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CHARACTER OF THE ENGLACIAL AND SUBGLACIAL DRAINAGE SYSTEM IN THE LOWER PART OF THE ABLATION AREA OF STORGLACIÄREN, SWEDEN, AS REVEALED BY DYE-TRACE STUDIES

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ABSTRACT. During the 1984 and 1985 melt seasons, flow velocities and dispersive characteristics of the englacial and subglacial hydraulic system on Storglaciären, a small valley glacier in northern Sweden, were studied with the use of dye-trace tests. Similar tests conducted on one of the two principal pro-glacial streams provided a basis for comparison of the combined englacial–subglacial system with the pro-glacial one. Velocities in the two systems were broadly comparable after compensating for the effect of slope differences. However, velocities in the glacial conduits increased almost linearly with discharge. Analysis suggests that this can be explained by an increase in water pressure in the conduits, combined with a decrease in effective sinuosity, as discharge increases. Dispersivity (the ratio of the dispersion coefficient to the water velocity) in the glacial system is high early in the season but decreases progressively during July. This is believed to reflect a change from an extensively braided to a more integrated drainage system. Dispersivity is only slightly lower in the pro-glacial streams than in the late-season glacial conduits, suggesting similar degrees of braiding. However, retardation of dye due to temporary storage is greater in the glacial conduits. This suggests that the glacial streams have a larger number of stable eddies in which dye can be trapped for extended periods of time.

INTRODUCTION

On a temperate glacier, melting at the ice surface is usually the most important source of water (Shreve, 1972; Theakstone and Knudsen, 1981; Collins, 1982), and this is the case on Storglaciären, a small sub-polar glacier in northern Sweden (Fig. 1) on which the present studies were conducted (Östling and Hooke, 1986). Rain also supplies a significant amount of water. Condensation makes a minor contribution.

On Storglaciären about 15% of this water leaves the glacier by way of supraglacial channels (Rudensky, unpublished). The rest is intercepted by moulins and crevasses, and thus finds its way into englacial and eventually subglacial conduits.

Following the work of Hodge (1974, p. 365) and Tangborn and others (1975, p. 194-95) it is now widely accepted that these conduits close and become deranged by plastic flow of the ice during the winter when water input is small or negligible. Then, when melting starts in the spring, water pours into the glacier faster than it can be transmitted through the shrunken conduits. The water thus backs up in the system, resulting in high water pressures at the bed. It is widely believed that these high pressures increase sliding speed, and that they are thus responsible for the rapid increases in surface velocity often observed early in the melt season (Hodge, 1974; Hooke and others, 1983[b]; Iken and others, 1983; among others). Eventually the energy dissipated by the flowing water enlarges the conduits by melting and water pressures are lowered. Sliding speeds then decrease.

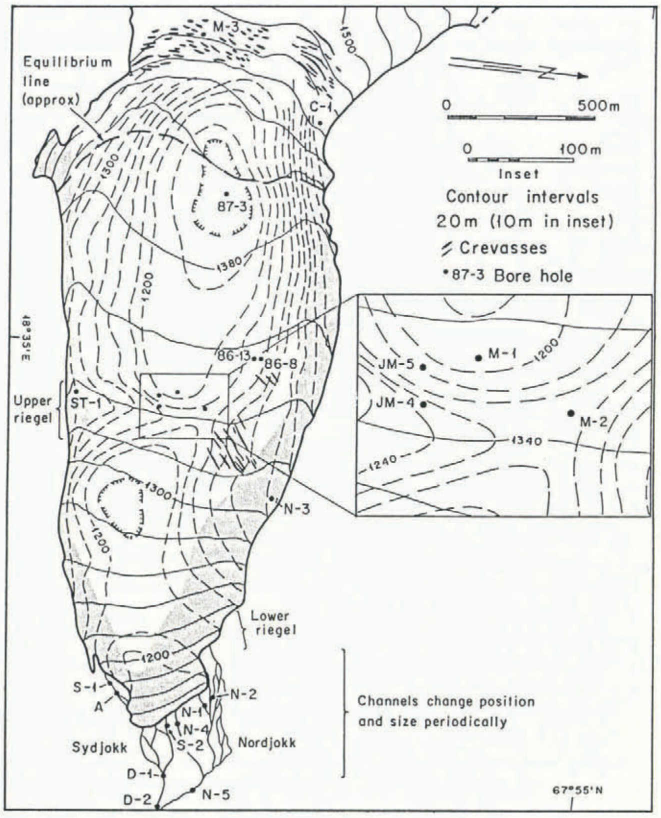


Fig. 1. Map of Storglaciären showing injection and sampling points for tracer tests described herein. Some of the points identified are discussed in a companion paper describing a test made in 1986 (Hooke and others, 1988). The flow passing N-4 was insignificant in 1984 and 1985. Surface contours are from a map based on 1969 aerial photography. Bed contours are from Björnsson (1981). Stippling indicates where ice is less than ~35 m thick and near-surface temperatures are below freezing in late summer. The glacier is inferred to be frozen to the bed in these areas.

In order to study the characteristics of these water conduits, and in particular the temporal variations in their characteristics throughout a summer melt season, dye-trace tests were done on Storglaciären during the summers of 1984 and 1985. Storglaciären is ~3.0 km long and 0.8 km wide, and has a maximum thickness of ~250 m (Björnsson, 1981). Temperature measurements suggest that it is at the pressure melting-point throughout, except for a ~35 m thick layer at the surface in the ablation area (Hooke and others, 1983[a]; hence the designation "sub-polar." Schytt (1959) has described the physical characteristics of the glacier in greater detail.

PREVIOUS WORK

For the past several years, structures formed either by refreezing of water in englacial conduits or by closure of such conduits have been mapped on Storglaciären as they were exposed by ablation (Holmlund, in press). This mapping has been supplemented by descents into moulins during the winter. These studies have shown that, upon reaching the bottom of a moulin, water forms a plunge pool. Upon overflowing from these pools, the water normally flows for some distance in channels that follow the crevasse in which the moulin was formed. The channels often meander; such meandering arises from an instability in the flow of water over ice, and implies rapid flow with Froude numbers in excess of 1 (Parker, 1975). The deepest structures exposed so far show that, in at least one instance, the water eventually encountered a nearly vertical conduit at a depth of ~40 m below the ice surface. Below this depth, virtually all knowledqe of the englacial water system is based on theoretical considerations and remote-sensing studies.

Theoretically, if englacial passages are completely filled with water, they should form an upward branching arborescent network oriented normal to equipotential planes in the ice (Shreve, 1972). The equipotential planes dip up-glacier at an angle of ~11 times the slope of the glacier surface. Along the bed, water courses should be orthogonal to contours formed by the intersection of these equipotential planes with the underlying topography.

However, many englacial and subglacial passages may not be completely full of water. Calculations suggest that, under ice less than a few hundred meters thick, discharges greater than a few tens of liters per second in a channel with a down-glacier slope of only a few degrees may melt conduit walls more rapidly than closure by plastic deformation can occur (Hooke, 1984). In such situations, the pressure in the conduits may range from atmospheric, if there is a connection to the glacier surface, to the triple-point pressure if there is no such connection.

