TOPOLOGICAL MAGNETIC FLUX PUMPING REVISITED

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ABSTRACT

Calculations by Drobyshevski and Yuferev (1974) suggest that magnetic flux is 'topologically pumped' to the bottom of a convecting Benard layer as the electrical conductivity is increased. This is shown to be mistaken: however, compressible flows where motion is mainly around field-lines can confine flux near their base.

INTRODUCTION

Observations of the Sun reveal a magnetic field heavily distorted by the convecting plasma which forms its outer layers. The conditions there are typically too extreme to be duplicated in terrestrial laboratory experiments, so our understanding is much helped by the use of anlaytic and numerical techniques.

As a first approximation we neglect the dynamical back-reaction of the magnetic field on the plasma, and consider the effect of arbitrary, but hopefully realistic flows on field distribution. Rolls of incompressible fluid, where there is dependence on only two spatial coordinates, have been well studied, see e.g. the recent review by Proctor and Weiss, 1982 (PW). Fully three-dimensional calculations were performed by Drobyshevski et al. (1974 = DY, 1980).

This paper summarises a study of a number of 3-D patterns, including the original DY one, carried to more extreme values of R_m (a measure of the importance of advection relative to diffusion) than in DY. A fuller account is to be published elsewhere (Arter, 1983).

This work and a similar study by Galloway and Proctor (1983), show DY's description of high R_m behaviour to be misleading. The high-R_m regime is characterised by the production of spirals of field by flow about axes oblique, but not perpendicular, to the initial imposed field direction, and there is no significant vertical asymmetry in the absolute averaged magnetic flux distribution. However, compressible velocity patterns do produce marked asymmetry.

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BRIEF STATEMENT OF THE PROBLEM

We solve the dimensionless magnetic induction equation,

$$\frac{\partial B}{\partial t} = R_m \, \mathrm{curl}(\underline{u}_\wedge \underline{B}) + \nabla^2 \underline{B}$$

for the magnetic field $\underline{B}$; t is time, $\underline{u}$ is a given velocity pattern with rectangular planform - see DY for further details. We note here only that the flux initially imposed in, say, the x-direction is conserved. In the anelastic approximation $\mathrm{div}\{e(z)\underline{u}\}=0$ where the density $e(z)=\exp(cz)$ depends on the vertical co-ordinate z, and c is the compressibility factor. We take

$$\underline{u} = \mathrm{curl\ curl}\ (S(x,y)\sin z\ \hat{\underline{z}}) + c(\mathrm{grad}S(x,y))\sin z.$$

In DY c=0 and S=cosx + cosy + ½cosxcosy. We also consider c=-1 with S=cosx + cos2y and S=cos2x + cosy. These will be referred to as DY, $C_{\parallel}$ and $C_{\perp}$. DY has the topological property - isolated regions of ascending flow are surrounded by falling fluid - but $C_{\parallel}$ and $C_{\perp}$ are roll-like with motion chiefly in planes parallel or perpendicular to the field respectively.

Furthermore, our R_m is defined using $|u_z|$, thus the largest R_m studied here, R_m=200, corresponds to R_m=66.7 in DY. The numerical techniques also differ. This paper, after Roberts and Weiss (1966), employs a primitive variable finite difference scheme, which turns out to preserve $\mathrm{div}\underline{B}=0$. The mesh size is 24×24×24 and integration to a steady solution typically takes about 3 minutes using an ICL DAP.

SUMMARY

At lower R_m the results here agree with DY's, but for $R_m>50$, measures of horizontal flux become negative in $z>\frac{1}{2}$ (Fig.1) indicating a preponderance of reversed field. Reconnection takes place at flow cell boundaries (Fig.2) rather than inside the eddies: contrast Fig.3 with e.g. Fig.1 of PW. However, the formation of persistent isolated loops of field in Fig.3 is due to a symmetry caused by the absence of net field in the y-direction: in general reconnection will involve at least two or more field-lines producing spirals. Either way, the unsigned flux is enhanced by a factor $R_m^{1/2}$ throughout most of the layer.

The topological property of $\underline{u}$ results in the formation of a flux-tube at the cell bottom, as opposed to a sheet at the top. Thus from PW, as $R_m\to\infty$ all the magnetic energy goes to the base. It is not clear how significant this is for dynamo action.

$C_{\parallel}$ (and $C_{\perp}$) are topologically different. $C_{\parallel}$ has flow mostly around the field-lines and we find B_x stays positive. It is enhanced where $\mathrm{div}\underline{u}\approx 0$, in falling fluid, and 80% of the flux resides at the layer bottom. Interestingly in $C_{\perp}$ flux is trapped at the top by the velocity shear in x.

