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FISCAL POLICY CHANGES AND LABOR MARKET DYNAMICS IN JAPAN'S LOST DECADE

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Japan's so-called Lost Decade of the 1990s presents a unique case study of an economy with a recent severe and prolonged recession, with large changes in the labor market and fiscal policy as the main policy available to the government. Japanese unemployment rate surged from 2.1% in 1991 to 5.4% in 2002. Meanwhile, the Japanese economy experienced a rise in government expenditures, while taxes remained fairly stable. This paper quantitatively evaluates the impact of these changes in fiscal policies on labor market variables, in particular the unemployment rate, during the 1990s. We build, calibrate, and simulate a dynamic general equilibrium model with search frictions in the labor market, a productive government sector, heterogenous government spendings, and different categories of taxes. Our model is able to reproduce the paths of the main labor market variables, and the counterfactual experiments show that the changes that took place in the different spending components affected the unemployment rate heterogeneously, although overall they kept unemployment lower than it could have been. We also find that had the government also implemented countercyclical tax policies, unemployment would not have risen as much as it did by 2002.

Keywords: Fiscal Policy, Search and Matching, Endogenous Job Destruction, Heterogenous Government Expenditure, Productive Government Sector, Japan's Lost Decade

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1. INTRODUCTION

Severe and prolonged recessions can have devastating effects on the economies of the world and their labor markets. During the Great Depression, unemployment in the USA was estimated to rise from 3.20% in 1929 to almost 25% in 1933. During the recent Great Recession, unemployment in the European Union almost doubled from 6.80% in 2008 to 11.00% in 2013. Since these recessions had severe and direct impacts on their labor markets, policy measures such as fiscal policy expansions, tax cuts, and monetary easing to mitigate their effects were called for. During the Great Recession, the US government increased its government spending by 14.2% from 2007Q4 to 2009Q4, the largest 2-year increase after the early 1950s [Oh and Reis (2012)]. Similarly, the UK increased its spending by 17.3% during the same period, while the European Union increased their spending by 11.8%.[1]

Although fiscal and monetary policies are typical candidates to mitigate negative impacts on labor markets, some of these measures may not always be viable. In particular, traditional monetary policy is infeasible when the economy is in a liquidity trap. In this case, fiscal policy is the main policy left to try to get the economy out of recession. More importantly, during severe recessions, governments have only limited resources to stimulate the economy. Therefore, it is imperative to allocate the additional government resources effectively to make the policy successful. Gomes (2015) argues that it is important to take into account the heterogenous components of government spending, such as consumption, wage, and investment by government, to study the effectiveness of a fiscal policy expansion because these components have different effects on the economy as a whole and the labor market in particular. Hence, studying the heterogenous effects of a fiscal expansion is necessary to understand the effective allocation of fiscal policy as a labor market policy.

The main purpose of our paper is to study the effectiveness of the fiscal policy expansions taking into account the heterogenous effects of the changes in the different components of government spending on unemployment and the other major variables in the labor market. For this goal, we study the experience of Japan during the 1990s, the so-called Lost Decade. During the decade of the 1990s, the Japanese average growth rate of output per capita was 0.50%, much lower than the average of the 1980s, 3.20%, and that of the USA during the same era, 2.60%, resulting in an almost three-fold increase in the unemployment rate from 2.13% in 1991 to 5.40% in 2002. During this period, the Japanese government found itself with limited policy resources when the interest rate came to a historical low of 0.50% in the mid-1990s and eventually hit the zero lower bound at the end of the decade, bringing the economy into a liquidity trap and rendering traditional monetary policy ineffective. Fiscal policy, therefore, was the main policy left to the Japanese government to try to get the economy out of this long and grave recession.

During the Lost Decade, the Japanese government increased the share of aggregate expenditures in gross domestic product (GDP) with heterogenous changes in different spending components: From 1991 to 2002, the share of government consumption in GDP increased from 7.88% to 11.10%, the share of government wage expenditure[2] rose slightly from 6.13% to 6.65%, and the share of government investment increased at the beginning of the decade, but dropped in the second half of the 1990s, which meant an overall small reduction from 6.35% to 6.32%. This rare scenario of prolonged low output growth, steady unemployment increase, and heterogenous expansion of fiscal policy as the sole traditional government policy available is ideal to study the effectiveness of fiscal policy.

To accomplish the goals, we build, calibrate, and simulate a dynamic general equilibrium model with search and matching frictions in the labor market. We extend the standard discrete-time neo-classical growth model to include production in both the private and the government sectors. Modeling a productive government sector allows our framework to explicitly study the role played by the three components of government spending: (i) wage expenditure changes affect the hiring in the government sector; (ii) changes in investment spending alter the accumulation of public capital and the productivity of both sectors; and (iii) variations in government consumption affect aggregate demand and production of the whole economy. In addition, our model includes three types of taxes, labor, capital, and consumption taxes, which the government uses to finance its expenditures. The model is calibrated to match the state of the Japanese labor market at the start of the Lost Decade, 1991. Using the solution method of a two-boundary problem, we solve and simulate the transition path of the economy from an initial steady state, assumed to be in 1991, to a new steady state far away in the future. The economy transitions from one steady state to the other led by the changes in TFP, government spending components, and maximum workable hours.

Our model is able to reproduce the changes in the main labor market variables, including the increase in unemployment and the changes in job finding and separation rates from 1991 to 2002. We then use our framework to perform counterfactual experiments where we fix the different components of government spending at the level of 1991 and study how the labor market would have evolved if such change in spending had not taken place over the Lost Decade. We find that the increase in government consumption was the one that had the biggest impact in controlling the rise in unemployment. Government wage expenditure did not have a major effect on the labor market, mainly because of the small size of its actual increase, whereas government investment spending could have potentially reduced unemployment had it not been reduced in the second half of the 1990s. We also find that had the largest increase in expenditures been devoted to government investment, rather than consumption, the unemployment rate would have increased less than it did. We perform another set of counterfactuals where we reduce each of the three tax rates in the model by 10% to understand the effect that such countercyclical policy could have had if it had been implemented. We find that such a hypothetical 10% drop in the tax rates would have led to a lower unemployment rate in 2002, with the biggest impact produced by the drop in labor income tax, followed by the consumption tax and finally by the capital tax. These results indicate the quantitative importance of not only the size but also the components of fiscal policy intervention for labor market dynamics.

Our paper is related to the recent stream of literature that quantitatively evaluates the impacts of fiscal policy on the aggregate economy and labor market [Rossi (2014), Mitra et al. (2019), Drygalla et al. (2020), and Yum (2020)]. For the USA, Monacelli et al. (2010), Bruckner and Pappa (2012), and Kuo and Miyamoto (2014) develop dynamic stochastic general equilibrium models with search and matching frictions and find that expansionary fiscal policy boosts output and reduces unemployment during recessions. Employing a similar framework, Kato and Miyamoto (2013) and Kato and Miyamoto (2015) evaluate the dynamic effects of changes in government spending on the Japanese labor market without a productive government sector. They find that expansionary government spending reduces unemployment, increases the job finding rate, and decreases the job separation probability. There is a part of this literature that has directly studied the effects of the different components of government spending on the labor market by employing a two-sector framework, such as Gomes (2015), Burgert and Gomes (2011), Quadrini and Trigari (2007), Afonso and Gomes (2014), Bermperoglou et al. (2017), and Meng (2015). These papers are closely related to our study, since they also introduce a productive public sector within a search and matching framework and study the impact of changes in the different types of government spending. These papers find that the presence of a public sector increases the volatility of unemployment [Quadrini and Trigari (2007)], that government wage shocks have heterogenous effects on private sector wages and employment [Afonso and Gomes (2014) and Bermperoglou et al. (2017)], and that the various components of government expenditures have different impacts in the labor market [Burgert and Gomes (2011), Gomes (2015) and Meng (2015)].

A common feature of this stream of literature is the use of a stochastic environment and a focus on the general business cycle fluctuations. Our paper, however, focuses on a specific period of time with a deep and prolonged recession, when fiscal policy was used intensively. In our analysis, we perform counterfactuals to compare the actual path of the economy with the one it could have taken if fiscal policy had been different. Therefore, we extend this stream of literature by providing a very direct comparison between the labor market as it was and as it would have been, had these policies been, or not been, implemented, which constitutes our first contribution to the literature.

Our study is also related to another stream of literature exploring the causes, consequences, and policies related to the poor performance of the Japanese economy in the 1990s, such as Hayashi and Prescott (2002), Peek and Rosengren (2005), Caballero et al. (2008), and Esteban-Pretel et al. (2010). The findings of this literature point at technological growth decline and credit constraints among the main causes of the Lost Decade. Various papers study the limited effect of monetary policy in Japan during these years (e.g. Dominguez et al. (1998), Eggertsson and Woodford (2003), and Flotho (2015)), but not much emphasis has been placed on fiscal policies. We complement this strand of literature by providing a quantitative analysis of the effects of different government expenditure and tax policies on the labor market during the 1990s, which constitutes our second contribution to the existing studies.

Of the previous literature, our paper is related the closest to Esteban-Pretel et al. (2010). We build on the framework and analysis of Esteban-Pretel et al. (2010), but we differ from it in several aspects: First, the focus of Esteban-Pretel et al. (2010) is to explain the cause of the changes in the labor market in Japan in the 1990s, whereas our goal is to go further and not only reproduce what took place over this decade but also examine the effects of fiscal policy changes on unemployment and other labor market variables. Second, our paper provides a more elaborate model with two productive sectors and disaggregated government expenditures that allow for a more in-depth analysis of the consequences of fiscal policy changes on the different parts of the labor market, which are our final two contributions to the literature.

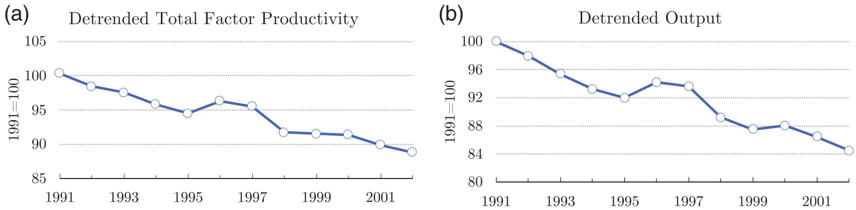
The remainder of this paper is organized as follows. Section 2 documents the stylized facts concerning the Japanese labor market and fiscal policy changes during the 1990s. In section 3, we develop a discrete time neo-classical growth model with search frictions and a productive government sector with rich specifications of fiscal policies. The model is calibrated in Section 4. Section 5 reports the benchmark simulation results. In Section 6, we examine the impact of changes in fiscal policy on unemployment through conducting counterfactual experiments. Section 7 concludes.

2. STYLIZED FACTS IN JAPAN'S LOST DECADE

The Lost Decade was the worst economic time in Japan's history since World War II, and as stated in the Introduction, it featured a unique blend of low growth, unemployment increase, and constraint government policy. To better understand the changes that we intend to analyze, in this section, we present the main stylized facts of the Japanese labor market during the 1990s, as well as the evolution of TFP and the focal fiscal policy variables.

Figures 1, 2, and 3 document the evolution of total factor productivity (TFP), GDP, the main labor market variables, the share of aggregate government spending in GDP, and the disaggregated shares of government wage, consumption, and investment in GDP, respectively, from 1991 to 2002.[3]

The level of detrended Japanese GDP in 2002 was 85% less than what it had been in 1991 as seen in 1b.[4] Hayashi and Prescott (2002) point to TFP growth decline as the main driving force of the recession of the 1990s. Figure 1a shows the evolution of detrended TFP since 1991. We observe that it declines by more than 10% from 1991 to 2002. Such drop in TFP is shown in Esteban-Pretel et al. (2010) to not only be the driver of the decline in output, as previously stated by Hayashi and Prescott (2002), but also of the changes in the labor market.



Notes: (i) Figures 1 (a) and 1 (b) show the detrended and normalized total factor productivity (TFP) and gross domestic product (GDP) in Japan's Lost Decade, respectively; (ii) We first detrend both series at 2% annual rate, which is the growth rate of the USA over the 20th century following Hayashi and Prescott (2002), and then we normalize the detrended series by their value in 1991.

FIGURE 1. Japan's detrended total factor productivity and output during the 1990s.

During the Lost Decade, the unemployment rate almost tripled, rising from 2.13% in 1991 to 5.40% in 2002, as shown in Panel 2a. We observe in Panel 2b that parallel to this increase in unemployment, real detrended wages for those workers who managed to keep their jobs decreased by about 20%. The rise in unemployment was also accompanied by large changes in the flows of workers. Panel 2c and Panel 2d show that the quarterly job finding probability decreased from 41% in 1991 to 27% in 2002 and the job separation probability increased from 0.87% to 1.87% during the same period. At the same time, an interesting phenomenon occurred, as shown in Panel 2e. While, as stated before, the probability that an individual worker would find a job in a given quarter decreased, the total number of workers finding jobs actually increased during this period. This is due to the large increase in the number of unemployed workers in the economy. Panel 2f shows that there was also an increase in the total number of workers who lost their job every quarter.