Tracer studies provide another means of investigating flow in glacial systems, and a number of researchers have employed this technique (Stenborg, 1969; Krimmel and others, 1973; Behrens and others, 1975; Lang and others, 1979; Theakstone and Knudsen, 1981; Collins, 1982; Burkimsher, 1983; Humphrey and others, 1986; Brugman, unpublished). These studies have revealed that in some cases water is delayed in its passage through a glacier, whereas in other cases it is not. In the latter situation, some authors have compared flow velocities and dispersion characteristics in the englacial–subglacial and the supraglacial systems, and concluded that the water flowed in open channels along most of its path after reaching the glacier bed (Krimmel and others, 1973; Behrens and others, 1975). However, alternative interpretations are possible (Collins, 1982, p. 533).

DRAINAGE OF STORGLACIÄREN

Storglaciären is drained by two principal streams, Nordjokk and Sydjokk (north and south stream, respectively) (Fig. 1). Both first emerge from the glacier along a lateral margin, some distance above the terminus. Nordjokk first appears near site N-3 (Fig. 1) on the up-glacier side of what may be an over-ridden terminal moraine extending out from the valley side. It then disappears, only to re-emerge on the stoss side of what seems to be a discontinuous bedrock-cored moraine ridge that projects beneath the glacier from both sides (Fig. 1, "lower riegel"). Sydjokk first surfaces at site S-1 on the stoss side of this same ridge on the south side of the glacier. While the flow of both streams is augmented by surface run-off near where they first appear, most of the flow is derived from englacial or subglacial sources. Below the lower riegel, both streams braid and some of the branches, often called anabranches in work on braided streams, plunge back beneath the glacier.

Of the several streams that finally emerge at the terminus, most are known or inferred to be anabranches that split from the streams as they passed over the lower riegel. The streams that pass sampling sites S-2 and N-4, however, seem to have sources farther up-glacier.

Of significance is the fact that Sydjokk is normally much dirtier than Nordjokk. Apparently, the water in Sydjokk has moved a considerable distance along the glacier bed while that in Nordjokk has been largely englacial.

Previous tracer tests on Storglaciären were conducted by Stenborg (1969) but without the benefit of the highly sensitive detection equipment now available. He concluded that a bilateral drainage system existed in the lowermost part of the glacier, owing to the occurrence of oblique crevasse fields on each side of its center line. Water intercepted by crevasses on the north side of the glacier appeared in Nordjokk, and conversely. Higher in the ablation area, however, there appeared to be a deviation from the bilateral nature of the drainage system due to the presence of a second riegel (Fig. 1, "upper riegel"). Drainage from the area up-glacier from this riegel in the northern half of the glacier did not contribute directly to discharge in Nordjokk. By comparing ablation and discharge data, Stenborg concluded that this area contributed indirectly to discharge in Sydjokk.

PROCEDURE

Dye tracing

The first series of dye-trace tests was conducted between 13 July and 13 August 1984. Building on the experience thus gained, a second series was initiated on 28 June and completed on 20 August 1985. Dates of the individual tests and other pertinent data are given in Table I.

Procedures were basically the same in all tests unless otherwise noted. A known quantity of 20% rhodamine WT was poured into a moulin at a pre-determined time by a person not involved in the sampling. The identities and locations of the moulins used are given in Table I and Figure 1, respectively. The principal sampling site was S-1 (Fig. 1), located on the south branch of Sydjokk. During the first several hours of a test, samples were taken there at intervals ranging from 1 to 30 min, depending on the rapidity with which dye concentrations appeared to be changing. Sampling was initiated before measurable concentrations of dye reached the sampling site. Site S-2, located on the north branch of Sydjokk, was also monitored, but at sampling intervals usually ranging from 5 to 60 min. Nordjokk was normally sampled at site N-1 once every 2 or 3 h.

Once concentrations had fallen below ~ 0.5 ppb, the sampling interval was increased and sampling was continued for at least an additional 12–16 h, and in some cases up to 24 h. By this time, dye concentrations had usually fallen to less than 0.1 ppb. Water levels (stages) were normally monitored at S-1 throughout the duration of a test.

Samples were analyzed in the laboratory at Tarfala Research Station which is about 1 km from the toe of the glacier. A Turner Designs model 10-005 fluorometer was used. The fluorometer was calibrated using standards of known concentration that were prepared by volumetric dilution, following an initial weight dilution to 1 ppm, using water from Sydjokk. Samples, standards, and blanks were kept at a constant, uniform temperature during analysis. Random samples were chosen for re-analysis and results proved to be reproducible to ~±2%. Standards were also read periodically to check for stability of the calibration.

The resulting concentration–time curves are presented in Figure 2. Of particular interest in these curves are: (1) the multiple peaks in tests 84-2 and 84-6, which we attribute to braiding of the englacial–subglacial conduit systems, (2) the decreasing breadth of the curves in the early part of the 1985 melt season, reflecting a decrease in

TABLE I. SUMMARY OF VELOCITY, DISCHARGE, AND DISPERSION RESULTS FOR S-1

Test	*Date*	*Input point*	*Discharge* m³/s Q_s	Q_m	Q_p	*Velocity*[1] m/s u	*Dispersion* m²/s D
84-1	13 Jul	JM-4	0.58		0.62	0.14	1.6
-2	18 Jul	JM-4	0.42		0.52[2]	0.10	0.8
-3	Test on different glacier						
-4	31 Jul	JM-5[3]	0.46		0.50	0.14	1.0
-5	3 Aug	M-1[3]	0.48		0.57	0.16	1.9
-6	13 Aug	JM-4	0.20		0.28[2]	0.11	0.6
85-1	28 Jun	M-1	0.23	0.32	0.38	0.044	3.6
-2	10 Jul	M-1	0.34	0.56	0.43	0.11	3.9
-3[4]	20 Jul	M-1 ST-1			0.49[5]	0.14	
-4	22 Jul	M-2	0.56	0.64	0.73[5]	0.16	4.7
-5	27 Jul	M-2	0.31	0.32	0.31	0.076	0.5
-6	1 Aug	M-3					
-7	5 Aug	M-1	0.32	0.44	0.33	0.092	0.4
-8	14 Aug	M-1	0.42	0.54[6]	0.50[6]	0.125	0.8
-9	20 Aug	M-1	0.10	0.12[6]	0.12[6]	0.032	0.1

[1] A sinuosity of 1.0 was assumed.

[2] Mean discharge at S-1 between time of injection and first peak.

[3] Moulin JM-4 was not receiving flow at the time of these tests.

[4] Use of two input points complicated the output curve so no further analysis was attempted.

[5] Adjusted for presumed 64 and 23% errors, respectively, based on mass-balance results.

[6] Values estimated based on average values of Q_s/Q_m and Q_p/Q_m.

dispersivity, and (3) the variations in the time to the peak concentration, which reflects variations in flow velocity. These points will be discussed further later, once we have discussed procedures for determining discharge and dispersivity.

