38. Radial Pressure in the Solar Nebula as Affecting the Motions of Planetesimals

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In the classical rotating Laplacian-type nebula, pressure gradients can develop radially to the protosun because of central radiation, particle ejection, and magnetic-field expansion or because of radial temperature or total gas density gradients. Except for the last two effects, the acting central acceleration for the gas is reduced from the gravitational value; the pressure gradient in the gas caused by temperature or density gradients may either add to or subtract from the gravitational acceleration, depending on the sense of the pressure gradient. Planetesimals in the nebula may thus experience tangential accelerations (+ or −) with respect to the gas because of the differential radial accelerations acting on the particles and the gas. As a consequence, the planetesimals may spiral outward or inward with respect to the protosun. The present paper deals with growing planetesimals and a range of drag laws depending on the Reynolds number and on the ratio of particle size to mean free path.

Particles spiral in the direction of positive pressure gradient, thus being concentrated toward toroidal concentrations of gas. The effect increases with decreasing rates of particle growth, i.e., with increasing time scales of planet formation by accretion. In the outer regions, where evidence suggests that comets were formed and Uranus and Neptune were so accumulated, the effect of the pressure gradient is to clear the forming comets from those regions. The large mass of Neptune may have developed because of this effect, perhaps Neptune's solar distance was reduced from Bode's "law," and perhaps no comet belt exists beyond Neptune. In the asteroid belt, on a slow time scale, the effect may have spiraled planetesimals toward Mars and Jupiter, thus contributing to the lack of planet formation in this region.

BASIC PRINCIPLES

LET US ASSUME THAT DURING PLANETARY FORMATION in our solar system, gas and dust grains constitute a Laplacian-type nebular disk rotating about a central mass, roughly that of the Sun today. The grains or planetesimals are generally accreting in all size ranges from atomic and molecular to large bodies. For smaller bodies, the gas, presumed to be approximately a solar mix, acts as a buffer against high velocities of encounter that might cause collisional destruction. The motions are fairly circular, controlled by a nearly inverse-square law of central force, while the gas and, at first, the grains are spread about the fundamental plane, held by the gas pressure against the surface gravity in the plane and the perpendicular component of the central force.

In such a system the gas pressure varies from

https://doi.org/10.1017/S0252921100049277 Published online by Cambridge University Press

point to point, depending on gravity and possible magnetohydrodynamic effects if a plasma is present. Radial variations in gas pressure affect the net central radial acceleration on the gas, causing the gas to deviate from the motion appropriate to the local gravitational acceleration and therefore appropriate to the motions of the solid grains, as shown in figure 1. The grains thus meet a resisting medium and tend to move in toward the protosun or are accelerated radially outward, depending on the sense of the effective radial acceleration acting on the gas.

Let Δg be the deviation from the central gravitational acceleration acting on the gas, whether caused by spiraling magnetic fields or by a radial pressure gradient in neutral gas, dP/dr, where P is the total gas pressure and r is the radial distance to the center of the protosun. For a gas of density ρ, the radial correction to the central acceleration is, by the classical formula,

$$\Delta g = +\frac{1}{\rho}\frac{dP}{dr} \tag{1}$$

Note that in equation (1) the sign is reversed from that for stellar interiors because the gas in the nebula is held in radial equilibrium by its

