IS COSMIC STRING DOMINATION OF THE UNIVERSE AVOIDABLE?

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We address, by means of Numerical Simulations, one of the main issues of the Cosmic String Galaxy Formation Scenario, namely the existence of a scaling solution, which is crucial to the very existence of the scenario. After a brief discussion of our numerical technique, we present our results which, though still preliminary, offer the best support to date of this scaling hypothesis.

INTRODUCTION

Cosmic strings are linear topological defects that are predicted to occur in many grand unified theories. They might be at the origin of the primordial density fluctuations that have led to the formation of galaxies and larger scale structures, and they are expected to be formed in great abundance when the universe goes through some spontaneous symmetry breaking phase transition. It has been shown that the majority of the string length at the time of formation is in the form of infinitely long strings. These infinitely long strings are a potential problem because their energy density naively seems to scale like non-relativistic matter implying that they will very quickly dominate the universe in the radiation-dominated era. This "disaster" is supposed to be avoided by the process by which the infinitely long strings cross themselves and break off small loops which can radiate away into gravitational radiation. This scenario has been studied analytically by Kibble (1985) and Bennett (1986a, b). This analytic work has shown that there are two possibilities: either the loop production is not sufficient to avoid a string-dominated universe, or the strings will settle down to a scaling solution in which the number of strings crossing a given horizon volume is fixed.

A great deal of work has already been done on the cosmic string theory of galaxy formation, *assuming* that a scaling solution does indeed exist, and there has been a great deal of speculation as to the *characteristics* of the assumed scaling solution, like the numerical value of the energy density in infinite strings, or the number of loops of a given length per horizon volume. So far, however, all of this work is on uncertain ground because the basic details of string evolution are not understood. Albrecht and Turok (1985) have published preliminary results from their simulation two years ago. However, their program was very crude, and these results were criticized as inconsistent on the basis of analytical work.

J. Audouze et al. (eds.), Large Scale Structures of the Universe, 289–291.

https://doi.org/10.1017/S0074180900136162 Published online by Cambridge University Press

NUMERICAL TECHNIQUE

To Generate the initial conditions, we follow the general procedure introduced by Vachaspati and Vilenkin (1984): one draws discretized random phases for each cell of a cubic periodic lattice; if their winding number around an edge is non-zero, the link is occupied. Sampling points and their attached pointers are then laid down accordingly. The one important improvement that we have made is to replace the generated sharp corners by arcs of circles. Physically, this seems to be a better representation, while numerically this helps to minimize the number of discontinuous derivatives that the program must deal with.

To evolve the generated configuration, we solve the partial differential equations describing the strings' motion. We use the gauge used by Turok and Bhattarcharjee (1984): $\ddot{\mathbf{x}}+2(\dot{a}/a)\dot{\mathbf{x}}(1-\dot{x}^2) = (\mathbf{x}\prime/\epsilon)\prime/\epsilon$, with $\dot{\epsilon} = -2(\dot{a}/a)\epsilon\dot{x}^2$, where dots denote "time" derivatives and primes partial derivatives along the string (a is the expansion factor of the metric). Spatial derivatives at mid-points are simply obtained by differences, while we use a modified leapfrog scheme for the time integration. ϵ is evolved according to $\epsilon = \epsilon(1-b)/(1+b), b = dt(\dot{a}/a)\dot{x}^2$, where $\dot{x}^2$ obtains by averaging over the end points. Each loop carries its one timestep satisfying the Courant condition. Our timestep halving routine preserve the O(2) accuracy of the overall integration.

A major difficulty that one encounters is that the strings have physical discontinuities in $\dot{\mathbf{x}}$ and $\mathbf{x}\prime$, which result from the constant crossings of the strings with each other. These "kinks" have a long lifetime, and have important implications for the pattern of loops produced. To avoid the development of instabilities near the kinks (where the damping term due to the expansion of the metric may go to zero), we introduce some numerical diffusion that we try to keep at a minimal level. This is accomplished by averaging the velocities over neighboring points, but *only* when wiggles start to develop, *i.e.* whenever the quantity $\dot{x}^2 + (x\prime/\epsilon)^2$ gets to be different by a few per cent from its correct value (which is conserved by the equations of motion). This procedure smooths out the wiggles without increasing the width of the discontinuities, and seems to preserve the kinks fairly well.