Stream rating

At various times during the summer, discharge measurements were made at S-1 and at a dam located lower on Sydjokk in order to determine stage–discharge relations for the two locations. Initially, the dam was at D-1 but a storm on 16 July 1985 washed out this dam and modified the channel geometry visibly. A new dam was constructed at D-2.

A fluorometric constant-injection-rate dye-dilution technique (Replogle and others, 1966) was used. A dye–water mixture was injected slightly up-stream from S-1 and samples were taken at point A (Fig. 1) and usually also at the dam. By taking samples at two points, discharges at both could be obtained from a single test. The standard error in these discharge measurements is estimated to be ±10% (Zimmerer, unpublished). Several slug-discharge tests (Replogle and others, 1966) were also performed in 1985 in order to supplement the Sydjokk discharge record after the storm.

Because gauge locations changed, separate rating curves were required for different time periods. Thus, although 12 usable constant-injection-rate tests were conducted, resulting in 19 discharge determinations, the individual rating curves (Fig. 3) are based on rather sparse data.

Water-level measurements were not made at S-2 during the dye-trace tests, because it was not possible to locate a gauge there such that the stage would be reasonably sensitive to variations in discharge. In 1984 the discharge there was estimated visually. In 1985 we made use of the fact that the flow at the dams is the sum of those at S-1 and S-2. Thus, the discharge at S-2 could be calculated for each of the seven discharge measurements made that year. It ranged from 17 to 54% of that at S-1, with five of the seven tests yielding values between 18 and 30%. Thus, in mass-balance calculations we assumed that the discharge at S-2 was 25% of that at S-1.

Determination of discharge during a dye-trace test

As discharge varied during a dye-trace test on the glacier, and as a considerable amount of time was required for the dye cloud to pass the sampling site, a systematic technique was required to obtain consistent measures of discharge. We first calculated an "average" discharge, Q_s, for Sydjokk (S-1 + S-2) by treating the test as a slug-discharge test. To do this, we had to assume: (1) that the discharge during the test was constant (which, of course, it was not), and (2) that the discharge at S-2 was 25% of the total, as discussed above. Thus, the Q_s values are not accurate averages. However, because stage data were not obtained in some tests, this was the only measure

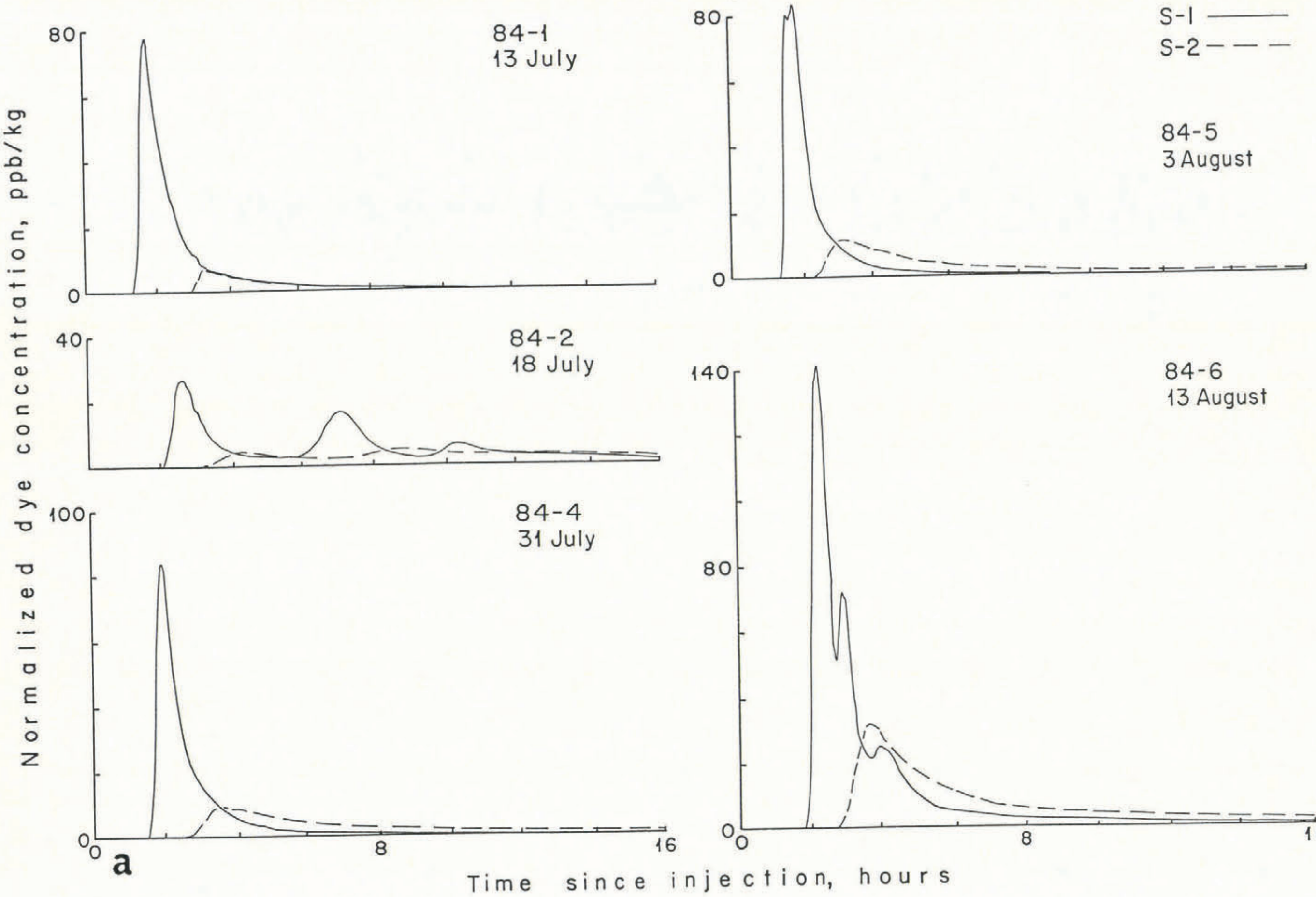


Fig. 2. Normalized dye concentration as a function of time at sites S-1 and S-2. a. 1984 tests. b. 1985 tests. Dotted curves are calculated from Equation (2).