In response to the slowdown of economic growth and upsurge of unemployment during the Lost Decade, the Japanese government increased its aggregate government spending to stimulate the economy and cushion the labor market. Figure 3a shows that the share of aggregate government expenditure in GDP increased from 20% in 1991 to 24% in 2002. Underlying this substantial 20% increase lies heterogenous movements of its different components, as shown in Panel 3b. From 1991 to 2002, the share of government wage expenditure increased by 8.5% , from 6.13% to 6.65% of GDP; the share of government consumption increased by more than 40% , from 7.88% to 11.10% of GDP; and the share of government investment was actually reduced by 0.64% , from 6.35% to 6.32% of GDP, although it went up in the first half of the 1990s and then dropped by 2002 to a slightly lower level than that of 1991. Finally, Panel 3c shows that during this period, the share of government employment in total employment increased from 8.1% to 8.7%. These numbers indicate that while the aggregate share of government spending in GDP increased during the 1990s, the shares of government wage, consumption, and investment were not always moving in the same direction and by equal magnitude. Given the different behaviors of these

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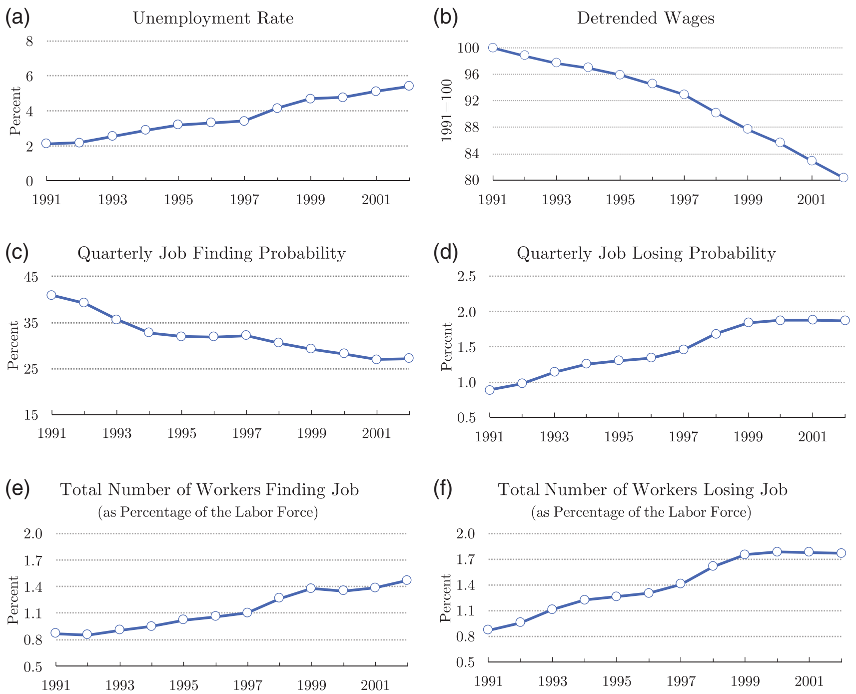


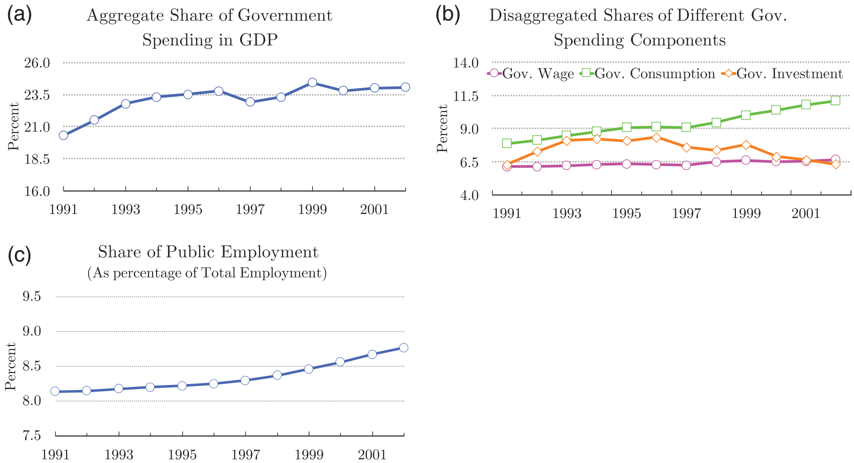
Notes: (i) Figure 2 (a) shows Japan's unemployment rate between 1991 and 2002. (ii) Figure 2 (b) displays the detrended and normalized real wages during the 1990s of Japan, we first detrend real wage at 2% annual rate, which is the growth rate of the USA over the 20th century as Hayashi and Prescott (2002) and then normalize the detrended wage series by the detrended wage in 1991. (iii) Figure 2 (c) exhibits the quarterly job finding probability in Japan from 1991 to 2002, where the job finding probability is defined as the number of workers moving from unemployment to employment divided by the number of unemployed workers in a given quarter. (iv) Figure 2 (d) depicts the quarterly job losing probability in Japan between 1991 and 2002, where the job losing probability is defined as the number of workers moving from employment to unemployment divided by the number of employed workers in a given quarter. (v) Figure 2 (e) shows the total number of workers finding jobs as a fraction of the labor force in a given quarter. (vi) Figure 2 (f) illustrates the total number of workers losing jobs as a fraction of the labor force in a given quarter.

FIGURE 2. Stylized facts of Japanese labor market during the 1990s.

government expenditure components, understanding how their changes affected the labor market is one of the issues addressed in this paper.

While government spending increased during the 1990s, as documented by Mendoza et al. (1994), taxes were relatively stable, and when changed, they did not move in the direction of counteracting the recession.[5] We study as one of the policy experiments in the paper how the labor market would have reacted if the government had implemented countercyclical changes in taxes.

In summary, the Japanese economy over the 1990s experienced a decline in TFP, an increase in unemployment, a rise in the job separation rate and a drop in



Notes: (i) Figure 3 (a) shows the share of aggregate government spending in GDP during the 1990s in Japan. (ii) Figure 3 (b) illustrates the disaggregated shares of different government spending components in GDP in Japan's Lost Decade, where the magenta (circles), green (squares), and orange (diamonds) lines indicate the shares of government wage, consumption, and investment in GDP, respectively. (iii) Figure 3 (c) exhibits the share of government employment as a percentage of total employment in Japan from 1991 to 2002.

FIGURE 3. Japanese government employment and spending during the 1990s.

the job finding rate, an upsurge in aggregate government spending, and stable tax rates. In the next section, we explain the model employed to examine the impact of fiscal policy changes on unemployment dynamics in Japan from 1991 to 2002.

3. MODEL

We use a discrete time neo-classical growth model, as in Cass (1965) and Koopmans (1965), augmented with a search and matching labor market with two productive sectors: a private and a government sector. There are three categories of infinitely lived agents in the economy: households/workers, firms, and the government.

Labor Market

The labor market is modeled in the style of the search and matching literature with random search, where recruiting firms and unemployed workers try to match and form employment relationships in a single labor market for both productive sectors. Such employment relationships are of one worker to one firm. Both private and government sector firms employ capital, labor, and technology for production. In what follows, private sector variables are indicated by the superscript p and government sector variables are denoted by g.

The economy-wide labor force is normalized to be 1. At period t, individuals are either private employees (n_t^p), or government employees (n_t^g), or unemployed (u_t); hence,

$$n_t^p + n_t^g + u_t = 1. \tag{1}$$

The number of vacancies posted in sector i is v_t^i, for each $i \in \{p, g\}$. Unemployed workers randomly search in a unified labor market, and matching occurs according to the standard Cobb-Douglas matching function

$$m_t = \eta (u_t)^{\mu} (v_t)^{1-\mu}, \tag{2}$$

where $v_t = v_t^p + v_t^g$. η measures matching efficiency and μ indicates the elasticity of the matching function with respect to unemployment. Under random search, there is only one matching function and matches in each sector are determined by the relative number of vacancies. The ratio of vacancies to unemployed workers is defined as labor market tightness, $\theta_t \equiv \frac{v_t}{u_t}$. The vacancy filling probability q_t and the sectoral job finding probabilities p_t^i are

$$q_t = \frac{m_t}{v_t}, p_t^i = \frac{m_t}{u_t}\frac{v_t^i}{v_t}, \text{ for each } i \in \{p, g\}. \tag{3}$$

Job separation is modeled as endogenous in the private sector and exogenous in the government sector. Assuming exogenous separations for the government is consistent with both the existent literature (e.g. Gomes (2015)) and the fact that employment in the Japanese government is almost always for life and workers cannot be fired at the will of the employer. However, firing in the private sector, while more difficult than in other industrialized countries, is not as restrictive as in the government sector and can be initiated by the firm, so we assume it to be endogenous.

Endogenous job destruction in the private sector firms is modeled by assuming that operating firms need to pay, in addition to the labor cost and capital cost, a non-productive intermediate input cost x_t, which is idiosyncratic to each match. This match-specific intermediate input cost is independent and identically distributed across firms and over time, with distribution function F: $[x_{\min}, x_{\max}] \rightarrow [0, 1]$. A new idiosyncratic intermediate cost is drawn each period by existing matches. The match is endogenously dissolved as a joint decision by the firm and the worker if this cost is too high under the available technology. The threshold value of x_t that dissolves the match is indicated by $\bar{x}_t$; thus, the job destruction probability is $1 - F(\bar{x}_t)$. Matches in the government sector are exogenously destroyed with rate λ^g.

The timing of the model is as follows. At the beginning of each period t, every matched firm draws a new idiosyncratic intermediate input cost. These, together with the exogenous policy variables and the available technology level, determine the number of productive and unproductive matches for period t, which in turn establish the levels of employment and unemployment. After this, production takes place at filled firms and search occurs in the labor market, where matches will become productive a period later. At the end of the period, wages are paid and profits are distributed to the household.

The evolution of unemployment and employment in each sector is characterized by the following equations:

$$u_t = \left[1 - p^p_{t-1} F(\bar{x}_t) - p^g_{t-1}\right] u_{t-1} + [1 - F(\bar{x}_t)]\, n^p_{t-1} + \lambda^g n^g_{t-1}, \tag{4}$$

$$n^g_t = (1 - \lambda^g)\, n^g_{t-1} + p^g_{t-1} u_{t-1}, \tag{5}$$

$$n^p_t = F(\bar{x}_t)\, n^p_{t-1} + p^p_{t-1} F(\bar{x}_t)\, u_{t-1}, \tag{6}$$

where $p^p_{t-1} F(\bar{x}_t)\, u_{t-1}$ and $p^g_{t-1} u_{t-1}$ are the number of unemployed workers who found a successful match in period $t-1$ in the private and government sectors, respectively; and $[1 - F(\bar{x}_t)]\, n^p_{t-1}$ and $\lambda^g n^g_{t-1}$ are the number of employed workers who lost their jobs in the private and government sectors.

Household

Following Merz (1995), the representative household consists of all individuals in the economy. Household members perfectly self-insure each other, which allows us to solve the model without having to track their employment and wealth distributions. The household owns private capital and rents it to private firms. It receives wage income from employed family members as well as the unemployment benefits from the unemployed individuals. The household has the following per period utility

$$u(c_t) + \nu\left(n^p_t, h^p_t, n^g_t, h^g_t, \bar{h}_t\right), \tag{7}$$

with

$$u(c_t) = \frac{c_t^{1-\sigma} - 1}{1 - \sigma}, \text{ and } c_t = \left[\phi\left(c^p_t\right)^{\frac{\zeta-1}{\zeta}} + (1-\phi)\left(c^g_t\right)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}}, \tag{8}$$

and

$$\nu\left(n^p_t, h^p_t, n^g_t, h^g_t, \bar{h}_t\right) = \psi\left[\left(n^p_t + n^g_t\right)\log\left(1 - \bar{h}_t\right) + n^p_t \log\left(\bar{h}_t - h^p_t\right) + n^g_t \chi_t \log\left(\bar{h}_t - h^g_t\right)\right], \tag{9}$$

where c_t is the effective consumption as in Bouakez and Rebei (2007), which is the aggregation of private consumption c^p_t and public goods produced in the government sector c^g_t. $\bar{h}_t$ is the exogenous maximum amount of hours that an individual can work. ψ represents the weight of leisure in the utility function, and χ_t allows for the disutility from work in the public sector to be different from work in the private sector.[6] σ is the inverse of the inter-temporal elasticity of substitution. ζ indicates the elasticity of substitution between private and public goods. $1 - \phi$ measures the weight of public goods in utility.

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The household chooses private consumption and savings each period $\{c^p_{t+j}, K^p_{t+1+j}\}_{j=0}^{\infty}$ taking $\{c^g_{t+j}, \bar{h}_{t+j}\}_{j=0}^{\infty}$ as given to maximize their life-time utility

$$\sum_{j=0}^{\infty}\beta^j\left[u\left(c_{t+j}\right)+\nu\left(n^p_{t+j},h^p_{t+j},n^g_{t+j},h^g_{t+j},\bar{h}_{t+j}\right)\right], \tag{10}$$

subject to

$$\begin{aligned}(1+\tau_c)\,c^p_{t+j}+K^p_{t+1+j}=&\,(1-\tau_n)\left(W^p_{t+j}+W^g_{t+j}\right)+u_{t+j}sz_{t+j}\\&+(1-\delta^p)\,K^p_{t+j}+r^p_{t+j}K^p_{t+j}-\tau_k\left(r^p_{t+j}-\delta^p\right)K^p_{t+j}\\&+\Pi_{t+j}-T_{t+j},\quad \text{given } K_0,\end{aligned} \tag{11}$$

for $j=\{0,1,\ldots,\infty\}$, where $\beta\in(0,1)$ is the discount factor of households. W^i_t is the total amount of wages paid to the individuals working in sector i, for each $i\in\{p,g\}$. τ_c is the tax rate on private consumption. τ_n is the tax rate on labor income, and τ_k is the capital income tax rate. s indicates the unemployment benefits or the value of home production. z_t is a variable that grows at the average growth rate of technology. Π_t is the profits from private sector firms. T_t is the lump sum taxes paid by the household. r^p_t is the real rental rate of private capital K^p_t, which depreciates at rate δ^p.

The problem of the household yields the following first-order conditions (FOCs)

$$\left[\phi\left(c^p_t\right)^{\frac{\zeta-1}{\zeta}}+(1-\phi)\left(c^g_t\right)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{1-\sigma\zeta}{\zeta-1}}\phi\left(c^p_t\right)^{\frac{-1}{\zeta}}=\varphi_t(1+\tau_c), \tag{12}$$

$$\beta\varphi_{t+1}\left[1+(1-\tau_k)\left(r^p_{t+1}-\delta^p\right)\right]=\varphi_t, \tag{13}$$

where φ_t is the marginal utility of private consumption.

Workers

Let $N^p_t(x_t)$ and N^g_t denote the values for a worker of being employed in a private sector firm with idiosyncratic input cost x_t and in a government sector job, respectively. Let U_t represents the value of being unemployed.