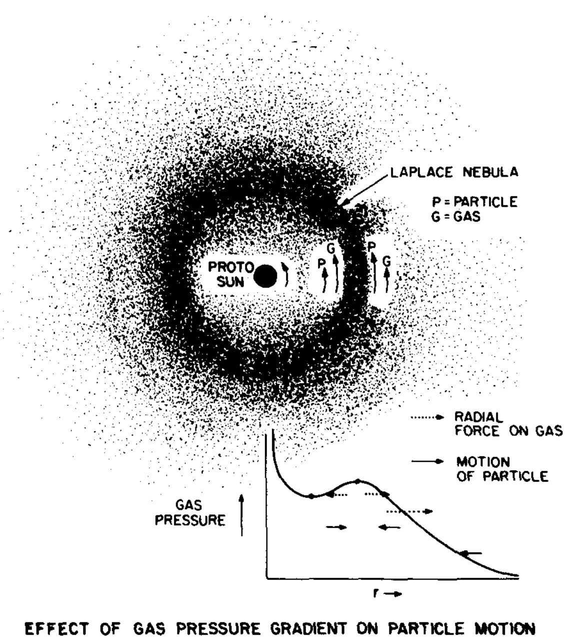

Figure 1.—Effect of gas pressure gradient on particle motion.

motion in the central gravitational field for $dP/dr=0$. A positive pressure gradient outward from the center increases the effective central acceleration from that of gravity alone and causes the gas in equilibrium motion to rotate more rapidly. In that case, the gas accelerates the grains in their near-circular motion and causes them to move radially outward. The opposite is true for $dP/dr<0$.

To determine the quantitative motion of the grains, note that the near-circular velocity change ΔV in the gas under a central acceleration g follows from the velocity law

$$V = g^{1/2} r^{1/2} \tag{2}$$

by differentiation, to the form

$$\Delta V = \frac{r^{1/2}\Delta g}{2g^{1/2}} \tag{3}$$

A spherical grain of radius s, density ρ_s, and mass m will experience a drag $(-)$ or accelerating $(+)$ force F by interaction with the gas at relative velocity v, where $v<\Delta V$. The drag equation in a neutral gas is formally stated as

$$F = \frac{C_D}{2}\pi s^2 \rho v^2 \tag{4}$$

where C_D is the dimensionless drag coefficient dependent on the Reynolds number R_e if the mean free path L of the gas atoms or molecules is small compared to s.

For gases the Reynolds number is defined by

$$R_e = \frac{2\rho v s}{\eta} \tag{5}$$

where η is the viscosity of the gas, given approximately by $\eta = \frac{1}{2}\rho\bar{v}L$, in which $\bar{v}$ is the mean kinetic speed of the atoms or molecules.

In case $L<s$, following the approximations by Probstein and Fassio (1969),

$$C_D = 24R_e^{-1} \quad \text{for} \quad R_e<1 \tag{6a}$$

$$C_D = 24R_e^{-3/5} \quad \text{for} \quad 1<R_e<10^3 \tag{6b}$$

$$C_D = 4/9 \quad \text{for} \quad R_e>10^3 \tag{6c}$$

where equation (6a) represents Stokes' law of drag and (6c) the Newtonian case, which holds fairly well for supersonic velocities $(v>\bar{v})$.

For the subsonic case when $L \geq s$, the drag equation becomes generally complicated, but for $v \ll \bar{v}$, the Epstein approximation has some validity:

$$F(\text{Epstein}) = A\bar{v}\rho s^2 v \qquad (7)$$

where $A \simeq 4\pi/3$ if the accommodation coefficient of atoms or molecules on the moving sphere is unity.

For our purposes, the various forms of the drag equation can be simplified by the introduction of the quantity *stopping time* t_e, which is the ratio of the body's speed to the absolute value of the deceleration from drag. Equations (6a), (6b), (6c), and (7), respectively, can then be expressed as follows:

$$R_e < 1 \qquad t_e = \frac{2\rho_s s^2}{9\eta} \qquad (6'a)$$

$$1 < R_e < 10^3 \qquad t_e = \frac{2^{3/5}\rho_s s^{8/5}}{9\eta^{3/5}\rho^{2/5}v^{2/5}} \qquad (6'b)$$

$$R_e > 10^3 \qquad t_e = \frac{6\rho_s s}{\rho v} \qquad (6'c)$$

$$\text{Epstein} \qquad t_e = \frac{\rho_s s}{\rho \bar{v}} \qquad (7')$$

where for all, the drag law becomes simply

$$\frac{F(v)}{m} = \frac{v}{t_e} \qquad (8)$$

In the case of equations (6′b) and (6′c), t_e has been changed from the *e*-folding decay time by a factor of less than 2 to produce the generality of equation (8), which is correct for all cases. Stokes' law (eq. (6′a)) attains an error of only a factor of 3 up to $R_e = 10^2$.