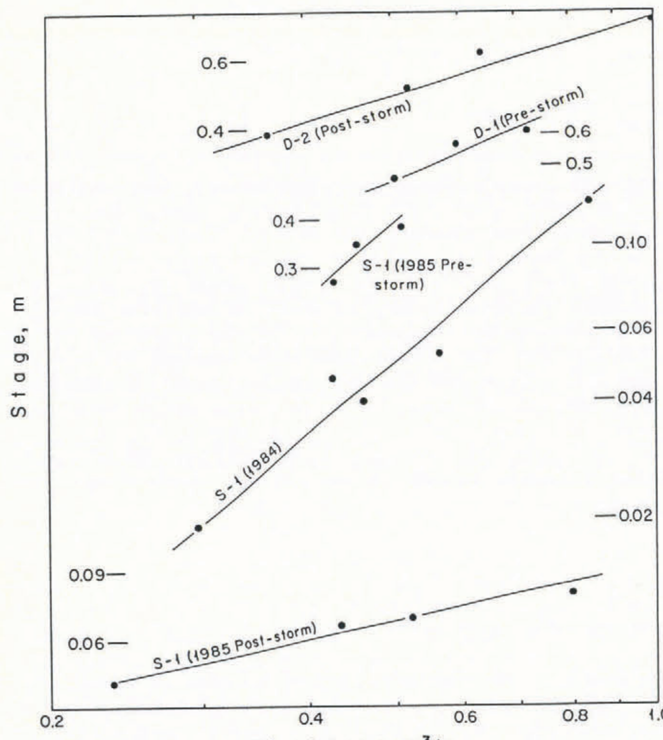


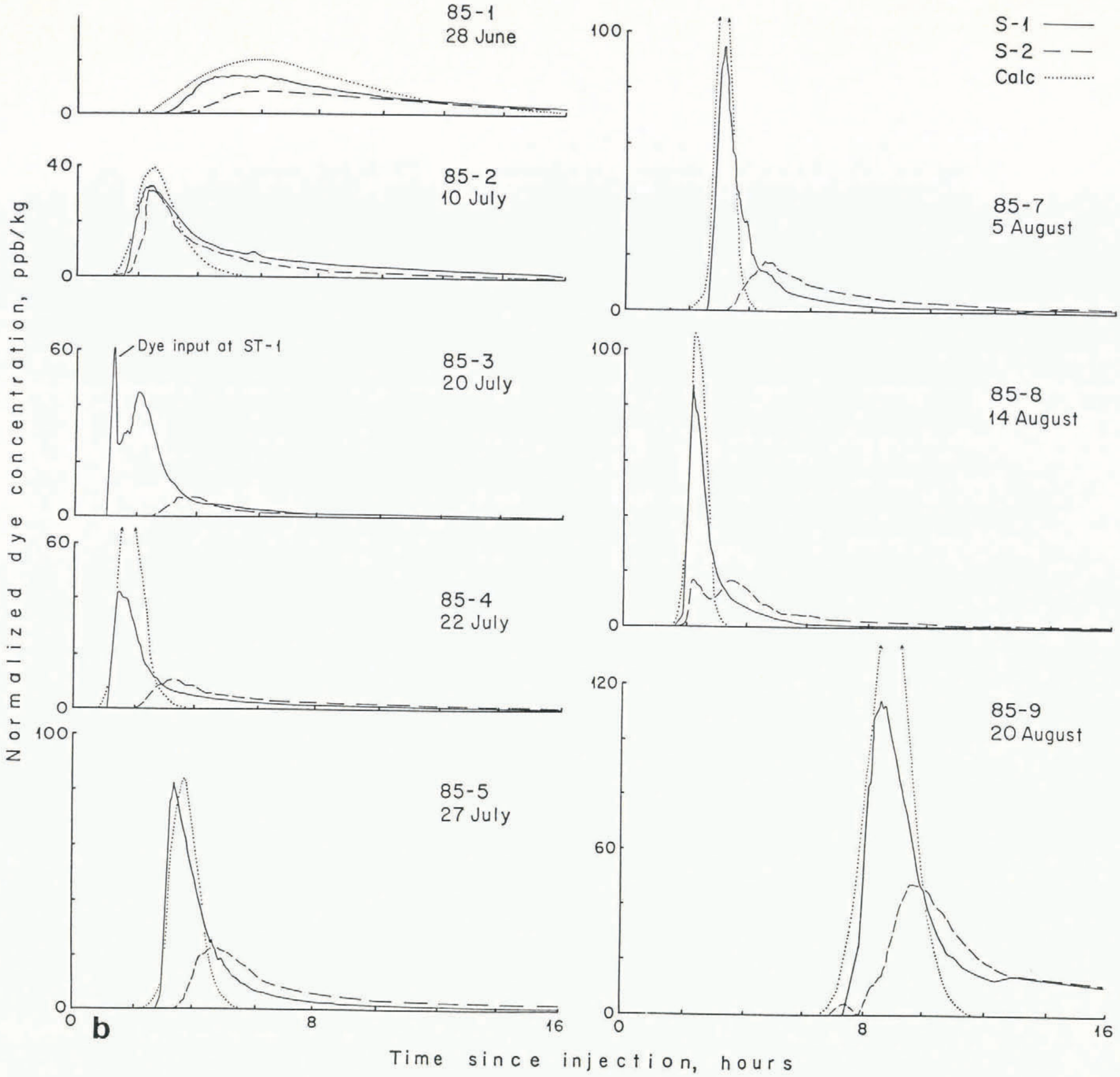
Fig. 3. Stage–discharge curves for site S-1 on Sydjokk and for the Sydjokk dams.

of discharge that could be obtained consistently for all tests.

We next used the stage–discharge relations for tests 85-1, -2, -4, -5, and -7 to calculate the mean or time-averaged discharge at S-1 during passage of the entire dye cloud, Q_m, and also to calculate the mean discharge between the time of injection and the time of passage of the peak, Q_p. Q_m and Q_p for the other tests were estimated from the ratios Q_m/Q_s and Q_p/Q_s in these five tests. Q_m/Q_s ranged from 1.0 to 1.6 and Q_p/Q_s from 0.7 to 1.65; the mean values of 1.3 and 1.2, respectively, were used in the calculations (Table I).

DYE RECOVERY

The weight of dye that passed Sydjokk dam during a tracer test was estimated using the measured concentrations and discharges. Table II lists the results. For the first four of the 1984 tests the calculations suggest that only ~85% of the dye was recovered, whereas for test 85-6 the recovery was more than 100%. These discrepancies are attributed to uncertainties in the rating curve for 1984, which was apparently yielding discharges that were too high at low stages and too low at high stages. For tests 85-1, 85-2, 85-5, and 85-7, it appears that all the dye was recovered, within the limits of uncertainty in the analyses. It is evident, however, that there is a discrepancy in the results for tests 85-3 and 85-4; both show recovery significantly greater than 100%. These two tests were conducted during the week following the 16 July storm. Slug-discharge tests conducted during this week suggest that the post-storm rating curves, which were based on measurements made later in the summer, yielded discharges that were systematically high during tests 85-3 and 85-4.



We infer that the channel was gradually adjusting during this period. The calculated recovery in test 85-4 is significantly closer to 100% than that for test 85-3, adding plausibility to this suggestion. It was not possible to calculate mass balances for tracer tests 85-8 and 85-9 as water-level data were not obtained during those tests.

Essentially no dye was recovered during test 85-6, in which moulin M-3 in the firn area (Fig. 1) was used as the injection point. This is probably because water from this area leaves the glacier by way of Nordjokk. Nordjokk was not being sampled regularly because no dye had been detected there in previous tests. However, two samples taken 13 h apart during test 85-6 contained 2.92 and 1.95 ppb dye, respectively. A dye-trace run in 1986 utilizing input point C-1, also in the firn area (Fig. 1), supports this conclusion (Hooke and others, 1988).

