DISTRIBUTION OF THE NUMBER OF CLAIMS IN MOTOR INSURANCE ACCORDING TO THE LAG OF SETTLEMENT

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1.1 *Introduction*

Let S^n be the set of motor claims S_i, $i = 1, 2, \ldots, N$ occurred during a given year n, namely S^n represents the set of claims relevant to the generation (or cohort) n.

If T^n is a subset (even empty) of claims resulting without payement (that is the set of zero-claims), the set $P^n = S^n - T^n$ shall denote the set of claims that should be settled.

For every $s_i \in P^n$, we can define the r.v. X_i which represents the period of time required for its settlement (namely the lag of settlement).

It is not sensible to deem that the r.v. X_i are equally distributed: as a matter of fact we know that the larger is the claim, the longer the lag of payement.

However, we can assume that in a subset U of P^n, the r.v. $X_i(U)$ have the same distribution function, which will be denoted by $F_U(x)$ or in short $F(x)$.

As $F(x)$ represents the probability that a claim $s_j \in U$ is settled within a period $0 - x$, the function $1 - F(x) = l(x)$ denotes the probability that the claim results unsettled after a lag x, that is the survival function of the claim.

In this study we intend to find an analytical expression of the function $1(x)$ on the basis of particular assumptions about the behaviour of the adjuster with regard to the settlement of claims.

The assumptions will be tested by fitting the function to some observed data.

1.2 *The assumptions*

On the analogy of the actuarial life theory, we shall consider the ratio

$$\mu(x) = -\frac{l'(x)}{l(x)} \tag{1}$$

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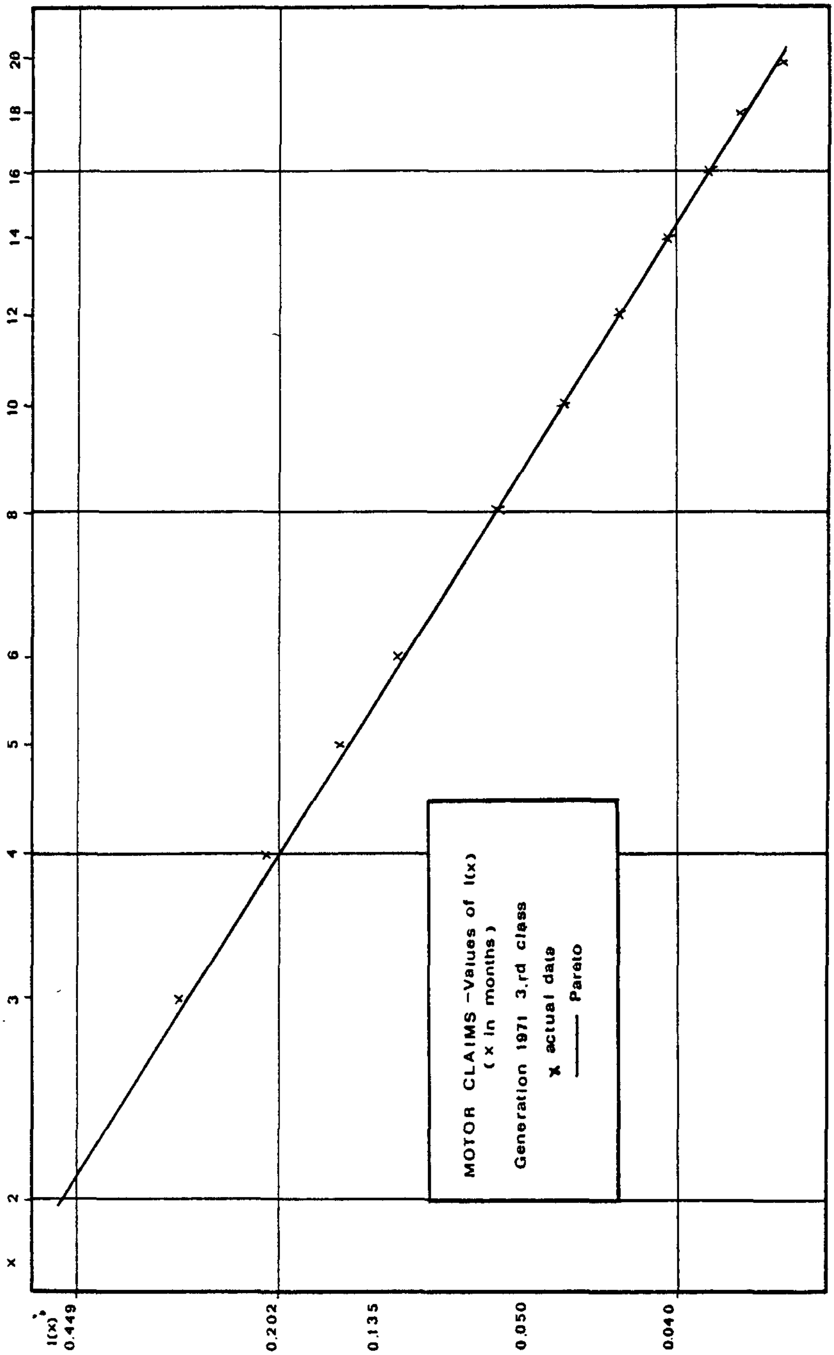
MOTOR CLAIMS – Values of I(x)
(x in months)
Generation 1971 3.rd class
x actual data
——— Pareto
x
2
3
4
5
6
8
10
12
14
16
18
20
I(x)
0.449
0.202
0.135
0.050
0.040