An unemployed worker receives sz_t units of consumption while unemployed. The worker matches with a firm in the private sector with probability p^p_t, and if the idiosyncratic intermediate input cost for the match x_t is below the destruction threshold $\bar{x}_{t+1}$, he becomes a private sector worker in the next period. If he matches with a firm in the government sector, with probability p^g_t, he starts working in the government sector in the following period.[7] If the unemployed worker does not match with a firm, he remains unemployed. The value of being unemployed in period t is

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$$U_t = sz_t + \beta \frac{\varphi_{t+1}}{\varphi_t} \left[p_t^p \int_{x_{min}}^{\bar{x}_{t+1}} N_{t+1}^p(x_{t+1})\, dF(x_{t+1}) + p_t^g N_{t+1}^g \right.$$
$$\left. + \left(1 - p_t^p F(\bar{x}_{t+1}) - p_t^g\right) U_{t+1} \right]. \quad \textbf{(14)}$$

The value of a job for a worker in the private sector depends on the idiosyncratic intermediate input cost x_t. A worker employed in a private firm obtains after-tax wage, enjoys utility from leisure for the hours he is not working, and gets the continuation value, which is the value of being employed if the match survives, or the value of unemployment if the match is destroyed. Therefore, the value of being employed in the private sector in period t is

$$N_t^p(x_t) = (1 - \tau_n)\, w_t^p(x_t)\, h_t^p + \psi \frac{\log\left(1 - \bar{h}_t\right)}{\varphi_t} + \psi \frac{\log\left(\bar{h}_t - h_t^p\right)}{\varphi_t}$$
$$+ \beta \frac{\varphi_{t+1}}{\varphi_t} \left[\int_{x_{min}}^{\bar{x}_{t+1}} N_{t+1}^p(x_{t+1})\, dF(x_{t+1}) + (1 - F(\bar{x}_{t+1}))\, U_{t+1} \right]. \quad \textbf{(15)}$$

Similarly to private sector workers, an individual employed in the government receives after-tax wage, enjoys the utility for his leisure hours, and obtains the continuation value. Thus, the value of being employed in the government sector in period t is

$$N_t^g = (1 - \tau_n)\, w_t^g h_t^g + \psi \frac{\log\left(1 - \bar{h}_t\right)}{\varphi_t} + \psi \chi_t \frac{\log\left(\bar{h}_t - h_t^g\right)}{\varphi_t}$$
$$+ \beta \frac{\varphi_{t+1}}{\varphi_t} \left[(1 - \lambda^g)\, N_{t+1}^g + \lambda^g U_{t+1} \right]. \quad \textbf{(16)}$$

Private Sector Firms

Private sector firms employ private physical capital, the household's labor, the capital stock of the public sector, and the available technology to produce output according to a constant returns to scale production function. Private physical capital is a choice variable for the firm, but hours worked is negotiated through Nash bargaining with the worker. Following Baxter and King (1993), we assume that public capital is productive not only to the government but also to the private sector. The production function of private firm is $y_t^p = a_t^p f\left(k_t^p, h_t^p, K_t^g\right)$, where a_t^p is the total factor productivity (TFP), y_t^p, k_t^p, h_t^p are output, private capital, and hours per private worker, respectively, and K_t^g is the capital stock of the public sector. Aggregate output and total private physical capital are related to y_t^p and k_t^p as follows:

$$Y_t = n_t^p y_t^p \quad \text{and} \quad K_t^p = n_t^p k_t^p. \quad \textbf{(17)}$$

Let V_t and $J_t(x_t)$ indicate the value of a vacant job and the value of an operating job, respectively. Firms in the private sector post vacancies in the labor market at

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a flow cost of $\iota^p z_t$. If the firm is matched and the idiosyncratic cost is low enough, the firm becomes productive in the next period, otherwise it remains vacant. The value of a vacancy is

$$V_t = -\iota^p z_t + \beta \frac{\varphi_{t+1}}{\varphi_t} \left[q_t \int_{x_{min}}^{\bar{x}_{t+1}} J_{t+1}(x_{t+1})\, dF(x_{t+1}) + (1 - q_t F(\bar{x}_{t+1}))\, V_{t+1} \right]. \tag{18}$$

In equilibrium, free entry of firms implies that $V_t = V_{t+1} = 0$; thus, the value of a vacancy is

$$0 = -\iota^p z_t + \beta \frac{\varphi_{t+1}}{\varphi_t} q_t \int_{x_{min}}^{\bar{x}_{t+1}} J_{t+1}(x_{t+1})\, dF(x_{t+1})\,. \tag{19}$$

If a private firm is matched with a worker, it implements optimal production schedules to maximize profits. In addition, it also pays wages, the cost of capital, and the intermediate input cost. When the idiosyncratic input cost x_{t+1} is below the threshold in the next period, the match survives; otherwise, it is destroyed and the firm becomes vacant. Hence, the value of an operating firm is

$$J_t(x_t) = \max_{k_t^p} \left\{ y_t^p - w_t^p(x_t)\, h_t^p - r_t^p k_t^p - x_t z_t + \beta \frac{\varphi_{t+1}}{\varphi_t} \int_{x_{min}}^{\bar{x}_{t+1}} J_{t+1}(x_{t+1})\, dF(x_{t+1}) \right\}. \tag{20}$$

The private firm chooses k_t^p to maximize the present discounted value of being filled, and the FOC is

$$a_t^p f_{k_t^p}\left(k_t^p, h_t^p, K_t^g\right) = r_t^p. \tag{21}$$

Equation (21) implies that the marginal product of private physical capital should be equal to its rental rate at the optimal.

Total profits of private firms are defined as

$$\Pi_t = n_t^p a_t^p f\left(k_t^p, h_t^p, K_t^g\right) - W_t^p - r_t^p n_t^p k_t^p - x_t^T z_t - \iota^p z_t v_t^p, \tag{22}$$

where x_t^T is the (detrended) total intermediate input cost paid by the firms, that is, $x_t^T = \frac{n_t^p}{F(\bar{x}_t)} \int_{x_{min}}^{\bar{x}_t} x_t dF(x_t)$, and the total wage paid to the private sector workers W_t^p is defined as

$$W_t^p = \frac{n_t^p h_t^p}{F(\bar{x}_t)} \int_{x_{min}}^{\bar{x}_t} w_t^p(x_t)\, dF(x_t)\,. \tag{23}$$

Surplus, Bargaining, Wages, Hours, and Destruction Threshold of Private Sector Firms

When a match becomes productive in the private sector, it creates a surplus $S_t(x_t)$ which is shared between the firm and the worker. The surplus S_t is the sum of the values of a filled job for an employed worker $N_t^p(x_t)$ and the firm $J_t(x_t)$ minus their outside options, that is, the value of being unemployed U_t and the value of a vacant job V_t. Because of free entry, $V_t = 0$ in equilibrium, so the joint surplus is $S_t(x_t) = N_t^p(x_t) + J_t(x_t) - U_t$.

Wages and hours worked in the private sector are determined through Nash bargaining between the workers and the firm in the private sector. In period t, private wages and hours worked are negotiated to maximize the Nash product

$$\max_{w_t^p(x_t), h_t^p} \left(N_t^p(x_t) - U_t\right)^{\xi} (J_t(x_t) - V_t)^{1-\xi}, \tag{24}$$

where $\xi \in (0, 1)$ is the worker's bargaining power.

The Nash solution to this bargaining problem implies that both private firms and workers receive a constant fraction of the surplus. The optimal sharing rules are:

$$N_t^p(x_t) - U_t = \frac{\xi(1-\tau_n)}{1-\xi\tau_n} S_t(x_t) \text{ and } J_t(x_t) = \frac{1-\xi}{1-\xi\tau_n} S_t(x_t). \tag{25}$$

Combining the optimal sharing rule (25) with the value equations (14), (15), (16), (19), and (20), we can express the surplus $S_t(x_t)$ as

$$\begin{aligned} S_t(x_t) = {} & y_t^p - \tau_n w_t^p(x_t)\, h_t^p - r_t^p k_t^p - x_t z_t - s z_t + \psi \frac{\log\left(1-\bar{h}_t\right)}{\varphi_t} + \psi \frac{\log\left(\bar{h}_t - h_t^p\right)}{\varphi_t} \\ & + \beta \frac{\varphi_{t+1}}{\varphi_t} \left(1 - p_t^p \xi \frac{1-\tau_n}{1-\xi\tau_n}\right) \int_{x_{min}}^{\bar{x}_{t+1}} S_{t+1}(x_{t+1})\, dF(x_{t+1}) \\ & - \beta \frac{\varphi_{t+1}}{\varphi_t} p_t^g \left(N_{t+1}^g - U_{t+1}\right). \end{aligned} \tag{26}$$

The division of the surplus between the private sector firms and workers yields the conditions that determine the wages paid to private sector employees and hours worked,

$$\begin{aligned} w_t^p(x_t)\, h_t^p = {} & \xi \left[y_t^p - r_t^p k_t^p - x_t z_t + \frac{p_t^p}{q_t} \iota^p z_t \right] + (1-\xi) \frac{1}{1-\tau_n} \\ & \times \left[s z_t - \psi \frac{\log\left(1-\bar{h}_t\right)}{\varphi_t} - \psi \frac{\log\left(\bar{h}_t - h_t^p\right)}{\varphi_t} \right. \\ & \left. + \beta \frac{\varphi_{t+1}}{\varphi_t} p_t^g \left(N_{t+1}^g - U_{t+1}\right) \right], \end{aligned} \tag{27}$$

$$\frac{1}{1-\tau_n} \psi \frac{\left(\bar{h}_t - h_t^p\right)^{-1}}{\varphi_t} = a_t^p f_{h_t^p}\left(k_t^p, h_t^p, K_t^g\right), \tag{28}$$

where equation (27) is similar to the wage equation in Pissarides (2000). Private sector workers are compensated for a proportion ξ of the firm's production and a measure of the saved vacancy posting cost. In addition, they are also compensated for the unemployment benefits, disutility of working, as well as the potential gains

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from working in the government sector, adjusted by labor income taxes. Hours worked in the private sector are determined by equation (28), which implies that the marginal cost of working should be equal to the marginal benefit of working at the optimal.

In the private sector, if the idiosyncratic intermediate input cost to the firm is so high that it drives the joint surplus to zero, a match is endogenously dissolved, a decision that benefits both the firm and the worker. The threshold intermediate input cost $\bar{x}_t$ that destroys the match is

$$\bar{x}_t z_t = y_t^p - \tau_n w_t^p(\bar{x}_t)\, h_t^p - r_t^p k_t^p - s z_t + \psi \frac{\log\left(1-\bar{h}_t\right)}{\varphi_t} + \psi \frac{\log\left(\bar{h}_t - h_t^p\right)}{\varphi_t}$$
$$+\beta \frac{\varphi_{t+1}}{\varphi_t}\left(1 - p_t^p \xi \frac{1-\tau_n}{1-\xi\tau_n}\right) \int_{x_{min}}^{\bar{x}_{t+1}} S_{t+1}(x_{t+1})\, dF(x_{t+1})$$
$$-\beta \frac{\varphi_{t+1}}{\varphi_t} p_t^g \left(N_{t+1}^g - U_{t+1}\right). \tag{29}$$

Government

The government in this economy is an active participant in the labor market and hires labor to produce public goods, c_t^g. Following Cortuk and Guler (2013) and Gomes (2015), we assume that government output is not sold and hence it is not a component of aggregate output. This public good is different from private goods: it is non-rival, non-excludable, and is supplied to the household for free. The public good c_t^g is used by the household as part of their consumption bundle, c_t, and is also utilized to pay the recruitment cost in the government sector. The government production function is specified as

$$c_t^g = a_t^g \left(K_t^g\right)^\gamma \left(n_t^g h_t^g\right)^{1-\gamma} - \iota^g v_t^g. \tag{30}$$

where a_t^g is the productivity in the government sector, K_t^g is the government physical capital, $n_t^g h_t^g$ are total hours worked by government employees, and $\gamma \in (0, 1)$ measures the elasticity of government output with respect to government physical capital. ι^g is the vacancy posting cost in the government sector, and v_t^g is the number of vacancies posted. Capital K_t^g is accumulated through government investment I_t^g and evolves as

$$K_{t+1}^g = (1-\delta^g)\, K_t^g + I_t^g. \tag{31}$$

The government imposes consumption tax, τ_c, labor tax τ_n, capital tax τ_k, and a lump sum tax T_t to finance its aggregate expenditure (compensation for its employees $W_t^g = w_t^g n_t^g h_t^g$, investment I_t^g, and consumption expenditure g_t^g). Thus,

$$\tau_c c_t^p + \tau_n\left(W_t^p + W_t^g\right) + \tau_k\left(r_t^p - \delta^p\right) K_t^p + T_t = w_t^g n_t^g h_t^g + I_t^g + g_t^g. \tag{32}$$

We assume that the lump-sum taxes T_t adjust to balance period-by-period government budget constraint.

There are multiple studies (e.g. Blanchflower (1996), Gregory and Borland (1999), and Morikawa (2014)) which show, including for Japan, that there is a wage premium in public sector jobs. Following this evidence and the work of Gomes (2015) and Michaillat (2011), we model government employee wages as a mark-up π^g over the average wage in the private sector[8]

$$w_t^g = \left(1 + \pi_t^g\right) w_t^p. \tag{33}$$

There is no strong evidence supporting large differences in hours work in Japan for private and government sectors workers. We, therefore, assume that hours of work in government jobs are not a choice variable and set them to be equal to those worked in the private sector.

Given the probability for a government sector vacancy to be filled, which is determined by the matching function (2), the government chooses how many vacancies to post so that the target of government expenditure in wages, W_t^g, is met every period. This target is a policy parameter, exogenous to the model, and the change of which we explore in Section 6 of the paper.

Equilibrium

The equilibrium in this economy is a perfect foresight competitive equilibrium defined below.