DISPERSION

Dispersion is a process by which a dye cloud (or other inhomogeneity) in a fluid is spread out as it travels down-stream. In a river channel, the primary mechanism responsible for dispersion is the spatial variation in velocity within the channel. Diffusion, which is the result of random molecular motions, has the same effect but it is of minor significance, by comparison, in a turbulent stream.

Taylor (1954) has shown that, with properly adjusted coefficients, longitudinal dispersion in a long, straight pipe may be characterized by the diffusion equation written in the form:

$$\frac{Dc}{Dt} = D\frac{\partial^2 c}{\partial x^2} \tag{1}$$

where c is the concentration of the dye, D is the coefficient of longitudinal dispersion for a coordinate system moving at the mean flow velocity, D/Dt is the material derivative, and t and x are time and distance along the conduit, respectively. This relation has been applied in studies of open-channel flow in flumes and natural streams by Elder (1959), Glover (1964), Fischer (1966, 1968, unpublished), Thomas (unpublished), among others. However, a complication arises when trying to apply it in the present situation because there are processes that result in temporary storage of some of the dye during its passage through the glacier. This dye is gradually released back into the main flow over a period of time that is considerably in excess of that required for the main dye cloud to pass through the glacier, and thus leads to a long tail in the dye-return

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TABLE II. MASS-BALANCE RESULTS*

1984 tests

Test number	84-1	84-2	84-4	84-5	84-6
Weight of dye input (g)	234.2	227.6	237.7	239.5	231.9
Weight of dye passing S-1 (g)	196.5	190.3	193.2	202.6	251.5
Weight of dye passing S-2† (g)	0.9	3.0	3.2	3.7	6.3
Per cent recovered	85	85	85	87	117

1985 tests

Test number	85-1	85-2	85-3	85-4	85-5	85-7
Weight dye input (g)	236.1	223.3	237.1	117.7	234.6	236.9
Weight dye passing S-1 (g)	190.8	167.8	353.6	128.0	145.0	185.1
Weight dye passing S-2† (g)	46.7	60.2	37.5	16.6	91.3	35.9
Per cent recovered	101	102	165	123	101	93

* Test 84-3 was on a different glacier. No dye was recovered during test 85-6. Insufficient water-level data are available for mass-balance calculations on tests 85-8 and 85-9.

† In 1984, discharge at S-2 was estimated visually. In 1985, it was assumed to be 25% of that at S-1 (see text).

curves (Fig. 2). Such storage is not included in Equation (1); an additional $\partial S/\partial t$ (S is storage) term would be required to include it (Brugman, unpublished, equation (5.16)).

The commonest storage mechanisms are diversion of flow into stable eddies or relatively low-velocity anabranches, and adsorption of dye on to sediments followed by subsequent release of this adsorbed dye. Temporary storage in eddies is a dispersive phenomenon that does not obey the one-dimensional diffusion equation. Such dye is returned to the flow slowly after the main dye cloud has passed. Passage of dye through anabranches is a dispersive phenomenon that would conform to the behavior described by the diffusion equation only if there were a continuum of anabranch sizes and lengths. This process can modify the curves significantly, as illustrated by the extreme cases of tests 84-2 and 84-6. Adsorption of dye on to sediments has a negligible effect at the relatively low sediment concentrations, typically less than 1 g/l, with which we were dealing (Brugman, unpublished, p. 32 and 61).

Because the processes contributing to the tails are not described by the diffusion equation, we need a method of estimating D that emphasizes the rising part of the concentration–time curve and the peak. An approach suggested by Brugman (unpublished, p. 132–33) satisfies this condition. In this approach the "instantaneous" pulse of dye injected at $t = 0$ is approximated by a delta function. Equation (1) then has the following analytical solution:

$$c(t) = \frac{u}{Q}\frac{V_0}{(4\pi Dt)^{\frac{1}{2}}} \exp - \frac{(x - ut)^2}{4Dt} \qquad (2)$$

(Behrens and others, 1975, p. 379; Brugman, unpublished, equations (5.5) and (5.10); Fischer, unpublished, p. 227). In this equation, Q is the discharge (assumed constant), u is the flow velocity, V_0 is the volume of dye injected, and the other symbols are as defined earlier. Let t_1 and t_2 be the times when c reaches half of its peak value, c_m, during the rise to the peak and during the following decline from the peak, respectively. Furthermore, let t_i (i = 1,2) be a general symbol for t_1 and t_2. Finally, use the approximation:

$$u = x/t_m \qquad (3)$$

where x is the straight-line distance from the injection point to S-1 (~1000 m). Then:

$$\tfrac{1}{2}c_m = uV_0/2Q(4\pi Dt_m)^{\frac{1}{2}} \qquad (2a)$$

where t_m is the time to the peak. t_m is not taken from field data but instead is treated as a variable to be obtained in the solution. Setting the right-hand sides of Equations (2) and (2a) equal to one another with t_i inserted for t in Equation (2), simplifying, taking logarithms, and solving for D, we obtain:

$$D = \frac{x^2(t_m - t_i)^2}{4t_m{}^2 t_i \ln\left[2(t_m/t_i)^{\frac{1}{2}}\right]} . \qquad (4)$$

Equation (4) represents two equations (for i = 1,2) that can be solved iteratively for D and t_m. The values of D obtained for the englacial–subglacial channel system are given in Table I, and those for the pro-glacial channels will be found in Table III.

Having obtained D in this manner, we then used Equation (2) to calculate concentration–time curves for the 1985 tracer tests. In our calculations we obtained u from Equation (3) and used the mean value of Q over the duration of the test, Q_m. The value of V_0 was based on the amount of dye that passed S-1. The results are shown as dotted lines in Figure 2b.

The calculated and measured concentration–time curves differ in two principal ways: (1) the calculated curves are

TABLE III. DISPERSION AND DISCHARGE RESULTS FOR SYDJOKK, 1985

Test	*Date*	*Discharge*	*Velocity*	*Dispersion*	*Dispersivity*
		m^3/s	m/s	m^2/s	m
		Q_s	u	D	α
85-P1	16 Jul	1.12	0.69	1.9	2.8
85-P2	17 Jul	0.38	0.52	2.7	5.1
85-P3	17 Jul	0.48	0.59	2.0	3.4
85-P4	17 Jul	0.40	0.59	0.9	1.5
85-P5	18 Jul	0.49	0.49	2.2	4.4
85-P6	19 Jul	0.33	0.30	0.8	2.6

narrower, higher, and lack long tails; and (2) the area under the calculated curves often differs from that under the measured ones. The latter is due to the necessity of using the average discharge in the calculations. The former, as noted, can be attributed to temporary storage of dye during passage through the glacier (Brugman, unpublished, p. 141-48).

The concentration–time curves for the slug-discharge tests in Sydjokk are also typically asymmetrical (Fig. 4), suggesting that temporary storage also plays an important role in controlling the shapes of these curves.