Fig. 1

that represents the force of mortality or (in our case) the force of settlement.

As it is known in life theory the assumptions concern the connection between $\mu(x)$ and x (age).

For our phenomenon it is not reasonable to link $\mu(x)$ directly to x (lag of the claim): we deem that μ depends on the function $l = l(x)$, that is

$$\mu = \mu(x) = \bar{\mu}[l(x)] = \bar{\mu}(l). \tag{2}$$

In fact it is sensible to think that in the settlement work the adjusters are more influenced by the (average) number of unsettled claims rather than by the "age" of the dossiers.

More precisely, we deem that the force of settlement $\bar{\mu}(l)$ is an increasing function of the (average) number l of unsettled claims. By assuming that the relative infinitesimal variation of the number of unsettled claims leads to a proportional relative infinitesimal variation of the force of settlement, we obtain

$$\frac{d\bar{\mu}(l)}{\bar{\mu}(l)} = \beta \frac{dl}{l} \quad \beta > 0 \tag{3}$$

and, by solving this differential equation, we may write

$$ln\bar{\mu}(l) = \beta ln\, l + c. \tag{4}$$

Hence

$$\bar{\mu}(l) = Kl^{\beta}, K > 0 \quad \beta > 0. \tag{5}$$

The value of the parameter β characterizes the pattern of the force of settlement μ. For $l \to 0$ (hence for $x \to +\infty$) the greater β is; the more rapidly $\bar{\mu}$ will tend to 0.

1.3 *The analythical expression of $l(x)$*

On the basis of this assumption from (2), we can write

$$\mu(x) = Kl^{\beta}(x) \tag{6}$$

where K and β are positive constants. From (1) we obtain

$$-\frac{l'(x)}{l(x)} = Kl^{\beta}(x). \tag{7}$$

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That is

$$- \frac{dl(x)}{l^{\beta+1}(x)} = K dx. \tag{8}$$

Since $l(0) = 1$, we find

$$l^{-\beta}(x) = \beta K \left(x + \frac{1}{\beta K}\right). \tag{9}$$

The survival function can be written as follows

$$l(x) = (\beta K)^{-(1/\beta)} \left(x + \frac{1}{\beta K}\right)^{-(1/\beta)} \tag{10}$$

or putting

$$x_0 = \frac{1}{\beta K}; \frac{1}{\beta} = \alpha$$

we find

$$l(x) = \left(\frac{x_0}{x + x_0}\right)^{\alpha} \quad x \geq 0, \qquad \alpha < 0, \; x_0 > 0. \tag{11}$$

2.1 *The statistical data*

In order to test our assumption we considered a particular portfolio of claims formed by material damages whose first evaluation was smaller than 300,000 Italian Lire (12,000 P.Escud.).