To determine if two string segments crossed during the time step, we check the volume of the tetrahedron spanned by the four points on the two segments. If it changed sign during the step, the configuration is checked at the time the volume is zero, to see if a crossing did really occur (the positions of the points are extrapolated linearly between time steps). This procedure is *exact*. Also, the internal dynamics of loops is on a much smaller timescale than the displacement of complete loops. A given loop is thus checked for self-crossings at each of its individual timesteps, while the crossings between loops are checked only at each system timestep, when all loops are synchronized.

Finally, when two segments have been determined to cross, we interchange partners, and average the positions and the velocities in the crossing region to help reduce the gauge condition violation. At this stage, we also update a "genealogical tree", which records the labels of the "parent" loops and of the "child" loops, as well as such relevant quantities as "birth" and "death" times, energies, center of mass position and velocity, etc...This enables us to get a posteriori a detailed picture of the string system evolution.

RESULTS

We have performed several runs in boxes as large in size as $28\xi_0$, where ξ_0 is the mesh size of the lattice used to generate the initial configuration (ξ_0 is thus

https://doi.org/10.1017/S0074180900136162 Published online by Cambridge University Press

the correlation length of the initial brownian walks). Typically, such runs will have $\sim$ 160,000 particles, $\sim$ 500 strings initially, and occupy $\sim$ 56Mbytes of memory. After 500 steps (which takes 7-8 hours of Cray-2 cpu), a very large number of small loops have been created (there are then $\sim$ 5500 strings, see Fig. 1), and the expansion factor a has been multiplied by 2.25 (a factor $\sim$ 5 in physical time). We also performed an extremely large simulation in a box measuring $36\xi_0$ on a side. In an effort to "bracket" the scaling solution, we evolved some configurations with different initial horizon size $h_0 = 2ct_0$, and thus different initial energy densities in long strings $\rho_L(t_0)$, to see if ρ_L behaves like radiation, *i.e.* if the evolved $\rho_L.t^2/\mu$ converge toward similar (constant) values. This does seem to happen after a fairly short time, as is illustrated in Fig. 2 (long strings are defined to be of length $> ct$). Runs with different numerical parameters (*e.g.* the numerical lower cutoff on the size of chopped loops) do exhibit similar behaviors, though the precise numerical values might be somewhat different. Other diagnostics, like the density of loops of a given size, also bring support to the hypothesis of existence of a scaling solution. Nevertheless, there still remains to feed back our results in the analytical formalism to check their consistency, and make absolutely sure that the apparent scaling behaviour is not the result of some funny numerical artefact.

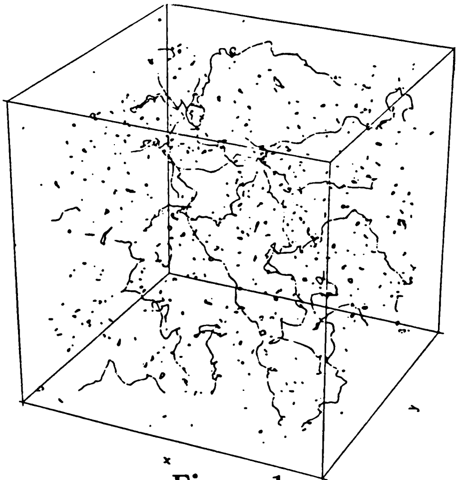

Figure 1
A final string configuration in a subvolume of side equals $ct/2$.

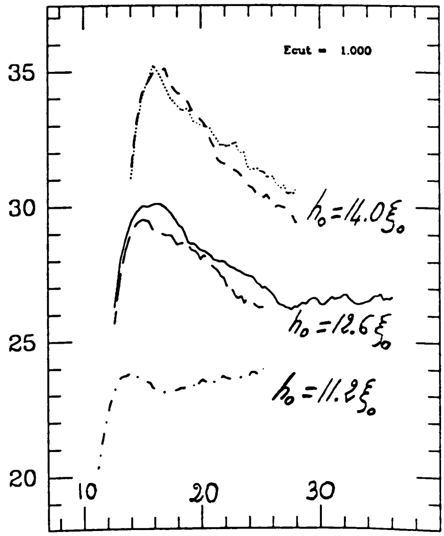

Figure 2
$\rho_L.t^2/\mu = f(h/\xi_0)$. The solid curve is for the run in a box of side $L=36\xi_0$, the others correspond to $L=28\xi_0$.

CONCLUSION

Our first results, though not definitive, lend support to the hypothesis of *existence* of a scaling solution. Nevertheless more work is necessary in order to be able to make reliable quantitative predictions on the accurate *characteristics* of the scaling regime, such as the number of strings of a given size per horizon volume, or their 2-point correlation function.

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