VARIATION OF VELOCITY WITH DISCHARGE

In comparing velocities with discharge, we were primarily interested in the value of Q between the time of injection and the time of peak concentration, Q_p, as this is approximately the time interval over which u is calculated (Equation (3)). Thus, in Figure 5 u is plotted aqainst Q_p. In tests conducted after early July, u increases with increasing Q_p as might be expected. However, in the earliest test run, test 85-1, the velocity was much lower than it was during comparable discharges later in the season.

Velocities in the pro-glacial streams are compared with the discharge obtained from the slug tests, Q_s. Here, velocities are generally higher than in the englacial–subglacial system. This is, at least in part, a consequence of the higher slopes in the former.

The difference in the rate of increase in u with Q in the englacial–subglacial tests and the pro-glacial tests is significant and will be taken up in the discussion.

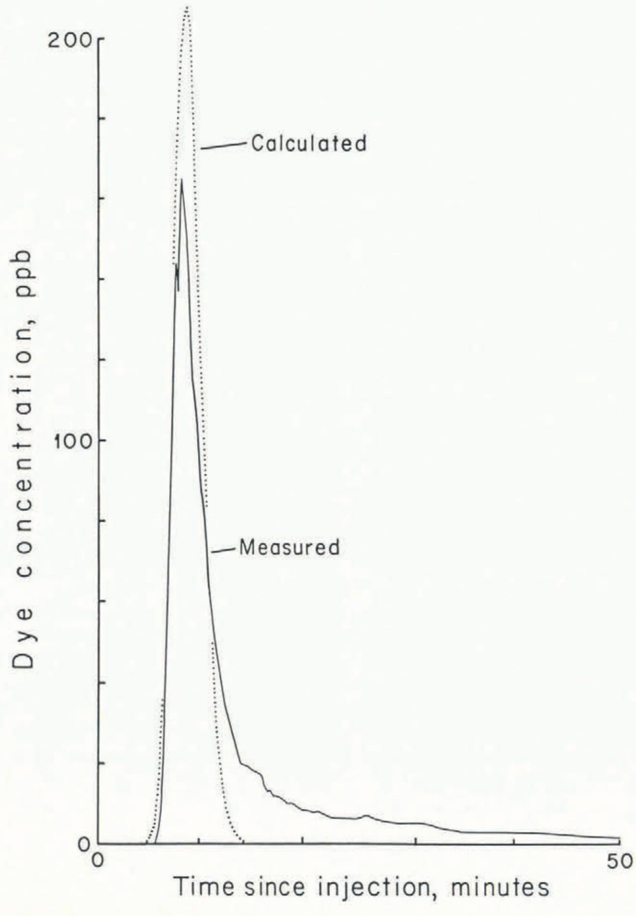

Fig. 4. Concentration–time curve from slug-discharge test 85-P3. Dotted curve is calculated from Equation (2).

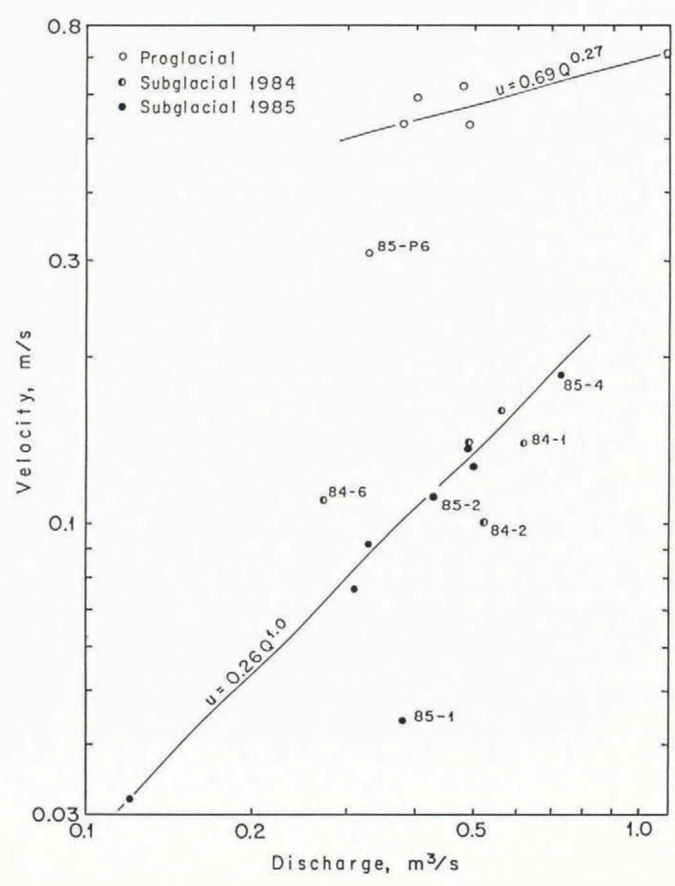

Fig. 5. Variation of flow velocity, u, with discharge, Q_p. (For pro-glacial tests Q_s is plotted rather than Q_p.)

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VARIATION OF DISPERSION WITH VELOCITY

In a given channel with a constant roughness, it has been found that D is approximately proportional to u (e.g. Fischer, unpublished, p. 2). We thus write $D = \alpha u$ and, following Brugman (unpublished, p. 160), will refer to α as the dispersivity.

In order to study changes in the conduit system during the melt reason, we have plotted D against u in Figure 6. For most of the englacial–subglacial experiments, D increases approximately linearly with u and $\alpha \simeq 6.5$ m. For the early season tests in 1985, however, α shows a progressive decrease from ~80 m during test 85-1 to ~30 m during test 85-4. In the pro-glacial tests, $\alpha \simeq 4$ m. For comparison, Behrens and others (1975) obtained a value of 4 m for the subglacial conduit system on Hintereisferner. The significance of these changes is discussed below.

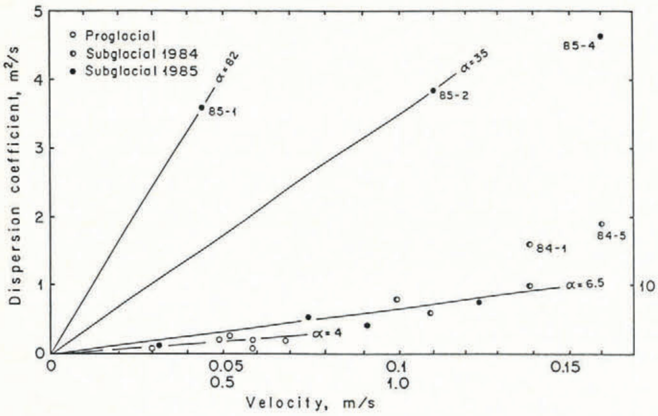

Fig. 6. Variation of dispersion coefficient with velocity, u. Ordinate scale on right and lower scale on abscissa apply to pro-glacial tests.