With the purpose of obtaining subsets such that the r.v. X_i are equally distributed, we subdivided furtherly the portfolio into six strata. The criterion of subdivision was based on the first evaluation of the claims. In fact, in our opinion, the first evaluation represents a way by which the adjuster graduates his judgement on the claim pattern. In other words, by expressing his first estimate the adjuster arranges the claim in a given class, characterized by particular severity, dispersion and lag of claims.

Our fitting was made on the generation 1971, which has been observed at the end of the year 1973.

2.2 *Research of the parameters*

The fitting of the function $l(x) = \left(\frac{x_0}{x + x_0}\right)^{\alpha}$ presents some difficulties: in fact $ln\ l(x)$ cannot be expressed in a linear form with

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respect to the parameter x_0. To overcome such drawback, we consider the translation $Y = X + x_0$ which allows us to express $l_y(x)$ as a Pareto function, that is

$$l_y(x) = \begin{cases} \left(\dfrac{x_0}{x}\right) & x \geq x_0 \\ 1 & 0 \leq x < x_0. \end{cases} \qquad (12)$$

In this way we consider a lag Y which presents a sure component x_0 (now undetermined) and the expression (12) represents the survival function at the "age" $Y = X + x_0$, where X is the further duration of the claim.

On the basis of our data and by means of the least square method, we found the values of parameters shown in table 1.

Since, in the generation considered we checked a posteriori that the values of $l(x_0)$ are sufficiently near to 1, we deem that our results are valid.

It is to be pointed out that, with the exception of the 5th stratum, the curve fits well the data and, therefore, the parameters can be used to forecast the further duration of claims relevant to our portfolio.

However, we intend to test our assumption on the basis of other generations and possibly to find an analythical procedure which allows us to determine the parameters directly from the expression (11).

TABLE 1

Fitting of Pareto curve to the distributions of the number of claims according to the lag of settlement.

$$l(x) = \left(\frac{x_0}{x}\right)^{\alpha}$$

x = time expressed in months (1 = 30 days);
$l(x)$ = average number of claims unsettled at time x.
Generation 1971 (on 31.12.1973)
Value of α and x_0.

Class of the 1st evaluation (thousands of It. Lire)	α	x_0	χ^2
0 — 50	1.252	0,89	1.68
50 — 75	1.465	1,11	6.56
75 — 100	1.242	1,10	2.49
100 — 175	1.114	1,10	4.06
175 — 250	0.976	1,21	12.39
250 — 300	0.749	1.21	9.69

TABLE 2

Generation 1971 — Motor Claims
3rd Class of first evaluation = 75 — 100 (thous. of It. lire)
Distribution of the number of claims according to the delay of settlement (time expressed in months 1 = 30 days); $\tilde{N}$ = total number of claims

Lag	Actual Claims	Expected Claims
x	$N \cdot l(x)$	$N \cdot l(x)$
2	43.887	46.838
3	28.749	28.311
4	20.497	19.804
5	15.538	15.010
6	12.207	11.969
8	8.388	8.372
10	6.310	6.345
12	5.039	5.059
14	4.161	4.178
16	3.538	3.539
18	3.052	3.057
20	2.616	2.682

$$\alpha = \frac{\Sigma\, ln\, l(x_i)\; \Sigma\, ln\, x_i - n\, \Sigma\, ln\, l(x_i)\; ln\, x_i}{n\, \Sigma\, (ln\, x_i)^2 - (\Sigma\, ln\, x_i)^2} = 1{,}242$$

$$ln\, x_0 = \frac{\alpha\, \Sigma\, [ln\, l(x_i)]^2 + \Sigma\, ln\, l(x_i)\; ln\, x_i}{\alpha\, \Sigma\, ln\, x_i} = 0.098$$

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