The motion of the spherical grain or planetesimal in the solar nebula under influence of the gas "wind" is generally complicated. The present paper will deal in detail with only two extreme and simple illustrative cases: (A) where the "drag" force F for $v \sim \Delta V$ is large so that the grain at velocity ΔV with respect to the gas would be stopped in a small fraction of the period of revolution T_p, i.e., $t_e \ll T_p/2\pi$; and (B) where the grain or planetesimal at relative velocity $v = \Delta V$ would not be stopped for a time comparable to or greater than the period of revolution, i.e., $t_e \gg T_p/2\pi$.

In case A, where $t_e \ll T_p/2\pi$, the grain will largely follow the rotational motion of the gas and thus experience a central acceleration reduced or increased by Δg. Hence, it will execute a radial motion at velocity v with respect to the gas derived from $F(v) \simeq m\,\Delta g$.

In case B, where $t_e \gg T_p/2\pi$, the much larger grain or planetesimal will experience a tangential drag or an accelerating force at velocity $\sim\Delta V$ and slowly spiral radially in or out with respect to the rotating system of gas according to the modified equations of motion in a central force.

EQUATIONS OF MOTION

Case A.—Grains carried along with the gas, $t_e \ll T_p/2\pi$. The relative velocity v of the grain in the gas is smaller than ΔV and very much smaller than the speed of sound. Thus, case A divides into only two subcases: case A1, when the mean free path L is larger than the particle radius s, requiring Epstein's law (eqs. (7), (7′), and (8)); and case A2, when $L \leq s$ and R_e is small, requiring Stokes' law (eqs. (6′a) and (8)).

Case A1.—$v \ll \Delta V$, $v \ll \bar{v}$, $L > s$, and $t_e \ll T_p/2\pi$. For a grain (radius s, mass m, density ρ_s) the terminal radial speed in the cloud under the acceleration Δg becomes from Epstein's law (eqs. (7′) and (8))

$$\frac{dr}{dt} = t_e\,\Delta g = \frac{\rho_s s}{\rho \bar{v}}\,\Delta g \qquad (9)$$

where Δg is given by eq. (1) for a radial pressure gradient in the gas or by the effect of a radial acceleration on the gas from some other source such as a magnetic field.

Case A2.—$v \ll \Delta V$, $v \ll \bar{v}$, $L \leq s$, and $t_e \ll T_p/2\pi$. Here the terminal radial speed of the grain is derived from Stokes' law (eqs. (6′a) and (8)), so that

$$\frac{dr}{dt} = t_e \Delta g = \frac{2\rho_s s^2\,\Delta g}{9\eta} \qquad (10)$$

where η is the viscosity of the gas.

Equation (10) is independent of the gas density so long as $L \leq s$ and holds fairly well to $R_e = 10$ and within a factor of 3 to $R_e = 10^2$.

Case B.—Grains and planetesimals meeting a resisting medium at $v \simeq \Delta V \ll V$. Here, $t_e \geq T_p$ and generally $L < s$, $v \ll \bar{v}$, while R_e may become

https://doi.org/10.1017/S0252921100049277 Published online by Cambridge University Press

appreciable for large bodies, although in most cases of interest R_e is small. Tisserand (1896) solved this two-body problem for small orbital eccentricity e and constant $v=\Delta V \ll V$. To the first order in e, $de/dt=0$, so that a small orbital eccentricity is retained either under resistance or acceleration for gas in nearly circular motion.