DISCUSSION

The englacial and subglacial channel network could consist of a simple arborescent system of channels. Alternatively, it might be a braided network with distributaries as well as tributaries. We have noted that, in the absence of temporary storage, a braided system with a continuum of anabranch sizes and lengths, herein called "homogeneously braided", would be expected to produce a concentration–time curve with a single broad peak that could be described by Equation (1) with an appropriately selected dispersion coefficient.

Multiple peaks in concentration–time curves

A surprising amount can be deduced about the geometry of the channel system through detailed analysis of the two tests which yielded triple peaks in the concentration–time curves, tests 84-2 and 84-6 (Fig. 2a). In these cases, the channel system was clearly not homogeneously braided. The simplest explanation for the pattern in test 84-2 is that two low-velocity anabranches diverged from and later rejoined the main channel somewhere between the injection point and the point where the flow to S-2 split off from the main channel. As test 84-1 was conducted under similar discharge conditions from the same moulin 5 d earlier, some change must have occurred during the intervening days. A storm resulting in high discharges occurred during this time period and may have been responsible for opening the new channels. Furthermore, because the moulin used for this test and test 84-4 were probably hydraulically connected, these channels must have become closed before test 84-1, 13 d later. These changes could have occurred in either the englacial or subglacial parts of the system. Englacial changes might have resulted, for example, from extension of old crevasses or opening of new ones that intercepted flow from the injection point and led it to different englacial passages. Later deepening of the channel by melting could have cut off flow to these distributaries.

The curves for test 84-6 are different, both in that the three peaks are much closer together, and in that clear secondary peaks occurred only at S-1. Important is the fact that the discharge during test 84-6 was about half that during test 84-2. A simple explanation for the curves from test 84-6 is that, as the discharge decreased, the flow in two anabranches decreased more than that in a third. Velocity differences among the anabranches could thus have become exaggerated. Alternatively, the number of connections between anabranches may have been reduced, resulting in less homogenization of the flow. The separation of the peaks is about 45 min, or about one-third of the time between injection and the first peak, so the anabranches must have been fairly long.

In 1985, dye concentrations at S-2 were typically ~25% of those at S-1 (Fig. 2b). If we assume that the dye was homogeneously distributed in the flow at the point where the channel to S-2 diverged from the main flow, the flow to S-2 must later have been augumented by at least a factor of 4. To the extent that the remaining flow to S-1 was increased down-stream from the split, this factor would be larger. Furthermore, as the discharge at S-2 is typically only ~25% of that at S-1, the fraction diverted to S-2 must have been equal to or less than ~6% (1/16) of the total immediately above the bifurcation. In the 1984 tests this fraction was ~4%.

The bifurcation to S-2 was above the point where the flow in the south lateral stream joined the main stream to S-1. This is indicated by the fact that dye injected at ST-1 (test 85-3) did not appear at S-2. The flow past ST-1 disappears in a moulin just below the upper riegel so the location of its junction with the main flow is not well constrained.

Late in 1985, at least one additional connection between the main stream and S-2 was apparently opened. This is suggested by the double peaks in the concentration–time curves for S-2 in tests 85-8 and 85-9.

In summary, down-glacier from the bifurcation leading to S-2, the normal late-season glacial drainage system apparently consists of a minimum of three main channels during low flows. There are, however, enough cross links between these channels so that the system behaves as a homogeneously braided stream during moderate to high flows. The main bifurcation to S-2 lies up-glacier from the junction between the channel leading to S-1 and the south lateral stream. Occasionally, there are other active cross links from S-1 to S-2. The volume of water thus diverted from S-1 to S-2 is, however, a small fraction of the total going to S-1, and is only part of the total appearing at S-2. During test 84-2, two additional flow paths were active up-glacier from the bifurcation leading to S-2. In short, the subglacial drainage system is decidedly braided and far from simple in geometry.

Hydraulics of the subglacial channels

Let us now turn our attention to an hydraulic comparison of the glacial and pro-glacial drainage systems, starting with the channel roughness, n, as defined by the Gauckler–Manning–Strickler equation:

$$u_{\mathrm{act}} = \frac{d^{2/3} S^{1/2}}{n} . \tag{5}$$

In this equation, u_{act} is the actual water velocity in the stream, d is the hydraulic radius which is essentially equal to the depth in the channels under consideration, and S is the hydraulic gradient driving the flow. Sydjokk is typically ~0.2 m deep and has an average slope of 0.21. Using a typical value of u_{act} of 0.55 m/s, n is found to be 0.3 m$^{-1/3}$ s. This is a high value; Fahnestock (1963), for example, found values in the range of 0.03–0.07 for channels of the braided White River which drains Emmons Glacier on Mount Rainier, and the values in engineering tables (e.g. Posey, 1950) do not go up this high. The high value in Sydjokk probably reflects the small size of the individual anabranches of the stream and the large size of the boulders comprising the channel boundaries.

For the glacial system, we do not have enough information to solve directly for the roughness, as we know neither the sinuosity nor the depth. Thus, our approach will

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be to assume a roughness, based on comparison with other data, and then explore the consequences of this assumption. Röthlisberger (1972) used a value of n of 0.1 $m^{-1/3}$ s to represent boulder-strewn channels at the bed of a glacier, and Posey (1950) suggested that, for streams with extremely bad alignment and deep pools, values as high as 0.15 $m^{-1/3}$ s would be appropriate. Our values are intended to represent flow along an irregularly sinuous course over a possibly boulder-strewn bedrock surface that is alternately smoothed by glacial abrasion and roughened by plucking. The conduits may be locally laterally bounded by ice and, when discharges are high enough to fill them, they will also be bounded by ice on the top. We conclude that n is probably not as low 0.1 $m^{-1/3}$ s but perhaps not as high as that in the pro-glacial stream. Furthermore, n probably varies with flow depth and with the degree to which water is in contact with the overlying ice. However, for purposes of illustration in the discussion below, we will assume a constant value of 0.2 $m^{-1/3}$ s.

Turning next to the velocity, we note that in the pro-glacial system u increases approximately as the 0.27 power of the discharge, Q (Fig. 5). This is typical of conditions on alluvial rivers; Fahnestock (1963) found an exponent of 0.27 for the braided channels of White River, which can widen rapidly by bank erosion when discharges increase, and Leopold and Maddock (1954) obtained a mean value of 0.34 for rivers of the south-western U.S. and Great Plains where greater bank stability inhibits such widening.

In contrast, the corresponding relation for the late-season glacial system has an exponent of 1.0. In interpreting this, we need to remember that u is not the actual water velocity in the channels, u_{act}, because the sinuousity of the channels, S_n, has not been taken into consideration. Instead, $u_{act} = uS_n$, or in terms of Q, using the regression relation in Figure 5:

$$u_{act} = 0.26 S_n Q^{1.0}. \tag{6}$$

If S_n is constant and, if the width, w, were also constant, then the depth, d, could not increase with Q as is readily seen from the continuity equation

$$Q = w d u_{act}. \tag{7}$$

However, from Equation (5) we see that, if S and n remain approximately constant, d would have to increase as $Q^{3/2}$ to account for all of the increase in u_{act} in this way. Apparently S_n, w, S, and n cannot all remain constant.