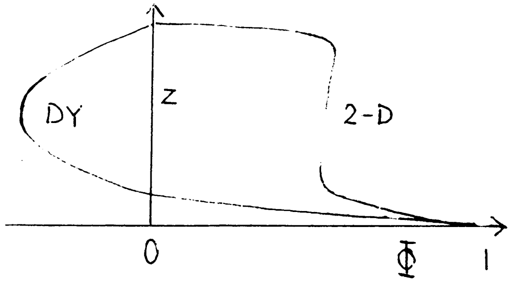

Fig.1 Sketch of $\Phi(z)=\int_V B_x\,dV$, where V is volume $1>\zeta>z$, for typical 2-D and 3-D flows, $R_m=150$.

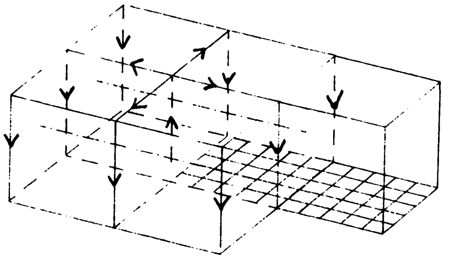

Fig.2 Schematic view of flow DY and field direction (dash-dot line).

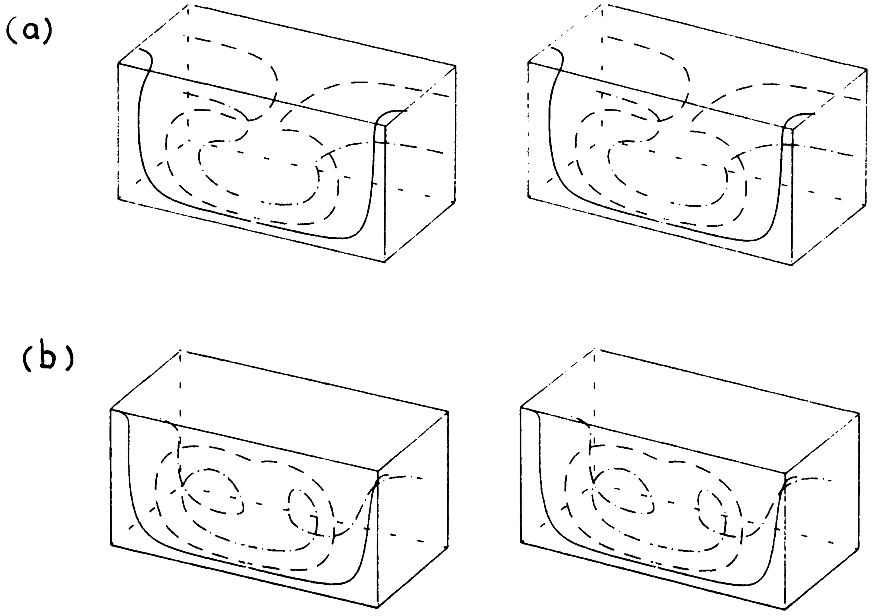

Fig.3 Stereo pairs showing field-lines reconnecting at $R_m=150$: time t increases from (a) to (b). The base is drawn hatched in Fig.2.

Now eddies in a strong dynamic magnetic field might look like $C_{\parallel}$, while others in the solar context might resemble $C_{\perp}$ due to the effect of rotation. Thus we have a mechanism which holds down strong field and expels weak field from a convective layer (cf. magnetic buoyancy). This needs further study, especially as the boundary conditions on $\underline{B}$ may strongly affect these results (Parker, 1975; Arter et al., 1983).

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REFERENCES

Arter, W.: 1983, in preparation.
Arter, W., Galloway, D.J. and Proctor, M.R.E.: 1982, submitted to Mon. Not. R. astr. Soc.
Drobyshevski, E.M., Kolesnikova, E.N. and Yuferev, V.S.: 1980, J. Fluid Mech. 101, pp.65-78.
Drobyshevski, E.M. and Yuferev, V.S.: 1974, J. Fluid Mech. 65, pp.33-44.
Galloway, D.J. and Proctor, M.R.E.: 1983, in preparation.
Parker, E.N.: 1975, Astrophys. J., 202, pp.523-527.
Proctor, M.R.E. and Weiss, N.O.: 1982, Rep. Prog. Phys., in press.
Roberts, K.V. and Weiss, N.O.: 1966, Math. Comp. 20, pp.272-299.

DISCUSSION

PROCTOR: Dr. Galloway and myself have done similar experiments only in a hexagonal geometry: we find similar results. It was not clear to me, though, how the compressibility of the field would affect the nature of the pumping in a general case.

GALLOWAY: The point of these calculations is the creation of large amounts of negative flux at the top of the layer (the input flux being positive), for high R_m. This looks nothing like the sun, where emerging flux is mainly vertical. Recently I have been looking at the effect of changing the top boundary condition so that the convecting layer lies underneath a current-free atmosphere. This implies that the *average* horizontal field has to vanish at this boundary, more like the sun. Preliminary results suggest that the flux is initially pumped to the bottom on a turnover timescale T, and subsequently decays on a longer timescale $R_m^{1/2}T$ as flux leaks out of the top (there is no steady state in this problem). So perhaps flux can be trapped at the base of the convection zone for an appreciable part of the solar cycle if the effective R_m is high enough.

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