DEFINITION 1. *Given the path of TFP* $\{a_t^p, a_t^g\}_{t=0}^{\infty}$, *the sequence of government taxes, government expenditure shares, public wage premiums, hours worked, and maximum workable hours,* $\{\tau_c, \tau_n, \tau_k, W_t^g, I_t^p, g_t^g, \pi_t^g, h_t^g, \bar{h}_t\}_{t=0}^{\infty}$, *and the initial capital stock* K_0, *a perfect foresight competitive equilibrium is a set of prices* $\{w_t^p(x_t), r_t^p, w_t^g\}_{t=0}^{\infty}$, *a sequence of quantities* $\{c_t^p, K_{t+1}^p, h_t^p, k_t^p, n_t^p, n_t^g, u_t, v_t^p, v_t^g, \theta_t, \bar{x}_t, Y_t, y_t^p, c_t^g\}_{t=0}^{\infty}$ *and* $\{K_t^g, m_t^p, m_t^g, p_t^p, p_t^g, S_t, \varphi_t, T_t, \chi_t\}_{t=0}^{\infty}$ *that satisfy:*

- (*a*) *Agents' optimization conditions:*
  - (*a*.1) *Household's maximization, as in equations (12) and (13);*
  - (*a*.2) *Value functions in the labor market satisfy (14), (15), (16), (19), (20);*
  - (*a*.3) *Private sector physical capital demand meets (21);*
  - (*a*.4) *Optimal surplus sharing rule in the private sector implies (25);*
  - (*a*.5) *Private wage satisfies (27) and hours worked satisfies (28);*
  - (*a*.6) *Destruction threshold meets (29);*
  - (*a*.7) *Equalization of ex-ante value of a job in the private and public sectors:* $\tilde{N}_t^p - U_t = N_t^g - U_t$.
- (*b*) *Markets clearing conditions:*
  - (*b*.1) *Consumption goods market:* $Y_t = c_t^p + K_{t+1}^p - (1 - \delta^p)K_t^p + v_t^p \iota^p z_t + n_t^p x_t z_t - u_t s z_t + I_t^g + g_t^g$;
  - (*b*.2) *Private capital market meets equation (17);*
  - (*b*.3) *Labor market flows meet (4), (5), and (1);*

(*c*) *Government behavior conditions:*
(*c*.1) *Government budget is balanced, as in equation (32);*
(*c*.2) *Government sector wages meet equation (33);*
(*c*.3) *Government production function is (30);*
(*c*.4) *Government capital evolves as in equation (31).*

To numerically solve the model, we rewrite the equilibrium conditions in terms of stationary variables, which are obtained by dividing each of the non-stationary variables by z_t. On the balanced growth path, z_t grows at the same rate of the TFP factor, $\left(a_t^p\right)^{\frac{1}{1-\alpha}}$, which is $\frac{\bar{\kappa}}{1-\alpha}$.[9] The steady state of the economy is a perfect foresight competitive equilibrium in which all stationary variables are constant. The characterization of this perfect foresight competitive equilibrium can be found in Appendix D.

4. CALIBRATION

We now describe the method employed to parameterize the model and explain the simulation techniques, as well as the assumptions associated with the exogenous variables of our model.

Calibration

We choose functional forms which are standard in the literature and then calibrate the model to match the stylized facts of Japan in 1991, which is specified as the initial steady state in our simulation. The length of a period is set to one quarter.

The utility function is given by equations (7), (8) and (9). The relative risk aversion is set at $\sigma = 1.00$, such that the utility function on consumption is logarithmic. We set the quarterly discount factor to $\beta = 0.9957$ to match the capital-output ratio for Japan in 1991, 7.83.

The share of private consumption is set to $\phi = 0.70$ as in Bruckner and Pappa (2012). Following the study by Garcia and Llopis (2005), we set the consumption substitutability ζ, at $\zeta = 2.00$.

According to Mendoza et al. (1994), the proportional labor tax τ_n and the proportional capital tax τ_k were relatively stable during the 1990s. These two tax rates are set at $\tau_n = 0.28$ and $\tau_k = 0.44$, which are the average from 1990 to 1996 from the extended data set of Mendoza et al. (1994). The consumption tax τ_c is set at $\tau_c = 0.03$, which is the average level from 1990 to 1997.[10]

The production function of the private sector firm is assumed to be $y_t^p = a_t^p \left(K_t^g\right)^{\gamma} \left(k_t^p\right)^{\alpha} \left(h_t^p\right)^{1-\alpha}$. Employing the extended data in Braun et al. (2006), we estimate the share of capital revenue in output α and the depreciation rate in private sector δ^p and set them at $\alpha = 0.383$ and $\delta^p = 0.028$, respectively. Following Kamps (2004), we set the depreciation rate of public capital to half of that in the private sector, thus, $\delta^g = 0.014$. The initial private sector technology a_{1991Q1}^p and initial government sector technology a_{1991Q1}^g are normalized to unity. The long run growth rate of TFP, $\bar{\kappa}$, is assumed to be 1.46%, which is the average in the data from 2002 to 2006. We set the value of home production or unemployment insurance, s, to be a fraction $s = 0.1$ of market production in the initial steady state, which is around 23% of what a worker could earn on average in the labor market.

In the labor market, the elasticity of matching with respect to unemployment, μ, is set at 0.5, which is conventional in the literature. Following what has also become standard in the literature, we set the bargaining power of the worker, ξ, to $\xi = 0.50$. The public vacancy posting cost ι^g is chosen such that it is about 15% of the private vacancy posting cost, in line with Gomes (2011), and set to $\iota^g = 0.1419$.

The intermediate idiosyncratic cost $x \in [x_{min}, x_{max})$ is assumed to follow an exponential distribution with probability density function $\frac{1}{\rho}e^{-\frac{x}{\rho}}$, where $x_{min} = 0$, $x_{max} = +\infty$. The mean of this distribution, ρ, is jointly calibrated along with other parameters and explained below.

According to the empirical study of Esteban-Pretel et al. (2017), we set the quarterly probability of job destruction in the government sector, λ^g, to 0.0039.[11] The share of public physical capital in the production functions γ is set to be $\gamma = 0.10$, which follows the estimates in Cubas (2011). All these exogenous parameters are listed in Table 1.

The mean of the distribution of the intermediate idiosyncratic cost, ρ, the matching efficiency parameter η, the private vacancy posting cost ι^p, and the weight of leisure in the utility function ψ, is jointly calibrated using the steady-state equilibrium of the model to match the unemployment rate and the job finding probability in 1991, the amount of hours worked in the private sector, as well as a normalized labor market tightness of unity. The calibrated values for those parameters are $\rho = 1.1604$, $\eta = 0.3957$, $\iota^p = 0.9459$, and $\psi = 0.1869$. All these endogenously calibrated parameters are reported in Table 2.

Simulation Technique and Path of Exogenous Variables

We simulate the above constructed model by postulating that the economy transitions from its initial steady state in 1991 to a final steady state at a point far enough away in the future.[12] The length of a period in our simulations is one quarter, although the data are later aggregated to annual frequency, as we are interested in the long-term transition of the economy rather than its short-run fluctuations. The simulation is deterministic, and the perfect foresight agents know the paths of exogenous variables. The paths of these exogenous variables in the data are described below.

- Growth rate of TFP, κ_t: The path of technology growth rate, κ_t, is calculated as the period to period change, that is, $e^{\kappa_t} = \frac{a_t^p}{a_{t-1}^p} = \frac{a_t^g}{a_{t-1}^g}$, from 1991 to 2002, and it remains constant at the average growth rate along the balanced growth path after 2002, $\bar{\kappa}$.[13]
- Maximum hours of work, $\bar{h}_t$: During the 1990s, the Japanese government introduced several measures to reduce the amount of hours that could potentially be

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TABLE 1. Exogenous parameters

Parameter	Notation	Value	Source
Private depreciation rate	δ^p	0.0281	Braun et al. (2006)
Capital revenue in output	α	0.3832	Braun et al. (2006)
Private consumption share	ϕ	0.7000	Bruckner and Pappa (2012)
Consumption substitutability	ζ	2.0000	Garcia and Llopis (2005)
Relative risk aversion	σ	1.0000	Esteban-Pretel et al. (2010)
Matching elasticity	μ	0.5000	Esteban-Pretel et al. (2010)
Public job destruction	λ^g	0.0039	Esteban-Pretel et al. (2017)
Public wage premium	π^g	0.1275	Morikawa (2014)
Public capital share	γ	0.1000	Cubas (2011)
Public depreciation rate	δ^g	0.0141	Kamps (2004)
Value of home production	s	0.1000	23% of average market wage
Worker's bargaining power	ξ	0.5000	$\xi = \mu$
Public vacancy cost	ι^g	0.1419	15% of private vacancy cost
Discount factor	β	0.9957	Esteban-Pretel et al. (2010)
Labor tax	τ_n	0.2800	Mendoza et al. (1994)
Capital tax	τ_k	0.4400	Mendoza et al. (1994)
Consumption tax	τ_c	0.0300	Mendoza et al. (1994)
Long run growth rate of TFP	$\bar{\kappa}$	0.0146	Esteban-Pretel et al. (2010)

TABLE 2. Endogenous parameters

Parameters	Notation	Value	Target
Mean of exponential distribution	ρ	1.1604	Job finding probability
Matching efficiency	η	0.3957	Labor market tightness
Private vacancy cost	ι^p	0.9459	Unemployment rate
Relative importance of leisure	ψ	0.1869	Private hours worked

worked by employees. These polices ranged from instituting new national holidays to forcing banks and public offices to close on Saturdays. We construct the time series of maximum hours of work by matching the hours of work in 1991 and the hours of work in 2002 of the private sector; the maximum hours of work between 1991 and 2002 are linearly interpolated.

- Wage premium in the public sector, π_t^g: Using OECD data, we let the wage premium in the public sector increase from 11% in 1991 to 31% in 2002.
- Share of government consumption expenditure, g_t^g: We construct this time series by combining the system of national accounts table and OECD data set and are defined as the ratio of the computed government consumption expenditure to GDP.[14] As evidenced in Panel 3b, the share of government consumption expenditure increased from 0.0788 to 0.111. It is assumed to remain constant at the final level after the last period.

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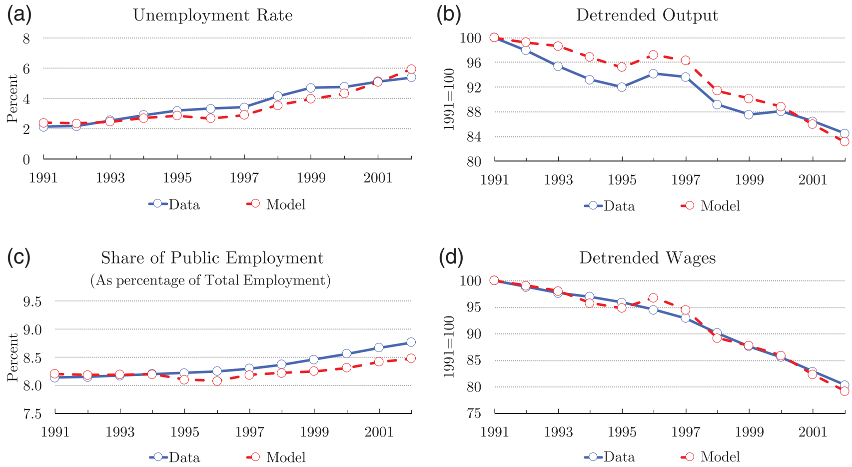

Notes: (i) Figure 4 (a) shows the historical and simulated unemployment rate of Japan from 1991 to 2002. (ii) Figure 4 (b) shows the historical and simulated GDP during the 1990s of Japan, where both series are detrended and normalized. (iii) Figure 4 (c) exhibits the historical and simulated share of government employment as a percentage of total employment in Japan from 1991 to 2002. (iv) Figure 4 (d) depicts the historical and simulated real wages during the 1990s of Japan, where both series are detrended and normalized.

FIGURE 4. Data and benchmark simulation—Main variables.

- Share of government wage expenditure, GWE_t: We construct this time series from the system of national accounts table and the OECD data set.[15] It is defined as the ratio of government wage expenditures to GDP. As we can see in Panel 3a, from 1991 to 2002, the share of government wage slightly increased from 0.0613 to 0.0665. It is assumed to be constant after the last period in our sample.
- Share of government investment expenditure, I_t^g: We construct this time series from the system of national accounts table. It is defined as the ratio of government investment to GDP. Panel 3b shows that from 1991 to 2002, the share of government investment slightly decreased from 0.0635 to 0.0632. We assume that it remains constant after the last period in our sample.

5. BENCHMARK SIMULATION

The results of our benchmark simulation are reported in Figures 4 and 5. Theses two figures demonstrate that our model is successful in replicating the transitional paths of the main labor market variables observed in the data from 1991 to 2002.[16]

Figure 4a shows that the unemployment rate increases from 2.1% in 1991 to 5.4% in 2002, which is attributable to the decline in TFP growth, the decrease in the maximum hours of work, the decrease in government investment spending, and no countercyclical tax policy.

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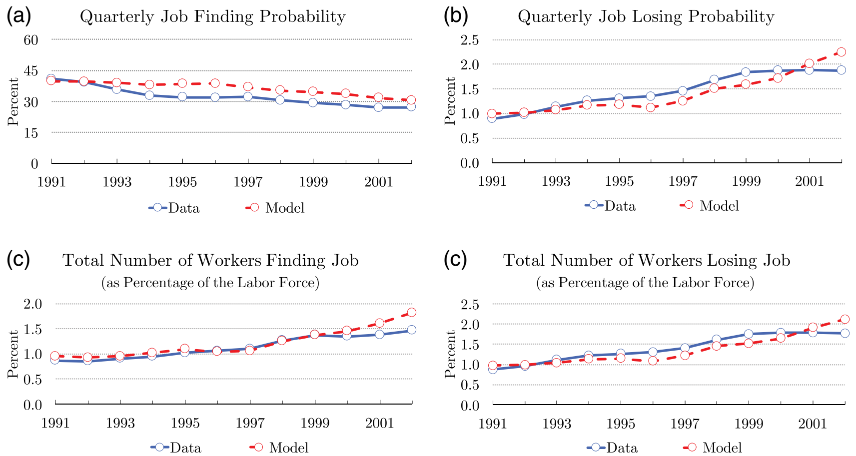

Notes: (i) Figure 5 (a) shows the historical and simulated quarterly job finding probability of Japan from 1991 to 2002. (ii) Figure 5 (b) shows the historical and simulated quarterly job losing probability during the 1990s of Japan. (iii) Figure 5 (c) exhibits the historical and simulated total number of workers finding jobs as a fraction of the labor force in Japan's Lost Decade. (iv) Figure 5 (d) depicts the historical and simulated total number of workers losing jobs as a fraction of the labor force in Japan's 1990s.