One resolution of this paradox might be to assume that at low flow the channel(s) are nearly full, and that as the discharge increases the channels become full and water then backs up in the hydraulic system, thus increasing the hydraulic gradient driving the flow. This is consistent with water-level recordings in bore holes somewhat down-glacier from the moulins used as injection points. These recordings show that, except during unusually cold periods, the water level varies diurnally from mid-July to late August (Hooke and others, 1987). Peak water levels occur in the late afternoon. On especially warm or rainy days, they may equal or locally exceed the overburden pressure. Minimum water levels occur between midnight and about 06.00 h. They are less than ~35 m above the bed and probably drop to the bed.

However, a simple calculation shows that this effect alone cannot explain the observed change in velocity with discharge. Assuming that the conduit system becomes full at a discharge of 0.1 m^3/s and that S is given by the mean slope from the lowest point on the upper riegel to S-1, 0.041, we find that raising the water level in the glacier to the point where the basal water pressure equals the overburden pressure would increase the velocity by a factor of 2, whereas the measured increase over the range of discharges studied is more than a factor of 6.

An alternative is to assume that the subglacial channels are open under daily average flow conditions, as suggested by Hooke (1984) on the basis of theoretical calculations, and that as discharge increases, low divides between channels are breached and the flow is able to follow an increasingly direct path between the injection point and S-1. In other words, S_n decreases with increasing Q.

For purposes of illustration, suppose we assume that S_n is proportional to $Q^{-0.5}$ and that $S_n = 1.0$ at a discharge of 0.75 m^3/s, which is close to the maximum observed (Fig. 5). For $Q = 0.1$ m^3/s, S_n then turns out to be 2.75. A few simple calculations, using Equations (5)–(7), then yield d = 0.057 m and w = 20 m for Q = 0.1 m^3/s, and d = 0.089 m and w = 42 m for Q = 0.75 m^3/s. Again, the values border on the unreasonable. The widths are high, even considering the presumed braided nature of the streams, and the rather small increase in d with Q makes it difficult to visualize how S_n could decrease so much as Q increases.

We conclude that, as Q increases, some anabranches of the braided stream become full so water backs up in the system. This increases the hydraulic gradient driving the flow in at least some anabranches. It also decreases the sinuosity, both because some sharp bends in more tortuous anabranches are cut off as water levels rise, and possibly also because less sinuous anabranches at higher levels on the irregular bed or entirely within the ice are occupied.

Dispersivity

We now return to a consideration of the dispersivity, α. α is expected to be dependent on a length scale of the system (Fischer, unpublished, p. 2). In a braided system, Brugman (unpublished, p. 173–74) suggested specifically that

$$\alpha = \tfrac{1}{2}(\sigma_I/t_I)^2 L_I \tag{8}$$

where L_I is the average length of anabranches, t_I is the average time required for flow through an anabranch, and σ_I is the standard deviation of these times. As $L_I = u_{act} t_I$, we can rewrite Equation (8) as:

$$\alpha = \tfrac{1}{2}(\sigma_I u_{act})^2 / L_I. \tag{9}$$

To the extent that u_{act} is constant, high values of dispersivity could result from small L_I or large σ_I. In other words, such values suggest more intensely braided conditions.

The high values of α early in the season (tests 85-1 through 85-4; Fig. 6) are thus consistent with a conceptual model in which the englacial–subglacial drainage consists of an intensely braided channel system with many anabranches of quite variable size. In such a system σ_I would be relatively large. As dominant passages are enlarged by melting and thus capture more of the flow (Röthlisberger, 1972; Shreve, 1972), σ_I should decrease and L_I should increase. The system reaches a stable configuration sometime in mid to late July and thereafter α remains constant.

Of interest is the observation that the 1984 tests generally follow the late-season curve in Figure 6. We infer that most of the transition from early season to late-season conditions occurred before 13 July (test 84-1) in 1984, although it did not occur until after 22 July (test 85-4) in 1985. From mid-May until the end of June there were 60% more degree-days above 0°C in 1984 than in 1985, so run-off during the early part of the season should have been higher in 1984. Thus, it is reasonable to expect that the drainage system acquired its late-season configuration earlier in 1984.

This model is consistent with the observation that u reaches its late-season values earlier than does α. We imagine that, as the drainage system develops, the most direct conduits are opened first, increasing u, while many of the more tortuous anabranches at higher levels remain small, contributing to high dispersion values.

The dispersivity in the pro-glacial streams is slightly less than that in the englacial–subglacial system. This suggests that the englacial–subglacial drainage is somewhat more intensively braided than the pro-glacial one. To provide a visual basis for such a comparison, the latter is shown in Figure 7. T.J. Hughes (personal communication) has suggested that the more intense braiding of the glacial system may reflect, primarily, its possible three-dimensional character with both englacial and subglacial anabranches. The question of whether anabranches can diverge from the ice–bed interface upward into the ice and then return to the bed further down-glacier does not have an obvious answer, although the greater density of water might make this unlikely.

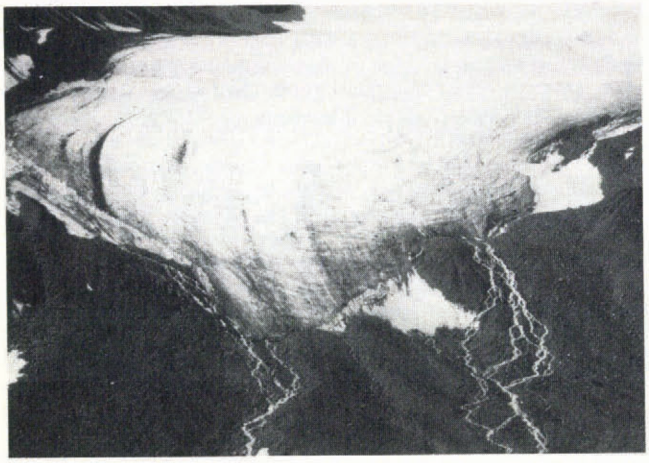

Fig. 7. Photograph of glacier snout showing Sydjokk on the left and Nordjokk on the right.

CONCLUSIONS

The drainage system leading from the moulins in the middle of the ablation area of Storglaciären to sampling site S-1 seems to consist of a single homogeneously braided stream under most conditions. The degree of braiding is apparently somewhat greater than that of pro-glacial Sydjokk down-stream from S-1. As discharges increase, individual low-lying anabranches of the streams become full, resulting in increased hydraulic gradients and hence increased velocities. At the same time, the overall sinuosity of the stream decreases as water overflows bends in more-sinuous anabranches and as less-sinuous higher-level conduits are occupied.

Early in the melt season, the individual anabranches are small. As low-lying anabranches are enlarged by melting, velocities increase. However, it is not until these anabranches are enlarged to close to their maximum size that dispersivity values drop to their late-season levels.

Much of the flow emerging at S-2, which constitutes about one-quarter of the flow in Sydjokk, follows a separate drainage system. This system is connected to the system leading to S-1 by at least one conduit early in the melt season and at least two later.

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