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Critical sets in latin squares and their application to minimal defining sets of designs

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This thesis is devoted to two closely related topics. These topics are critical sets in latin squares and the very similar concept, minimal defining sets for block designs.

The first chapter presents the necessary background material and a view of the previous results in each of these areas. It also provides some results used in the third and fourth chapters.

The second and fifth chapters are entirely devoted to critical sets in latin squares and the weaker concept of partial latin squares with unique completion. This work involves taking two known partial latin squares with unique completion, or critical sets in latin squares, and using a product construction to produce new partial latin squares with unique completion, or new critical sets in larger latin squares.

The results of the fifth chapter are specifically for critical sets of the latin squares which are the direct product of two latin squares, one of which is associated with the cyclic group of order 2 and the other with a cyclic group of odd order.

Chapters 3 and 4 of this thesis examine defining sets of some Steiner triple systems (or $2-(v, 3, 1)$ designs). The Steiner triple systems studied are those of two infinite families; one is the family of triple systems corresponding to the projective geometries over the Galois field of order 2 and the other is the family of triple systems from the affine geometries over the Galois field of order 3. In each case the geometric structure is used to choose the blocks for the defining sets.

A defining set of a design is analogous to a partial latin square with unique completion and a minimal defining set is analogous to a critical set of a latin square. But this is not the only connection between the two topics of this thesis. Results about partial latin squares with unique completion (some of which are new results of my own and some of which were established by others) are used to show that the chosen subsets

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of these designs are defining sets. Finally properties of the geometry are used to show that the defining sets are minimal defining sets.

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