For circular velocity V of mass m at distance r from the effective center of mass M, and a resisting or accelerating force F, the classical two-body solution takes the form

$$\frac{1}{r}\frac{dr}{dt}=\frac{2F(\Delta V)}{mV}=\frac{2\Delta V}{t_e V}=\frac{1}{t_e}\frac{\Delta g}{g} \qquad (11)$$

by equations (2), (3), (5) and (8).

The appropriate expression for t_e can be chosen from equation (6′a), (6′b), or (6′c), depending on the value of R_e for $v=\Delta V$ (eqs. (5) and (4)). Since the spiraling rate is extremely small for large bodies with large values of R_e, only the Stokes' law (eq. (6′a)) equation will be presented explicitly, viz:

$$\frac{dr}{dt}=\frac{9\eta r}{2\rho_s s^2}\frac{\Delta g}{g} \qquad (12)$$

The intermediate case between A and B, when $t_e \sim T_p/2\pi$, has been solved by F. Franklin (private communication) for Epstein's and Stokes' laws, the drag being proportional to v. The maximum rate of orbital change is $dr/dt=0.5\Delta V$ at $t_e=0.8T_p/2\pi$. When $t_e=T_p$, dr/dt equals 0.8 the value given by equation (11), approaching equation (11) as a limit for larger values of t_e. For $t_e<T_p/30$, equations (9) and (10) are correct within about 20 percent and improve for smaller t_e. The intermediate case thus gives values of dr/dt varying from $0.3\Delta V$ at $t_e=T_p/30$ to a maximum of $0.5\Delta V$ at $t_e=T_p/8$ and dropping to $0.3\Delta V$ at $t_e=T_p$. Near but outside these limits, equations (9) and (10) or equation (11) gives fairly satisfactory values for dr/dt.

EFFECTS OF GRAIN GROWTH ON RADIAL MOTION

We wish to consider grains that are growing by the accretion of atoms and molecules from the nebula. For atoms or molecules that can "freeze" on the grain, constituting a fraction f_i of the total density by weight, with an accommodation coefficient α_i, a mean molecular velocity $\bar{v}_i$, and a mean diffusion coefficient D_i, a spherical grain of radius s and density ρ_s grows at a rate

$$\frac{ds}{dt}=\frac{\alpha_i f_i \bar{v}_i \rho}{4\rho_s}\left(1+\frac{\bar{v}_i s}{4D_i}\right)^{-1} \qquad (13)$$

The second term becomes significant when the grain has reached such a size that for a velocity v through the gas small compared to $\bar{v}_i$, grain growth is inhibited by gas diffusion of the appropriate atoms and molecules. Approximately, $D_i=1.4\eta_i/\rho$, where η_i is the viscosity appropriate to the "freezing" atoms and molecules. Hence, from the second term of equation (13), the grain growth rate slows when $s \gg 5.6\eta_i/\bar{v}_i\rho$, and is given by

$$\frac{ds}{dt}=\frac{1.4\alpha_i f_i \eta_i}{\rho_s s} \qquad (14)$$

becoming independent of the general gas density and varying inversely as the radius of the grain.

The transition from rapid grain growth (first term of eq. (13); e.g., Kuiper, 1951) to diffusion-limited grain growth (eq. (14)) begins when the mean free path L_i becomes small compared to s, or roughly when the drag law at low grain velocities changes from the Epstein law (eq. (9)) to Stokes' law (eq. (10) or (12)). Hence, for the Epstein case (A1) we keep only the first term in equation (13), so that a very small grain grows to radius s_1 in time t_1 given by

$$t_1=\frac{4\rho_s}{\alpha_i f_i \bar{v}_i \rho} s_1 \qquad (15)$$

Again for the Epstein case, by equations (9) and (15), the change in radial distance from r_0 at $t=0$ to r_1 at t_1 and s_1 becomes