FIGURE 5. Data and benchmark simulation—Flows in and out of unemployment.

Our model can successfully reproduce the actual path of the unemployment rate over the 1990s. It is also able to reproduce well the other empirical facts presented in Section 2, such as the drop of detrended output and wages as shown in Panels 4b and 4d, respectively. Similarly, it is able to replicate the empirically observed increase in the share of public employment. The intuition is as follows: The decrease in the growth rate of TFP leads to a drop in the detrended TFP level, which together with the fact that firms in the private sector could not have their employees work longer because of the exogenous decrease in maximum number of hours, reduces private firm's profits. The drop in profits implies that productive private firms have lower incentives to remain operational, which increases the probability of job destruction, as shown in Figure 5b. At the same time, potential entrants expect lower profits, and thus less vacancies are posted, which together with the increase in unemployment reduces the job finding probability, as shown in Figure 5a. The drops in productivity, employment, and hours also lead to the decreases in output and in wages observed in Figure 4b and 4d. While the job separation and job finding rates move in opposite directions, the total number of people losing and finding jobs in this period increase, as shown in Figure 5c and 5d. The increase in the number of people losing jobs during the recession is natural, as the job separation rate increases. However, while the increase in the number of workers finding jobs may seem counter-intuitive, it occurs because the effect of the increase in unemployment is larger than the effect of the drop in the job finding rate. Finally, the increase in public employment is explained by

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the increase in government expenditures, especially in government consumption, where the intuition for the effect of the rise in government consumption on the labor market is described in more detail in the following section.

The benchmark simulations presented above show that our model is able to reproduce the observed paths of the variables of interest in the data. In the next section, we use the previous benchmark as a basis for comparison to what could have happened if the Japanese government had implemented different fiscal policies.

6. EFFECTS OF FISCAL POLICY CHANGES

We now investigate the impact of fiscal policy changes on the unemployment rate over the Lost Decade. We perform counterfactual experiments using our calibrated model and changing the paths of government spending and taxes to analyze the impact of such changes on the evolution of unemployment and its final level in 2002.

We first consider the fiscal policy change from the spending side. In Experiment I, we examine what the unemployment rate in 2002 would have been if the government had not changed the share of government wage, government consumption, and government investment spending during the Lost Decade. This counterfactual experiment is implemented by fixing one by one the share of these components at their 1991Q1 level and otherwise simulating the economy as in the benchmark case.

Then, in Experiment II, we study what would have happened if all changes in government spending had been concentrated into only one of the components. To this end, we fix two of the components to their initial level and allocate the whole increase of government spending observed in the data to the third component.

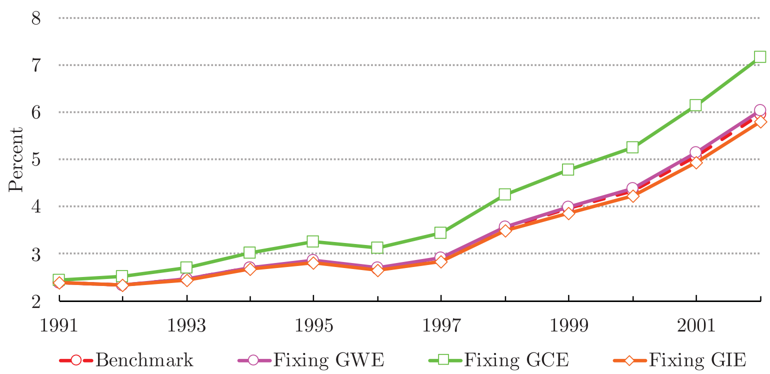
Finally, we investigate the impact of fiscal policy changes from the tax perspective, which we label as Experiment III. We examine what the level of unemployment would have been by the year 2002 if the government had adopted countercyclical tax policies during the Lost Decade. This counterfactual experiment is implemented by reducing one by one the labor tax, capital tax, and consumption tax by 10% in 1991 and keeping the new tax rate constant during the lost decade.

Figure 6, Table 3 and Table 6 report the simulation results from the first government spending counterfactual, Experiment I. From our experiments, we conclude the following: First, if the Japanese government had not increased government consumption expenditure as it did, that is, if the share of government consumption had been fixed at the 1991's level, the unemployment rate in 2002 would have been 7.17%, or what is the same, 20.45% higher than in the benchmark simulation. The intuition for this result is the following. In a typical Real Business Cycle model, employment increases in response to an increase of government spending because of a negative wealth effect: the anticipation of future tax increases leads to a negative wealth effect, private consumption falls, and thus agents increase their

TABLE 3. Unemployment rate: Benchmark and Experiment I

Year	Benchmark	Fixing GWE	Fixing GCE	Fixing GIE
1991	2.39	2.39	2.46	2.40
1992	2.35	2.35	2.53	2.35
1993	2.46	2.47	2.71	2.45
1994	2.70	2.70	3.03	2.68
1995	2.85	2.86	3.25	2.81
1996	2.69	2.70	3.14	2.65
1997	2.89	2.91	3.43	2.84
1998	3.56	3.58	4.26	3.48
1999	3.97	4.00	4.79	3.87
2000	4.33	4.38	5.25	4.22
2001	5.07	5.14	6.14	4.94
2002	5.95	6.04	7.17	5.80

Notes: The counterfactual experiments maintain the same parameter values and path of exogenous variables as in the benchmark simulation, except for one share of government expenditures which is kept fixed at the 1991 level.

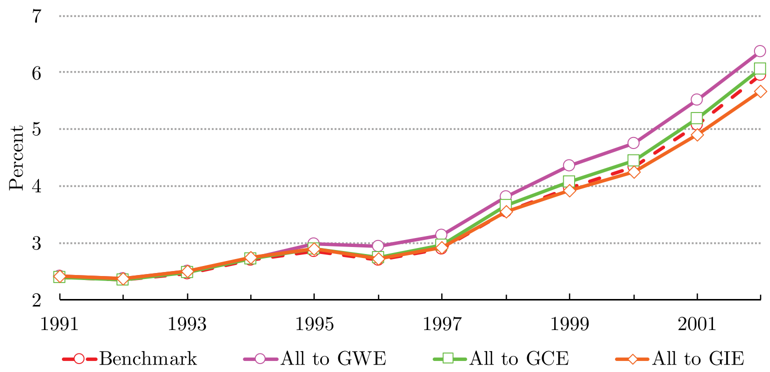


Notes: The counterfactual experiments maintain the same parameter values and path of exogenous variables as in the benchmark simulation, except for the one share of government expenditures which is kept fixed at the 1991 level.

FIGURE 6. Benchmark and counterfactual's unemployment rate: Experiment I.

labor supply. In our model with the utility function with complementarity between private and public goods consumption, an increase in government expenditures increases the marginal utility from private goods and thus their consumption. This leads to a strong wealth effect and thus large increase of labor input, stimulating output and production. This, in turn, means that if such increase had not taken place, unemployment would have been higher by 2002.

Second, if the government had not increased government wage expenditure, the unemployment rate in 2002 would have been 6.04%, only 1.55% higher than that of the benchmark. The small effect of the increase in government wage



Notes: The counterfactual experiments maintain the same parameter values and path of exogenous variables as in the benchmark simulation, except for the share of government expenditures. In this experiment, two of the components of government spending are fixed to the 1991 level and the complete change in expenditures is allocated to the third of the components, either government wage, consumption, or investment expenditures.

FIGURE 7. Benchmark and counterfactual's unemployment rate: Experiment II.

is attributable to the fact that government wage expenditure did not increase very much during this period, so keeping it constant would not have had such a substantial effect.

Third, if the government had not decreased government investment, the unemployment rate at the end of the Lost Decade would have been 5.80%, or 2.59% lower than in the benchmark simulation. So the drop in government investment had a small, but negative effect on unemployment, and the labor market would have been less affected if such drop had not taken place. This is intuitive because again the drop in government investment led to less needed production of investment equipment and a subsequent drop in output and an increase in unemployment. This effect can also be seen in the early years of the 1990s, when government investment did increase and our counterfactual experiment shows that for those early years, unemployment would have been slightly higher if it had remained low.

Figure 7, Table 4 and Table 6 show the results from Experiment II, where we try to answer the question of how the unemployment rate would have evolved had the complete change of government expenditures been allocated to only one of the components, leaving the other two components at the 1991 level. This experiment allows us to see that while the increase in government consumption was important to keep unemployment from increasing to even higher levels than what was seen in the economy, as we showed with Experiment I, such an increase in expenditures would have been more effective in reducing unemployment had it been allocated to government investment. We see that the unemployment rate would have been lowest among the three by 2002 if the complete change in expenditures had been devoted to government investment. This corroborates the intuition developed earlier that investment by the government is important to the economy,

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Table 4. Unemployment rate: Benchmark and Experiment II

Year	Benchmark	All to GWE	All to GCE	All to GIE
1991	2.39	2.42	2.39	2.40
1992	2.35	2.36	2.34	2.38
1993	2.46	2.50	2.47	2.50
1994	2.70	2.73	2.72	2.73
1995	2.85	2.97	2.89	2.88
1996	2.69	2.94	2.74	2.72
1997	2.89	3.14	2.96	2.91
1998	3.56	3.82	3.65	3.56
1999	3.97	4.35	4.07	3.93
2000	4.33	4.76	4.44	4.24
2001	5.07	5.51	5.18	4.91
2002	5.95	6.38	6.07	5.68

Notes: The counterfactual experiments maintain the same parameter values and path of exogenous variables as in the benchmark simulation, except for fixing two of the components to their initial level and allocating the whole increase of government spending observed in the data to the third component.

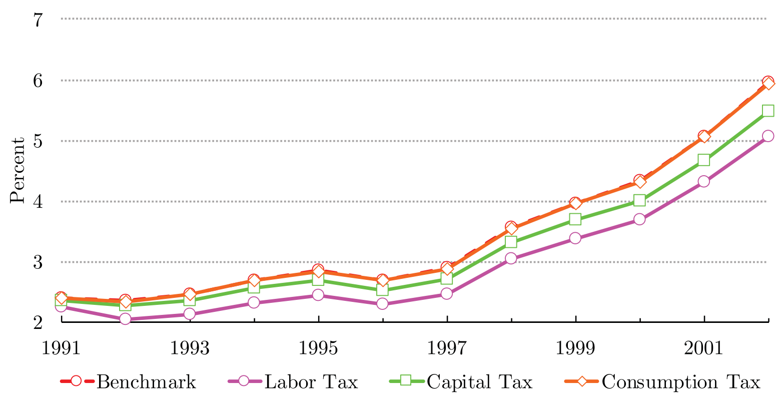
and reducing that component during the second half of the 1990s had a negative impact on unemployment, which is in part due to the fact that increasing public investment raises public capital, which not only increases the labor productivity in the public sector but also in the private sector.[17] It also shows that part of the effect that we observed in Experiment I was due to the size of increase, and not so much to the importance of that component. Finally, we also learn from this experiment that if all the increase in government expenditures had been devoted to hiring, via increases in wage expenditure, that would not have been all that beneficial to keep the unemployment rate down. The reason is that a substantial increase in hiring by the government sector, where wages are higher and job destruction is lower, would have led many workers to prefer those types of jobs and become more selective while unemployed. In particular, given the better conditions of public sector jobs, the bargaining position of workers would be strengthened and average wages in the private sector would rise, resulting in less hiring and more firing in the private sector. This would lead to increases in their unemployment spells and raises in the unemployment rate as observed in Figure 7.

The results of the previous counterfactual experiments show that the government spending changes in Japan over the Lost Decade did have an impact on unemployment and that overall they kept it lower than it could have been, although the drop in government investment in the second half of the 1990s acted in the opposite direction and had the complete increase in expenditures been devoted to government investment, the unemployment rate would have increased even less than it did. Overall, our results reinforce the intuition that increases in government spending reduce the unemployment rate, whereas decreases in government spending increase it. Therefore, government spending did serve as a valid policy

TABLE 5. Unemployment rate: Benchmark and Experiment III

Year	Benchmark	Labor tax	Capital tax	Consumption tax
1991	2.39	2.26	2.37	2.39
1992	2.35	2.05	2.28	2.34
1993	2.46	2.12	2.37	2.46
1994	2.70	2.32	2.57	2.69
1995	2.85	2.45	2.70	2.84
1996	2.69	2.30	2.54	2.69
1997	2.89	2.47	2.71	2.89
1998	3.56	3.04	3.32	3.55
1999	3.97	3.38	3.68	3.95
2000	4.33	3.69	4.01	4.32
2001	5.07	4.32	4.68	5.06
2002	5.95	5.07	5.47	5.93

Notes: The counterfactual experiments maintain the same parameter values and path of exogenous variables as in the benchmark simulation, except for one tax rate (labor, capital or consumption tax), which is reduced by 10% from the baseline parameterization.



Notes: The counterfactual experiments maintain the same parameter values and path of exogenous variables as in the benchmark simulation, except for one tax rate (labor, capital and consumption tax), which is reduced by 10% from the baseline parameterization.

FIGURE 8. Benchmark and counterfactual's unemployment rate: Experiment III.

instrument for the Japanese government to combat unemployment increases during the Lost Decade, although it may have been slightly mis-allocated.

