There is no room to indicate the experimental means by which the method may be executed, nor to show how the various instruments may be introduced, their laws discovered, and constants calculated; but these matters will be fairly obvious.

Eighth Meeting, 13*th June* 1890.

R. E. ALLARDICE, Esq., M.A., Vice-President, in the Chair.

Note on the Orthomorphic Transformation of a circle into itself.

From a letter by Professor CAYLEY.

"The following is, of course, substantially well-known, but it strikes me as rather pretty :—to find the Orthomorphic transformation of the circle

$$x^2 + y^2 - 1 = 0$$

into itself. Assume this to be

$$x_1 + \iota y_1 \frac{A(x + \iota y) + B}{1 + C(x + \iota y)}.$$

Then, writing A′, B′, C′ for the conjugates of A, B, C, we have

$$x_1 - \iota y = \frac{A'(x - \iota y) + B'}{1 + C'(x - \iota y)};$$

and then

$$x_1^2 + y_1^2 = \frac{AA'(x^2 + y^2) + AB'(x + \iota y) + A'B(x - \iota y) + BB'}{1 + C(x + \iota y) + C'(x - \iota y) + CC'(x^2 + y^2)},$$

which should be an identity for $x^2 + y^2 = 1$, $x_1^2 + y_1^2 = 1$.

Evidently $C = AB'$, whence $C' = A'B$, and the equation then is

$$1 + AA'BB' = AA' + BB',$$

i.e.,

$$(1 - AA')(1 - BB') = 0.$$

But $BB' = 1$ gives the illusory result

$$x_1 + \iota y_1 = B,$$

therefore

$$1 - AA' = 0,$$

and the required solution thus is

$$x_1 + \iota y_1 = \frac{A(x + \iota y) + B}{1 + AB'(x + \iota y)};$$

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where A is a unit-vector (say $A = \cos\lambda + \iota\sin\lambda$) and B, B′ are conjugate vectors. Or, writing $B = b + \iota\beta$, $B' = b - \iota\beta$, the constants are λ, b, β; 3 constants as it should be."

Quaternion Synopsis of Hertz' View of the Electrodynamical Equations.

By Professor TAIT.

Note on Menelaus's Theorem.

By R. E. ALLARDICE, M.A.

§ 1. The object of this note is, in the first place, to show that Menelaus's Theorem, regarding the segments into which the sides of a triangle are divided by any transversal, is a particular form of the condition, in trilinear co-ordinates, for the collinearity of three points; and, in the second place, to point out an analogue of Menelaus's Theorem in space of three dimensions.

§ 2. In the usual system of areal co-ordinates, the x-co-ordinate of P (fig. 52) is $\Delta PBC/\Delta ABC$, that is PD/AD. Now let D, E, F, be three points in BC, CA, AB, respectively, dividing these sides in the ratios l_1/m_1, l_2/m_2, l_3/m_3; then the co-ordinates of D, E, F, are proportional to $(0, m_1, l_1)$, $(l_2, 0, m_2)$, $(m_3, l_3, 0)$. Hence the condition that D, E, F, lie on the straight line $Ax + By + Cz = 0$ is

$$\begin{vmatrix} 0 & m_1 & l_1 \\ l_2 & 0 & m_2 \\ m_3 & l_3 & 0 \end{vmatrix} = 0,$$

that is, $l_1 l_2 l_3 + m_1 m_2 m_3 = 0$, which is Menelaus's Theorem.

§ 3. In space of three dimensions we may use the corresponding system of tetrahedral co-ordinates, and obtain a theorem analogous to that of Menelaus.

Let BCD (fig. 53) be one of the faces of the tetrahedron; and put $a_2 = PB'/BB' = \Delta PCD/BCD$, $a_3 = PC'/CC' = \Delta PDB/\Delta CDB$, etc. Then the co-ordinates of P, Q, R, S, points in the four faces of the tetrahedron, may be written $(0, a_2, a_3, a_4)$, $(b_1, 0, b_3, b_4)$, etc.; and the condition that these four points be coplanar is

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