$$r_1-r_0=\frac{\alpha_i f_i \bar{v}_i \Delta g}{8\bar{v}} t_1^2=\frac{2\rho_s^2 \Delta g s_1^2}{\alpha_i f_i \bar{v}_i \bar{v} \rho^2} \qquad (16)$$

applicable when $v \ll \bar{v}$, $t_e \ll T_p/2\pi$, and $L \leq s$ and when changes in ρ and g with r are neglected. Equation (16) applies well in the assumed low-density region of the solar nebula beyond Uranus, where cometary accretion is expected. Note that exhaustion of condensable gas slows the growth rates and thereby increases the total amount of spiraling.

Within Jupiter's orbit, the relatively high gas density usually assumed and the consequent rapid growth rates carry the grain through the regimes

https://doi.org/10.1017/S0252921100049277 Published online by Cambridge University Press

of Epstein's and Stokes' laws rather quickly. Thus, equation (12) can be used if applicable, and we will introduce the equations for large Reynolds numbers (t_e from eq. (6′c) and dr/dt from eq. (11)) and larger bodies, even though the relative velocity ΔV (eq. (3)) may be small. On this basis, the slow growth from s_2 to s_3 will take place by equation (14) in time $t_3 - t_2$ given by

$$t_3 - t_2 = \frac{\rho_s(s_3^2 - s_2^2)}{2.8\alpha_i f_i \eta_i} \tag{17}$$

while

$$\frac{1}{r}\frac{dr}{dt} = \frac{\rho(\Delta V)^2}{3\rho_s V}\frac{1}{s} \tag{18}$$

where the sign of dr/dt is that of ΔV, given by equation (3), $\Delta V = 2^{-1} r^{1/2} g^{-1/2}\, \Delta g$.

APPLICATION IN THE "COMETARY REGION"

Let us arbitrarily define the "cometary region" at $r = 25$ AU, with a typical density in the plane of $10^{-11.4}$ g cm^{-3}, $T = 55°$ K, $P = 10^{-2.1}$ dyn cm^{-2}, giving a total areal density across the plane of $10^{2.3}$ g cm^{-2} (2×10^{-5} solar mass per square AU) for a solar mixture by weight of gases (H, He, Ne, Ar) 0.9803, ices (C, N, O plus H) 0.0175, and Earthy materials (heavy elements) 0.0022. The corresponding viscosity will be $\eta = 10^{-4.3}$ cgs, $\bar{v} = 10^{4.85}$ cm s^{-1} for a mean molecular weight of 2.34, and $\bar{v}_i = 10^{4.41}$ cm s^{-1} for the ices plus Earthy material of mean molecular weight 18.4 and a mass fraction $f_i = 0.0197$.

The peak of the gas pressure and density will have been farther out, near $r = 30$ AU at Neptune's present solar distance, but the order-of-magnitude calculations at $r = 25$ AU will illustrate the nature of the spiraling phenomena caused by a negative pressure gradient outward near the edge of the nebula. Suppose the pressure falls linearly with r to 0 at 50 AU. Then $dP/dr = -10^{-16.7}$ cgs, while $\Delta g = 10^{-5.3}$ cm s^{-2} by equation (1) and $\Delta V = 10^{3.2}$ cm s^{-1} by equation (3) for a central solar mass. A forming icy grain of radius s and density 0.1 g cm^{-3} thus meets a resisting velocity of 16 m s^{-1}, small compared to molecular velocities. The Reynolds number for equation (5) becomes $R_e = 10^{-3.6}\, s$ (cm), so that we are in the realm of Epstein's and Stokes' laws of drag up to "cometesimals" of radius 100 m or larger at great solar distances. Correspondingly, the mean free path of the gas molecules is of the order of 1 m or more so that little error is made in applying Epstein's law up to perhaps $s = 10$ m, while to about the same limit the diffusion slowing of the accretion rate by the second term of equation (13) can be neglected. Hence, by equation (15), the time t_1 for a tiny grain to grow to a radius s_1 becomes

$$t_1 = 6.3 s_1(\text{cm})\,\text{yr} \tag{19}$$

if we accept an accommodation coefficient $\alpha_i = 1.0$.