Figure 8, Table 5 and Table 6 summarize the simulation results from the tax rate counterfactual, Experiment III. Three salient characteristics are revealed: First, if the proportional labor income tax had been reduced by 10%, from 28% to 25.20%, in 1991 and stayed at that level, the unemployment rate at the end of the Lost Decade would have been 5.07%, 14.90% lower than the benchmark. The reason is that reducing the labor tax increases the value of working and hence the surplus

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TABLE 6. Percentage changes of the unemployment rate in 2002

Experiment I	Fixing GWE	Fixing GCE	Fixing GIE
Change in unemp. rate	1.55	20.45	−2.59
Experiment II	All to GWE	All to GCE	All to GIE
Change in unemp. rate	7.20	2.00	−4.57
Experiment III	Labor Tax	Capital Tax	Consumption Tax
Change in unemp. rate	−14.90	−7.83	−0.34

Notes: This table shows the percentage changes of the unemployment rate in 2002 under each of the three experiments that we conducted.

of an operating match, which reduces destruction of jobs and encourages vacancy posting and hiring, thus increasing output and reducing unemployment. Second, if the proportional capital income tax had been reduced by 10%, from 44% to 39.6%, in 1991 and stayed at that level, the unemployment rate in 2002 would have been 7.83% lower compared to the benchmark. The intuition is that if the capital tax is reduced, firms have higher incentives to invest in physical capital. This leads to increases in the future capital stock and the marginal product of labor, which encourages hiring, discourages firing, and reduces unemployment. Third, if the proportional consumption tax had been reduced by 10%, from 3% to 2.7%, in 1991 and stayed at that level after that, the unemployment rate at the end of the Lost Decade would have decreased by 0.34% and been at 5.93% in 2002. This result is due to the fact that reducing the consumption tax increases the value of working, leading to increases in the surplus. This increase encourages vacancy posting and hiring, reduces separations, and thus increases output and decreases unemployment. Therefore, while countercyclical tax policies were not used by the government during the Lost Decade, they would have also been effective policies to control the increase in unemployment over this period.

The counterfactual experiments shown in this section demonstrate that during the 1990s, changes in different government spending components affected the unemployment rate heterogeneously. The increases in government consumption expenditure and government wage prevented the unemployment rate during the Lost Decade to rise to a higher level than it did, but the decrease in government investment expenditure contributed to the surge of the unemployment rate in the 1990s. We also show that the most effective way to keep unemployment down would have been to devote more government resources to investment, rather than to consumption and especially government wage expenditures. In addition, the unemployment rates in the 1990s would have been lower than the actual ones if the Japanese government had adopted countercyclical tax policies.

7. CONCLUSION

Japan during the 1990s presents a very interesting and unique case study for the effects of fiscal policy in the labor market during a prolonged and severe recession. The unemployment rate in Japan increased from 2.13% in 1991 to 5.40% in 2002, and during the same period, the job finding probability decreased and the job losing probability increased. Meanwhile, the Japanese government increased government expenditures, but did not change tax rates to counteract the recession.

In this paper, we build, calibrate, and simulate a neo-classical growth model with search and matching frictions, a productive government sector, and different categories of taxes to evaluate the impact of changes in fiscal policies on labor market variables in the 1990s of Japan. The paths of TFP, maximum hours worked, government wage, government consumption, and government investment are fed into the deterministic simulations of our model.

We find that of the changes in spending implemented by the government, the one that had the largest impact in controlling unemployment was the increase in government consumption. Had government consumption remained constant at the pre-Lost Decade level, the unemployment rate would have been 20.45% higher by the year 2002. Government wage expenditure played a very small role due to its limited increase and had it remained fixed at the 1991 level, the unemployment rate would have only increased by 1.55%. On the other hand, the drop in government investment expenditures did have an effect in unemployment, which would have been 2.59% lower had investment spending remained at the level of 1991. We find, however, that had the government expenditures been allocated more toward investment, the unemployment rate would have been even further controlled. We also find that if the Japanese government had also used countercyclical policies in terms of taxes, unemployment would have been lower than it ended up being. Reducing the labor, capital or consumption taxes by 10% would have reduced the unemployment rate in 2002 by 14.90%, 7.83%, and 0.34%, respectively. The intuition for the effects of these countercyclical fiscal polices on unemployment has to do with the increase in production and profitability of firms when these policies are implemented and their subsequent drop in job separation and increase in vacancy posting.

Our study, therefore, shows that the implemented countercyclical fiscal policies did contribute to reducing the unemployment rate during the 1990s in Japan, although both increases in government spending and reductions in taxes could have been employed as policy instruments to cushion the labor market during the Lost Decade.

NOTES

1. According to the IMF Fiscal Monitor, the share of government spending in GDP for the Euro Area increased from 46.8% in 2007 to 52.3% in 2009 [Tanzi (2015)].
2. Note that when we mention the shares of government wage, investment, and consumption later in this paper, we mean the shares of government wage, investment, and consumption in GDP, rather than those in total government spending.
3. Here, we follow Esteban-Pretel et al. (2010), where the Lost Decade is defined as 1991 to 2002 because the TFP and main labor market variables either stabilize or reverse their trend after 2002.

4. We detrend output at 2% annual rate, which is the growth rate of the world's leading economy, the USA, over the 20th century, and is the one used by Hayashi and Prescott (2002).

5. For instance, the consumption tax rate was raised from 0.03 to 0.05 in April 1997.

6. χ_t is used in the model to ensure that in equilibrium, unemployed workers find it optimal to take jobs in the public sector.

7. Note that the parameter governing the disutility of working in the public sector, χ_t, is allowed to vary so that in equilibrium, an unemployed worker always finds in optimal to take a job in the public sector. In particular, we assume that an unemployed worker is ex-ante indifferent between being employed in the private or the public sectors, which in equilibrium implies that the value for a worker of being employed in either sector is the same, or $\tilde{N}_t^p - U_t = N_t^g - U_t$, where $\tilde{N}_t^p$ is the ex-ante average value of a private job for a worker $\tilde{N}_t^p = \frac{1}{F(\bar{x}_t)} \int_{x_{min}}^{\bar{x}_t} N_t^p(x_t)\, dF(x_t)$. This condition ensures that $(N_t^g - U_t)$ is always positive in equilibrium and an unemployed worker will always be willing to take a job in the public sector.

8. The average wage in the private sector is defined as $w_t^p = \frac{1}{F(\bar{x}_t)} \int_{x_{min}}^{\bar{x}_t} w_t^p(x_t)\, dF(x_t)$.

9. Here, $\bar{\kappa}$ is the average growth rate of TFP, a_t^p and a_t^g, along the balanced growth path.

10. Although the consumption tax rate was increased from 0.03 to 0.05 in 1997, we choose $\tau_c = 0.03$ as the value used in our benchmark calibration. Our results are also robust to alternative values of consumption tax, such as $\tau_c = 0.05$.

11. For Japan, Esteban-Pretel et al. (2011) show that the quarterly separation rate in the private sector before the Lost Decade was about 3 times higher than that of the public sector (0.012 versus 0.0039). Fontaine et al. (2020) find that the separation rate of private sector workers is also around 3 times as large as those employed by the government in countries such as France, the UK, Spain, and the USA.

While our calibration strategy does not explicitly target the separation rate of the private sector, which in the model is $1 - F(\bar{x}_t)$, the baseline simulation does a good job at delivering a separation rate for the private sector of similar magnitude to that observe in the data (the initial steady-state private separation rate is 0.011, which is very close to the empirical one of 0.012).

12. The model is simulated using the equations exhibited in Appendix D, the calibrated parameters in our previous subsection, as well as the paths of exogenous variables that we explained in this subsection. The simulations were performed using the Dynare package with MATLAB, version 4.5.6.

13. TFP growth data are an extended version of the one used by Braun et al. (2006).

14. Note that for our model to be consistent with the data, in calibration and simulation, we are targeting the ratios of government wage, consumption, and investment to what can be called aggregate private output in the model, which is, $GDP - w_t^g n_t^g h_t^g$.

15. We employ the share of government wage expenditure in the OECD data for Japan to disaggregate the raw government consumption expenditure found in the system of national accounts table into two components: government wage expenditure and government consumption expenditure. We are grateful to Bermperoglou Dimitrios, Pappa Evi, and Vella Eugenia for sharing the disaggregated time series of Japanese government expenditures with us. Their source for the disaggregated time series for the different government expenditure components is the OECD Economic Outlook N.90, whose official webpage is http://www.oecd-ilibrary.org/economics/data/oecd-economic-outlook-statistics-and-projections_eo-data-e.

16. Note that the simulation is performed at quarterly frequency, but the results are displayed aggregated annually, since we are interested in the long-term transition of the variables and not in its short-run fluctuations. This implies, however, that for some variables, such as for instance unemployment, the calibration target is the 1991q1 level, but the simulation shows the average for 1991. This implies that the data and simulation results in Figures 4 and 5 may differ for the initial year, despite the fact that they were both the same at their 1991Q1 values.

17. It is worth noting that the drop in the unemployment rate when all government expenditures changes were allocated to government investment is primarily due to the model assumption that public capital enters the production function of the private sector. We believe that this is a reasonable assumption if one takes into account all the infrastructure investments done by the government and

how beneficial they are to the private sector. However, in a version of the model where γ in the private production function is set to zero, and therefore, public capital does not affect the private sector production, the effect of the counterfactual where all government expenditures are allocated to government investment does not display a lower path of the unemployment rate over the 1990s. The other results in the paper do not qualitatively change under the assumption that γ in the private production function is equal to zero. This alternative counterfactual experiment is available from the authors upon request.

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A DATA SOURCE AND DESCRIPTION

SOURCES OF DATA

The data employed in our study comes from five sources: (I) System of National Accounts (SNA), Cabinet Office, Japan; (II) Labor Force Survey (LFS), Statistics Bureau, Ministry of Internal Affairs and Communications, Japan; (III) Data for Hayashi and Prescott, "The 1990s in Japan: A Lost Decade"; (IV) The data file that accompanies the published paper Esteban-Pretel et al. (2010); (V) OECD Economic Outlook N.90.

CONSTRUCTION OF DISAGGREGATED GOVERNMENT SPENDINGS

We acknowledge Bermperoglou Dimitrios (Universitat Autònoma de Barcelona, UAB), Pappa Evi (European University Institute), and Vella Eugenia (European University Institute) for sharing the disaggregated time series of Japanese government expenditures with us. They obtained these disaggregated time series of different government expenditures from OECD Economic Outlook N.90, whose official webpage is http://www.oecd-ilibrary.org/economics/data/oecd-economic-outlook-statistics-and-projectionseo-data-e.

We employ the share of government wage expenditure in the OECD data for Japan in order to disaggregate the raw government consumption expenditure from the system of national accounts table into two components: the government wage expenditure and government consumption expenditure.

B PROOF OF KEY EQUATIONS

PROOF OF EQUATION (12)

The Lagrangian of the household's problem is

$$\mathcal{L} = \sum_{j=0}^{\infty}\beta^j \left\{ \frac{\left\{\left[\phi\left(c_{t+j}^p\right)^{\frac{\zeta-1}{\zeta}} + (1-\phi)\left(c_{t+j}^g\right)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}}\right\}^{1-\sigma} - 1}{1-\sigma} \right.$$
$$+ \psi\left(n_t^p + n_t^g\right)\log\left(1-\bar{h}_t\right) + \psi n_t^p \log\left(\bar{h}_t - h_t^p\right) + \psi\chi_t n_t^g \log\left(\bar{h}_t - h_t^g\right)$$
$$+ \varphi_{t+j}\left[(1-\tau_n)\left(W_{t+j}^p + W_{t+j}^g\right) + u_{t+j}s_{t+j}z_{t+j} + \left(1-\delta^p\right)K_{t+j}^p\right.$$
$$\left.\left. + r_{t+j}^p K_{t+j}^p - \tau_k\left(r_{t+j}^p - \delta^p\right)K_{t+j}^p + \Pi_{t+j} - T_{t+j} - (1+\tau_c)c_{t+j}^p - K_{t+1+j}^p\right]\right\}. \quad \textbf{(B.1)}$$

The first-order conditions are characterized by

$$c_t^p: \left[(\dot{c}_t)^{\frac{\zeta}{\zeta-1}}\right]^{-\sigma}(\dot{c}_t)^{\frac{1}{\zeta-1}}\phi\left(c_t^p\right)^{\frac{-1}{\zeta}} = \varphi_t(1+\tau_c), \quad \textbf{(B.2)}$$

$$K_{t+1}^p: \varphi_t = \beta\varphi_{t+1}\left[1 + (1-\tau_k)\left(r_{t+1}^p - \delta^p\right)\right], \quad \textbf{(B.3)}$$

where

$$\dot{c}_t = \phi\left(c_t^p\right)^{\frac{\zeta-1}{\zeta}} + (1-\phi)\left(c_t^g\right)^{\frac{\zeta-1}{\zeta}}. \tag{B.4}$$

PROOF OF EQUATION (25) AND EQUATION (26)

From the Nash product (24), (15), (14), (20), the FOC w.r.t. private wage $w_t^p(x_t)$ is

$$\xi\left(N_t^p(x_t) - U_t\right)^{\xi-1}\left(J_t(x_t) - V_t\right)^{1-\xi}\frac{\partial N_t^p(x_t)}{\partial w_t^p(x_t)} \tag{B.5}$$

$$= -(1-\xi)\left(N_t^p(x_t) - U_t\right)^{\xi}\left(J_t(x_t) - V_t\right)^{-\xi}\frac{\partial J_t(x_t)}{\partial w_t^p(x_t)}. \tag{B.6}$$

As $\frac{\partial N_t^p(x_t)}{\partial w_t^p(x_t)} = (1-\tau_n)\, h_t^p$, $\frac{\partial J_t(x_t)}{\partial w_t^p(x_t)} = -h_t^p$, combing (B.5) with (B.6), and canceling common terms,

$$\xi(1-\tau_n)\, J_t(x_t) = (1-\xi)\left(N_t^p(x_t) - U_t\right). \tag{B.7}$$

Therefore, the optimal sharing rules are $N_t^p(x_t) - U_t = \frac{\xi(1-\tau_n)}{1-\xi\tau_n} S_t(x_t)$, and $J_t(x_t) = \frac{1-\xi}{1-\xi\tau_n} S_t(x_t)$.

