductory chapter, Chapter 1. In Chapter 2 we review the properties of area preserving mappings as well as dissipative mappings. These are the respective analogues of Hamiltonian dynamical systems and dissipative dynamical systems. We also discuss the properties of measure preserving mappings which are a generalisation of area preserving mappings.

In Chapter 3 we introduce the concept of reversibility for a mapping of the plane and discuss its consequences for the mapping's dynamics. We show how non area preserving reversible mappings and non area preserving involutions can be derived from certain second order difference equations. We then look at the problem of how to decide whether a given mapping is reversible.

In Chapter 4 we discuss a special case of reversible mappings of the plane, integrable reversible mappings. These are the most ordered of reversible mappings in the sense that all of their orbits lie on curves in the plane. Therefore these reversible mappings do not have chaotic orbits or attractors and repellors. We present a large class of integrable

Received 19th July, 1990.
Thesis submitted to University of Melbourne, January 1990. Degree approved May 1990. Supervisors: Professor C.J. Thompson and Dr. G.R.W. Quispel.

reversible mappings discovered by us and study their features. We prove that although these reversible mappings are not area preserving they are still basically conservative in nature because they are measure preserving. We conclude with an example of an integrable reversible mapping arising from the Heisenberg Spin Chain.

Large classes of non integrable and non measure preserving reversible mappings with attractors are presented in Chapter 5. We give systematic methods that we have found for creating such mappings. These mappings are particularly useful for introducing dissipative behaviour into area preserving mappings, such as the well known Hénon mapping, whilst still retaining the KAM regions associated with such area preserving mappings. Moreover, reversibility implies that if there is an attractor in our mappings then there is also a repellor.

In Chapter 6 we concentrate on the conservative features of these reversible mappings via a study of the breaking up of their KAM curves and of the period doubling of their symmetric periodic orbits — that is, those periodic orbits left invariant by the involutions of the mapping. We find that the critical exponents associated with these two processes are the same as in area preserving mappings.

In Chapter 7 we concentrate on the dissipative features, that is, the attractors, in our reversible mappings. These features are necessarily asymmetric. We find that the critical exponents associated with the period doubling of asymmetric periodic orbits are the same as those found in dissipative systems. Furthermore, this period doubling can lead, upon accumulation, to the appearance of strange attractors in reversible mappings.

In Chapter 8 we discuss the interaction of the conservative and dissipative features of our mappings. This highlights their truly hybrid nature as dynamical systems and provides some interesting possible directions for future research.

Institute for Theoretical Physics
University of Amsterdam
Valckenierstraat 65
1018 XE Amsterdam
The Netherlands