Thus, our cometesimal will grow to a radius of 10 m in some 10^4 yr. Its loss of solar distance from r_0 to r_1, given by equation (16), then becomes

$$r_0 - r_1 = 1.2\times10^{-5}\, s_1^2 \text{ AU} \tag{20}$$

so that the cometesimal spirals in "nominally 12 AU" by the time it has grown to a radius of 10 m, neglecting significant changes in density and other quantities depending on solar distance.

Note that the time varies inversely as the nebular density ρ and that the amount of spiraling varies as ρ^{-2}, so that the inward spiraling for a constant $\Delta g = dP/dr\rho$ is increased for cometesimals of a given size at greater solar distances or at lower solar densities, even though the time scale increases as ρ^{-1}. Reduction in growth rates by any cause increases the total amount of spiraling for a given gas density and gradient.

The sizable magnitude of the spiraling rate for cometesimals growing at the edge of the solar nebula may explain three facets of the present solar system: (a) the comparable masses of Neptune and Uranus, (b) the reduction of Neptune's solar distance from Bode's "law," and (c) the still unobservable "comet belt" beyond Neptune, expected by the writer (Whipple, 1964).

If we assume (Kuiper, 1951; Cameron, 1962; Whipple, 1964) that Uranus and Neptune are aggregates of cometesimals, then it is otherwise surprising that there should have been enough material at Neptune's distance to make a planet as large as Uranus. The spiraling effect, however, could bring in the cometesimals to a solar distance where the pressure gradient was smaller and pile up the cometary material from greater distances for accretion at Neptune, producing no sizable planet (Pluto?) beyond. Neptune's mean solar distance may also have been reduced. Thus, my expectation of a comet belt beyond Neptune is perhaps unwarranted. In fact, the observations indicate that less than one Earth mass exists in a ring beyond Neptune to a solar distance of 50 AU (Hamid, Marsden, and Whipple, 1968).

APPLICATION IN THE ASTEROIDAL REGION

We may take a "typical" asteroidal region at $r=2.5$ AU, with density in the plane of $10^{-8.4}$ g cm^{-3} or a pressure of $10^{-4.1}$ atm, $T=550°$ K, $P=10^{+1.9}$ dyn cm^{-2}, giving a total areal density across the plane of $10^{4.3}$ g cm^{-2} (2×10^{-3} solar mass per square AU), $\eta=10^{-3.8}$ cgs, $\bar{v}=10^{5.35}$ cm s^{-1}, and $\bar{v}_i=10^{4.78}$ cm s^{-1} for Earthy molecules of mean molecular weight 33.5 and a mass fraction $f_i=0.0022$. The assumed temperature is that derived by Larimer and Anders (1967), supported by the further conclusions of Keays, Ganapathy, and Anders (1971) that in 13 chondrites at formation, $T=530°{}^{+80°}_{-60°}$ K and $P=10^{-4\pm2}$ atm. The atmospheric model is here calculated to allow for one central solar mass plus the additional surface pressure derived from the gravitational attraction of the gas on itself.

Suppose, for illustration, that the gas pressure doubles in 1 AU toward the Sun so that $dP/dr=-10^{-11.3}$ cgs, $\Delta g=-10^{-2.9}$ cm s^{-2} by equation (1), and $\Delta V=10^{4.1}$ cm s^{-1} by equation (3). In the gas the mean free path is 0.2 cm and $R_e=0.64s$. This puts us squarely in the Newtonian range of drag (eq. (6′c)) and the range of slow grain growth (eq. (15)) for sizable planetesimals of $s\gtrsim10$ m. Growth to such sizes for $\rho_s=3$ occurs rapidly, $\sim$1.4 cm yr^{-1}, for a short time and then slows. As we have seen, the radial spiraling rate for a change approximately fifty fold in radius by Stokes' law averages about 0.3 ΔV or $dr/dt=10^{-2.1}$ AU yr^{-1}. Thus, the reduction in r is a fraction of 1 AU while the planetesimal is growing to meter dimensions.