Since $S_t(x_t) = N_t^p(x_t) + J_t(x_t) - U_t$, combining it with (14), (15), and (20) delivers

$$\begin{aligned} S_t(x_t) = {} & y_t^p - \tau_n w_t^p(x_t)\, h_t^p - r_t^p k_t^p - x_t z_t - s_t z_t + \psi\frac{\log\left(1-\bar{h}_t\right)}{\varphi_t} \\ & + \psi\frac{\log\left(\bar{h}_t - h_t^p\right)}{\varphi_t} + \beta\frac{\varphi_{t+1}}{\varphi_t}\int_{x_{min}}^{\bar{x}_{t+1}} S_{t+1}(x_{t+1})\, dF(x_{t+1}) \\ & - \beta\frac{\varphi_{t+1}}{\varphi_t}\left[p_t^p\int_{x_{min}}^{\bar{x}_{t+1}}\left(N_{t+1}^p(x_{t+1}) - U_{t+1}\right) dF(x_{t+1}) + p_t^g\left(N_{t+1}^g - U_{t+1}\right)\right]. \end{aligned} \tag{B.8}$$

Employing the optimal sharing rule and rearranging yield

$$\begin{aligned} S_t(x_t) = {} & y_t^p - \tau_n w_t^p(x_t)\, h_t^p - r_t^p k_t^p - x_t z_t - s_t z_t + \psi\frac{\log\left(1-\bar{h}_t\right)}{\varphi_t} + \psi\frac{\log\left(\bar{h}_t - h_t^p\right)}{\varphi_t} \\ & + \beta\frac{\varphi_{t+1}}{\varphi_t}\left(1 - p_t^p\xi\frac{1-\tau_n}{1-\xi\tau_n}\right)\int_{x_{min}}^{\bar{x}_{t+1}} S_{t+1}(x_{t+1})\, dF(x_{t+1}) - \beta\frac{\varphi_{t+1}}{\varphi_t}p_t^g\left(N_{t+1}^g - U_{t+1}\right). \end{aligned} \tag{B.9}$$

PROOF OF EQUATIONS (27) AND (28)

From the optimal sharing rule $N_t^p(x_t) - U_t = \frac{\xi(1-\tau_n)}{1-\xi\tau_n} S_t(x_t)$, the value equations (14), (15), and the surplus (B.9), we have

$$\begin{aligned} & (1-\tau_n)\, w_t^p(x_t)\, h_t^p + \psi\frac{\log\left(1-\bar{h}_t\right)}{\varphi_t} + \psi\frac{\log\left(\bar{h}_t - h_t^p\right)}{\varphi_t} - s_t z_t \\ & + \beta\frac{\varphi_{t+1}}{\varphi_t}\left(1-p_t^p\right)\frac{\xi(1-\tau_n)}{1-\xi\tau_n}\int_{x_{min}}^{\bar{x}_{t+1}} S_{t+1}(x_{t+1})\, dF(x_{t+1}) - \beta\frac{\varphi_{t+1}}{\varphi_t}p_t^g\left(N_{t+1}^g - U_{t+1}\right) \end{aligned}$$

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$$= \frac{\xi(1-\tau_n)}{1-\xi\tau_n}\left[y_t^p - \tau_n w_t^p(x_t)\, h_t^p - r_t^p k_t^p - x_t z_t - s_t z_t + \psi\frac{\log\left(1-\bar{h}_t\right)}{\varphi_t} + \psi\frac{\log\left(\bar{h}_t - h_t^p\right)}{\varphi_t}\right.$$
$$\left. + \beta\frac{\varphi_{t+1}}{\varphi_t}\left(1 - p_t^p \xi\frac{1-\tau_n}{1-\xi\tau_n}\right)\int_{x_{min}}^{\bar{x}_{t+1}} S_{t+1}(x_{t+1})\, dF(x_{t+1}) - \beta\frac{\varphi_{t+1}}{\varphi_t}p_t^g\left(N_{t+1}^g - U_{t+1}\right)\right]. \tag{B.10}$$

Combining terms and rearranging gives

$$w_t^p(x_t)\, h_t^p$$
$$= \xi\left[y_t^p - r_t^p k_t^p - x_t z_t + \beta\frac{\varphi_{t+1}}{\varphi_t}p_t^p\frac{1-\xi}{1-\xi\tau_n}\int_{x_{min}}^{\bar{x}_{t+1}} S_{t+1}(x_{t+1})\, dF(x_{t+1})\right]$$
$$+ \frac{1-\xi}{1-\tau_n}\left[s_t z_t - \psi\frac{\log\left(1-\bar{h}_t\right)}{\varphi_t} - \psi\frac{\log\left(\bar{h}_t - h_t^p\right)}{\varphi_t} + \beta\frac{\varphi_{t+1}}{\varphi_t}p_t^g\left(N_{t+1}^g - U_{t+1}\right)\right]. \tag{B.11}$$

Employing the free entry condition and rearranging delivers equation (27)

$$w_t^p(x_t)\, h_t^p = \xi\left[y_t^p - r_t^p k_t^p - x_t z_t + \frac{p_t^p}{q_t}\iota^p z_t\right]$$
$$+ (1-\xi)\,\frac{1}{1-\tau_n}\left[s_t z_t - \psi\frac{\log\left(1-\bar{h}_t\right)}{\varphi_t} - \psi\frac{\log\left(\bar{h}_t - h_t^p\right)}{\varphi_t}\right.$$
$$\left. + \beta\frac{\varphi_{t+1}}{\varphi_t}p_t^g\left(N_{t+1}^g - U_{t+1}\right)\right]. \tag{B.12}$$

The FOC with respect to private hours worked h_t^p is

$$\xi\left(N_t^p(x_t) - U_t\right)^{\xi-1}(J_t(x_t) - V_t)^{1-\xi}\,\frac{\partial N_t^p(x_t)}{\partial h_t^p} \tag{B.13}$$
$$= -(1-\xi)\left(N_t^p(x_t) - U_t\right)^{\xi}(J_t(x_t) - V_t)^{-\xi}\,\frac{\partial J_t(x_t)}{\partial h_t^p}, \tag{B.14}$$

which simplifying delivers

$$\xi J_t(x_t)\,\frac{\partial N_t^p(x_t)}{\partial h_t^p} = -(1-\xi)\left(N_t^p(x_t) - U_t\right)\frac{\partial J_t(x_t)}{\partial h_t^p}. \tag{B.15}$$

Finally, employing (25), the value equations (20), and (15), we get equation (28)

$$\frac{1}{1-\tau_n}\psi\frac{\left(\bar{h}_t - h_t^p\right)^{-1}}{\varphi_t} = a_t^p f_{h_t^p}\left(k_t^p, h_t^p, K_t^g\right). \tag{B.16}$$

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C THE NON-STATIONARY EQUILIBRIUM

HOUSEHOLD OPTIMAL CONDITIONS

FOC w.r.t. private consumption:

$$\left[\phi\left(c_t^p\right)^{\frac{\zeta-1}{\zeta}} + (1-\phi)\left(c_t^g\right)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{1-\sigma\zeta}{\zeta-1}} \phi\left(c_t^p\right)^{\frac{-1}{\zeta}} = \varphi_t(1+\tau_c)\,. \tag{C.1}$$

FOC w.r.t. private saving:

$$\beta\varphi_{t+1}\left[1+(1-\tau_k)\left(r_{t+1}^p-\delta^p\right)\right] = \varphi_t. \tag{C.2}$$

Aggregate resource constraint:

$$Y_t = c_t^p + K_{t+1}^p - (1-\delta^p)K_t^p + v_t^p\iota^p z_t + n_t^p x_t z_t - u_t s_t z_t + I_t^g + g_t^g. \tag{C.3}$$

FIRM EQUATIONS

Aggregate output:

$$Y_t = n_t^p a_t^p\left(K_t^g\right)^{\gamma}\left(k_t^p\right)^{\alpha}\left(h_t^p\right)^{1-\alpha}. \tag{C.4}$$

Aggregate private physical capital:

$$K_t^p = n_t^p k_t^p. \tag{C.5}$$

Optimal physical capital rental rate for the private sector firm:

$$r_t^p = \alpha a_t^p\left(K_t^g\right)^{\gamma}\left(k_t^p\right)^{\alpha-1}\left(h_t^p\right)^{1-\alpha}. \tag{C.6}$$

Average intermediate input cost in productive matches:

$$x_t^a = \frac{1}{F(\bar{x}_t)}\int_{x_{min}}^{\bar{x}_t} x_t dF(x_t)\,. \tag{C.7}$$

LABOR MARKET CONDITIONS

Optimal private sector wages:

$$w_t^p(x_t)\,h_t^p = \xi\left[y_t^p - r_t^p k_t^p - x_t z_t + \frac{p_t^p}{q_t}\iota^p z_t\right] + (1-\xi)\,\frac{1}{1-\tau_n}$$
$$\left[s_t z_t - \psi\frac{\log\left(1-\bar{h}_t\right)}{\varphi_t} - \psi\frac{\log\left(\bar{h}_t-h_t^p\right)}{\varphi_t} + \beta\frac{\varphi_{t+1}}{\varphi_t}p_t^g\Delta_{t+1}^g\right]. \tag{C.8}$$

where $\Delta_t^g = N_t^g - U_t$.

Optimal private sector hours worked:

$$\frac{1}{1-\tau_n}\psi\frac{\left(\bar{h}_t-h_t^p\right)^{-1}}{\varphi_t} = (1-\alpha)\,a_t^p\left(K_t^g\right)^{\gamma}\left(k_t^p\right)^{\alpha}\left(h_t^p\right)^{-\alpha}. \tag{C.9}$$

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Free entry condition:

$$0 = -\iota^p z_t + \beta \frac{\varphi_{t+1}}{\varphi_t} q_t \frac{1-\xi}{1-\xi\tau_n} \int_{x_{min}}^{\bar{x}_{t+1}} S_{t+1}(x_{t+1})\, dF(x_{t+1})\,. \tag{C.10}$$

Destruction threshold:

$$\begin{aligned}\bar{x}_t z_t = {}& y_t^p - \tau_n w_t^p(\bar{x}_t)\, h_t^p - r_t^p k_t^p - s_t z_t + \psi \frac{\log\left(1-\bar{h}_t\right)}{\varphi_t} + \psi \frac{\log\left(\bar{h}_t - h_t^p\right)}{\varphi_t} \\ &+\beta \frac{\varphi_{t+1}}{\varphi_t}\left(1 - p_t^p \xi \frac{1-\tau_n}{1-\xi\tau_n}\right) \int_{x_{min}}^{\bar{x}_{t+1}} S_{t+1}(x_{t+1})\, dF(x_{t+1}) - \beta \frac{\varphi_{t+1}}{\varphi_t} p_t^g \Delta_{t+1}^g.\end{aligned} \tag{C.11}$$

Evolution of unemployment:

$$u_t = \left[1 - p_{t-1}^p F(\bar{x}_t) - p_{t-1}^g\right] u_{t-1} + [1 - F(\bar{x}_t)]\, n_{t-1}^p + \lambda^g n_{t-1}^g. \tag{C.12}$$

The probability of leaving unemployment:

$$p_t^{ue} = p_{t-1}^p F(\bar{x}_t) + p_{t-1}^g. \tag{C.13}$$

Evolution of government employment:

$$n_t^g = m_{t-1} \frac{v_{t-1}^g}{v_{t-1}} + (1-\lambda^g)\, n_{t-1}^g. \tag{C.14}$$

Evolution of private employment:

$$n_t^p = 1 - n_t^g - u_t. \tag{C.15}$$

Labor market tightness:

$$\theta_t = \frac{v_t^p + v_t^g}{u_t}. \tag{C.16}$$

The matching function:

$$m_t = \eta (u_t)^\mu (v_t)^{1-\mu}\,. \tag{C.17}$$

Private job finding probability:

$$p_t^p = \frac{m_t}{u_t} \frac{v_t^p}{v_t}. \tag{C.18}$$

Government job finding probability:

$$p_t^g = \frac{m_t}{u_t} \frac{v_t^g}{v_t}. \tag{C.19}$$

Government employment value premium:

$$\begin{aligned}\Delta_t^g = {}& (1-\tau_n)\, w_t^g h_t^g + \psi \frac{\log\left(1-\bar{h}_t\right)}{\varphi_t} + \psi \chi_t \frac{\log\left(\bar{h}_t - h_t^g\right)}{\varphi_t} - s_t z_t \\ &+\beta \frac{\varphi_{t+1}}{\varphi_t}\left[\left(1-\lambda^g - p_t^g\right) \Delta_{t+1}^g - p_t^p \frac{\xi(1-\tau_n)}{1-\xi\tau_n} \int_{x_{min}}^{\bar{x}_{t+1}} S_{t+1}(x_{t+1})\, dF(x_{t+1})\right].\end{aligned} \tag{C.20}$$

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The joint surplus:

$$S_t(x_t) = y_t^p - \tau_n w_t^p(x_t)\, h_t^p - r_t^p k_t^p - x_t z_t - s_t z_t + \psi \frac{\log\left(1-\bar{h}_t\right)}{\varphi_t} + \psi \frac{\log\left(\bar{h}_t - h_t^p\right)}{\varphi_t}$$
$$+\beta \frac{\varphi_{t+1}}{\varphi_t}\left(1 - p_t^p \xi \frac{1-\tau_n}{1-\xi\tau_n}\right) \int_{x_{min}}^{\bar{x}_{t+1}} S_{t+1}(x_{t+1})\, dF(x_{t+1}) - \beta \frac{\varphi_{t+1}}{\varphi_t} p_t^g \Delta_{t+1}^g. \tag{C.21}$$

The cumulative distribution function of the non-productive idiosyncratic input cost:

$$F(\bar{x}_t) = 1 - e^{-\frac{\bar{x}_t}{\rho}}. \tag{C.22}$$

GOVERNMENT EQUATIONS

Government production:

$$c_t^g = a_t^g \left(K_t^g\right)^\gamma \left(n_t^g h_t^g\right)^{1-\gamma} - \iota^g v_t^g. \tag{C.23}$$

Public physical capital evolution:

$$K_{t+1}^g = (1-\delta^g)\, K_t^g + I_t^g. \tag{C.24}$$

Government budget constraint:

$$w_t^g n_t^g h_t^g + I_t^g + g_t^g = \tau_c c_t^p + \tau_n\left(W_t^p + W_t^g\right) + \tau_k\left(r_t^p - \delta^p\right) K_t^p + T_t. \tag{C.25}$$

Government sector wage:

$$w_t^g = (1+\pi^g)\, w_t^p. \tag{C.26}$$

Government hours worked:

$$h_t^g = GWE_t / \left(w_t^g n_t^g\right). \tag{C.27}$$

D THE STATIONARY EQUILIBRIUM

In this section, we write down the stationary equilibrium conditions that characterize the model. Define κ_t as the growth rate of TFP, that is, $e^{\kappa_t} = \frac{a_t^p}{a_{t-1}^p} = \frac{a_t^g}{a_{t-1}^g}$, and the TFP factor as $\left(a_t^p\right)^{\frac{1}{1-\alpha}}$, where we assume that the technology in the government sector grows at the same rate as TFP to guarantee the existence of the balanced growth path.

To get the stationary equilibrium, we employ z_t, a variable that grows at the average growth rate of the TFP factor along the balanced growth path, $\frac{\bar{\kappa}}{1-\alpha}$. The growing variables in the economy can be detrended as stationary by dividing them by z_t as follows:

$$\tilde{Y}_t = \frac{Y_t}{z_t}, \tilde{K}_t^p = \frac{K_t^p}{z_t}, \tilde{c}_t^p = \frac{c_t^p}{z_t}, \tilde{c}_t^g = \frac{c_t^g}{z_t}, \tilde{k}_t^p = \frac{k_t^p}{z_t}, \tilde{y}_t^p = \frac{y_t^p}{z_t},$$
$$\tilde{K}_t^g = \frac{K_t^g}{z_t}, \tilde{I}_t^g = \frac{I_t^g}{z_t}, \tilde{g}_t^g = \frac{g_t^g}{z_t}, \tilde{w}_t^g = \frac{w_t^g}{z_t}, \tilde{\varphi}_t = \frac{\varphi_t}{z_t}.$$

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We assume that technology is labor augmenting, and the production function of the private sector firm is specified as $y_t^p = a_t^p \left(K_t^g\right)^\gamma \left(k_t^p\right)^\alpha \left(h_t^p\right)^{1-\alpha}$. We define the detrended TFP as $\tilde{a}_t^p \equiv \frac{a_t^p}{z_t^{1-\alpha}} = \tilde{a}_{t-1}^p e^{(\kappa_t - \bar{\kappa})}$. In the simulations, we normalize the initial technology to be one, that is, $a_0^p = \tilde{a}_0^p = 1$ and $a_0^g = \tilde{a}_0^g = 1$.

Define the stationary average surplus, average idiosyncratic productivity, average wage, and public sector working premium, conditional on being productive, as $\tilde{S}_t = \frac{1}{F(\bar{x}_t)} \frac{1}{z_t} \int_{x_{min}}^{\bar{x}_t} S_t(x_t)\, dF(x_t)$, $x_t^a = \frac{1}{F(\bar{x}_t)} \int_{x_{min}}^{\bar{x}_t} x_t dF(x_t)$, $\tilde{w}_t^p = \frac{1}{F(\bar{x}_t)} \frac{1}{z_t} \int_{x_{min}}^{\bar{x}_t} w_t^p(x_t)\, dF(x_t)$, $\tilde{\Delta}_t^g = \frac{1}{z_t}\left(N_t^g - U_t\right)$.

We can define a perfect foresight stationary competitive equilibrium, for a given path of exogenous TFP growth rate $\{\tilde{a}_t^p, \tilde{a}_t^g\}_{t=0}^{\infty}$, government policy $\{\tau_c, \tau_n, \tau_k, \tilde{W}_t^g, \tilde{I}_t^p, \tilde{g}_t^g, \pi_t^g, h_t^g, \bar{h}_t\}_{t=0}^{\infty}$, and K_0, as $\{\tilde{c}_t^p, \tilde{c}_t^g, \tilde{K}_{t+1}^p, \tilde{K}_t^g, h_t^p, \tilde{k}_t^p, n_t^p, n_t^g, m_t^p, m_t^g, p_t^p, p_t^g, p_t^{ue}, u_t, v_t^p, v_t^g, \theta_t, \bar{x}_t, \chi_t, x_t^a, \tilde{Y}_t, \tilde{S}_t, \tilde{\varphi}_t, r_t^p, \tilde{w}_t^p, \tilde{w}_t^g, \tilde{T}_t, \tilde{\Delta}_t^g, F(\bar{x}_t)\}_{t=0}^{\infty}$ which satisfy:

HOUSEHOLD OPTIMAL CONDITIONS

FOC w.r.t. private consumption:

$$\left[\phi\left(\tilde{c}_t^p\right)^{\frac{\zeta-1}{\zeta}} + (1-\phi)\left(\tilde{c}_t^g\right)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{1-\sigma\zeta}{\zeta-1}} \phi\left(\tilde{c}_t^p\right)^{\frac{-1}{\zeta}} = \tilde{\varphi}_t(1+\tau_c)\,. \quad \textbf{(D.1)}$$

FOC w.r.t. private saving:

$$\beta\frac{\tilde{\varphi}_{t+1}}{\tilde{\varphi}_t} e^{-\frac{\bar{\kappa}}{1-\alpha}}\left[1+(1-\tau_k)\left(r_{t+1}^p - \delta^p\right)\right] = 1. \quad \textbf{(D.2)}$$

Aggregate resource constraint:

$$\tilde{Y}_t = \tilde{c}_t^p + \tilde{K}_{t+1}^p e^{\frac{\bar{\kappa}}{1-\alpha}} - (1-\delta^p)\tilde{K}_t^p + v_t^p \iota^p + n_t^p x_t^a - u_t s_t + \tilde{I}_t^g + \tilde{g}_t^g. \quad \textbf{(D.3)}$$

FIRM EQUATIONS

Aggregate output:

$$\tilde{Y}_t = \tilde{a}_{t-1}^p e^{(\kappa_t - \bar{\kappa})} n_t^p \left(\tilde{K}_t^g\right)^\gamma \left(\tilde{k}_t^p\right)^\alpha \left(h_t^p\right)^{1-\alpha}. \quad \textbf{(D.4)}$$

Aggregate private physical capital:

$$\tilde{K}_{t-1}^p e^{-\frac{\bar{\kappa}}{1-\alpha}} = n_t^p \tilde{k}_t^p. \quad \textbf{(D.5)}$$

Optimal physical capital rental rate for the private sector firm:

$$r_t^p = \alpha \tilde{a}_{t-1}^p e^{(\kappa_t - \bar{\kappa})} \left(\tilde{K}_t^g\right)^\gamma \left(\tilde{k}_t^p\right)^{\alpha-1} \left(h_t^p\right)^{1-\alpha}. \quad \textbf{(D.6)}$$

Average intermediate input cost in productive matches:

$$x_t^a = \frac{1}{F(\bar{x}_t)} \int_{x_{min}}^{\bar{x}_t} x_t dF(x_t)\,. \quad \textbf{(D.7)}$$

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LABOR MARKET CONDITIONS

Optimal private sector wages:

$$\tilde{w}_t^p h_t^p = \xi \left[\tilde{a}_{t-1}^p e^{(\kappa_t - \bar{\kappa})} \left(\tilde{K}_t^g\right)^{\gamma} \left(\tilde{k}_t^p\right)^{\alpha} \left(h_t^p\right)^{1-\alpha} - r_t^p \tilde{k}_t^p - x_t^a + \frac{p_t^p}{q_t} \iota^p \right] + (1-\xi) \frac{1}{1-\tau_n} \left[s_t - \psi \frac{\log\left(1-\bar{h}_t\right)}{\tilde{\varphi}_t} - \psi \frac{\log\left(\bar{h}_t - h_t^p\right)}{\tilde{\varphi}_t} + \beta \frac{\tilde{\varphi}_{t+1}}{\tilde{\varphi}_t} p_t^g \tilde{\Delta}_{t+1}^g \right]. \tag{D.8}$$

Optimal private sector hours worked:

$$\frac{1}{1-\tau_n} \psi \frac{\left(\bar{h}_t - h_t^p\right)^{-1}}{\tilde{\varphi}_t} = (1-\alpha)\, \tilde{a}_{t-1}^p e^{(\kappa_t - \bar{\kappa})} \left(\tilde{K}_t^g\right)^{\gamma} \left(\tilde{k}_t^p\right)^{\alpha} \left(h_t^p\right)^{-\alpha}. \tag{D.9}$$

Creation condition:

$$0 = -\iota^p + \beta \frac{\tilde{\varphi}_{t+1}}{\tilde{\varphi}_t} q_t \frac{1-\xi}{1-\xi\tau_n} \tilde{S}_{t+1} F\left(\bar{x}_{t+1}\right). \tag{D.10}$$

Destruction threshold:

$$\bar{x}_t = \tilde{a}_{t-1}^p e^{(\kappa_t - \bar{\kappa})} \left(\tilde{K}_t^g\right)^{\gamma} \left(\tilde{k}_t^p\right)^{\alpha} \left(h_t^p\right)^{1-\alpha} - \tau_n \tilde{w}_t^p h_t^p - r_t^p \tilde{k}_t^p - s_t + \psi \frac{\log\left(1-\bar{h}_t\right)}{\tilde{\varphi}_t} + \psi \frac{\log\left(\bar{h}_t - h_t^p\right)}{\tilde{\varphi}_t} + \beta \frac{\tilde{\varphi}_{t+1}}{\tilde{\varphi}_t} \left(1 - p_t^p \xi \frac{1-\tau_n}{1-\xi\tau_n}\right) \tilde{S}_{t+1} F(\bar{x}_{t+1}) - \beta \frac{\tilde{\varphi}_{t+1}}{\tilde{\varphi}_t} p_t^g \tilde{\Delta}_{t+1}^g. \tag{D.11}$$

Evolution of unemployment:

$$u_t = \left(1 - p_t^{ue}\right) u_{t-1} + \left[1 - F(\bar{x}_t)\right] n_{t-1}^p + \lambda^g n_{t-1}^g. \tag{D.12}$$

The probability of leaving unemployment:

$$p_t^{ue} = p_{t-1}^p F(\bar{x}_t) + p_{t-1}^g. \tag{D.13}$$

Evolution of government employment:

$$n_t^g = m_{t-1} \frac{v_{t-1}^g}{v_{t-1}} + \left(1 - \lambda^g\right) n_{t-1}^g. \tag{D.14}$$

Evolution of private employment:

$$n_t^p = 1 - n_t^g - u_t. \tag{D.15}$$

Market tightness:

$$\theta_t = \frac{v_t^p + v_t^g}{u_t}. \tag{D.16}$$

The matching function:

$$m_t = \eta (u_t)^{\mu} (v_t)^{1-\mu}. \tag{D.17}$$

Private job finding probability:

$$p_t^p = \frac{m_t}{u_t} \frac{v_t^p}{v_t}. \tag{D.18}$$

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Government job finding probability:

$$p_t^g = \frac{m_t}{u_t}\frac{v_t^g}{v_t}. \tag{D.19}$$

Government employment value premium:

$$\tilde{\Delta}_t^g = (1-\tau_n)\,\tilde{w}_t^g h_t^g + \psi\,\frac{\log\left(1-\bar{h}_t\right)}{\tilde{\varphi}_t} + \psi\,\chi_t\frac{\log\left(\bar{h}_t - h_t^g\right)}{\tilde{\varphi}_t} - s_t$$
$$+\beta\,\frac{\tilde{\varphi}_{t+1}}{\tilde{\varphi}_t}\left[\left(1-\lambda^g-p_t^g\right)\tilde{\Delta}_{t+1}^g - p_t^p\,\frac{\xi(1-\tau_n)}{1-\xi\tau_n}\tilde{S}_{t+1}F(\bar{x}_{t+1})\right]. \tag{D.20}$$

The joint surplus:

$$\tilde{S}_t = \tilde{y}_t^p - \tau_n\tilde{w}_t^p h_t^p - r_t^p\tilde{k}_t^p - x_t - s_t + \psi\,\frac{\log\left(1-\bar{h}_t\right)}{\tilde{\varphi}_t} + \psi\,\frac{\log\left(\bar{h}_t - h_t^p\right)}{\tilde{\varphi}_t}$$
$$+\beta\,\frac{\tilde{\varphi}_{t+1}}{\tilde{\varphi}_t}\left(1-p_t^p\xi\,\frac{1-\tau_n}{1-\xi\tau_n}\right)\tilde{S}_{t+1}F(\bar{x}_{t+1}) - \beta\,\frac{\tilde{\varphi}_{t+1}}{\tilde{\varphi}_t}p_t^g\,\tilde{\Delta}_{t+1}^g. \tag{D.21}$$

The cumulative distribution function of the non-productive idiosyncratic input cost:

$$F(\bar{x}_t) = 1 - e^{-\frac{\bar{x}_t}{\rho}}. \tag{D.22}$$

GOVERNMENT EQUATIONS

Government production:

$$\tilde{c}_t^g = \tilde{a}_{t-1}^g e^{(\kappa_t-\bar{\kappa})}\left(\tilde{K}_t^g\right)^{\gamma}\left(n_t^g h_t^g\right)^{1-\gamma} - \iota^g v_t^g. \tag{D.23}$$

Public physical capital evolution:

$$\tilde{K}_{t+1}^g e^{\frac{\bar{\kappa}}{1-\alpha}} = (1-\delta^g)\,\tilde{K}_t^g + \tilde{I}_t^g. \tag{D.24}$$

Government budget constraint:

$$\tilde{w}_t^g n_t^g h_t^g + \tilde{I}_t^g + \tilde{g}_t^g = \tau_n c_t^p + \tau_n\left(\tilde{w}_t^p n_t^p h_t^p + \tilde{w}_t^g n_t^g h_t^g\right) + \tau_k\left(r_t^p - \delta^p\right)\tilde{K}_t^p + \tilde{T}_t. \tag{D.25}$$

Government sector wage:

$$\tilde{w}_t^g = (1+\pi^g)\,\tilde{w}_t^p. \tag{D.26}$$

Government hours worked:

$$h_t^g = GWE_t/\left(\tilde{w}_t^g n_t^g\right). \tag{D.27}$$

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