For larger bodies, $s>10$ m, we can apply equation (18) (Newtonian drag and the classical resisting medium) to find the spiraling distance Δr in time Δt for constant s (cm)

$$\frac{\Delta r}{r}=-\frac{1.22}{s}\Delta t(\text{yr}) \qquad (21)$$

Appreciable change of solar distance occurs when Δt in years becomes comparable to the radius of the planetesimal measured in centimeters and varies directly as the nebular density (eq. (18)). The growth rate, however, is really quite uncertain. The assumed value of ΔV is still small (less than 1 percent) compared to the orbital velocity but is some 7 percent of the mean molecular velocity. Turbulent motion of the gas might easily produce random relative velocities of the order of 1 km s^{-1} so that the faster growth rate might be applicable (first term of eq. (13)). In that case, the change of r for a further growth to kilometers might not be significant. The slow growth rate, however, would make a significant change in r quite likely.

We may conclude, therefore, that effects of pressure gradients in the asteroid belt could be very important in shifting planetesimals toward Mars or Jupiter, provided the growth rates of the asteroids are not too rapid. For the basic development of the asteroids on a short time scale of 10^3 to 10^4 yr, the effect would be minor. However, should the nebular density be 10^{-5} atm or less in the asteroid region and the growth time 10^6 to 10^8 yr, pressure gradients could well have decimated the asteroid region by spiraling the planetesimals toward Mars or Jupiter, depending on the distribution of gas and the location of original grain growth. Possibly this effect contributed to the failure of an Earth-sized planet to develop between Mars and Jupiter.

GENERAL REMARKS

My attention was first directed to systematic interactions between gas motions and growing planetesimals in a postulated solar nebula by Hoyle's (1960) theory involving the expansion of a centrally condensed nebula by the outward force of spiraling magnetic fields from the Sun. In this process, the plasma would be pressed outward from the Sun, carrying with it the neutral gas and, according to Hoyle, also the planetesimals. In fact (Whipple, 1964), the reduced effective gravity on the gas will, following the arguments of this paper, cause the planetesimals to spiral inward toward the Sun rather than outward.

Cameron (1969) mentions the effect of a radial pressure gradient in producing an inward spiraling of planetesimals but does not discuss the alternative possibility, viz, outward spiraling, should a positive outward pressure gradient occur. Generally, if a toroid of higher density occurs in the solar nebula, the growing planetesimals are drawn toward it from the inside as well as from the outside, increasing the growth rate of an accreting planet. The importance of the effect, as we have seen above, depends mostly upon the nebular density and the time rates of chemical condensation. For a short time scale ($\sim 10^3$ to 10^4 yr) such as Cameron envisages, the pressure-gradient effect would not be important.

For long time scales of planetary formation, 10^6 to 10^8 yr, as are frequently postulated, the pressure-gradient effect could be highly significant. It may well have drawn in the cometary material from far beyond Neptune, adding materially to Neptune's mass and reducing the solar distance from Bode's "law." It may have greatly reduced the comet production in these regions of space so that no significant comet belt exists beyond Neptune. Conceivably, the pressure-gradient effect may have assisted in the building of Mars and Jupiter, at the expense of the asteroid belt. A lower surface gas density between the regions of Mars and Jupiter might not, in itself, have prevented the accretion of a sizable terrestrial planet between Mars and Jupiter without the action of the pressure-gradient effect.

ACKNOWLEDGMENT

I am indebted to Fred A. Franklin for his assistance with regard to the mathematical solutions noted in this paper.

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