EFFECTIVE LYAPUNOV NUMBERS AND CORRELATION DIMENSIONS IN A 3-D HAMILTONIAN SYSTEM

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Transport in Hamiltonian systems, in the case of strong perturbation, can be modeled as a *diffusion process*, with the diffusion coefficient being constant and related to the maximal Lyapunov number (Konishi 1989). In this respect the relation found by Lecar et al. (1992) between the escape time of asteroids, T_E, and the Lyapunov time, T_L, can be easilly recovered (Varvoglis & Anastasiadis 1996). However, for moderate perturbations, chaotic trajectories may have a peculiar evolution, owing to *stickiness* effects or migration to adjacent stochastic regions. As a result, the function $\chi(t)$, which measures the exponential divergence of nearby trajectories, changes behaviour within different time intervals. Therefore, trajectories may be divided into segments, $i = 1, ..., n$, each one being assigned an "Effective" Lyapunov Number (ELN), $\lambda_i = \chi(t_i)$.

We study trajectories in a well known 3-D Hamiltonian system (Contopoulos & Barbanis 1989)

$$H = \frac{1}{2}({p_x}^2 + {p_y}^2 + {p_z}^2) + \frac{1}{2}Ax^2 + \frac{1}{2}By^2 + \frac{1}{2}Cz^2 - \epsilon xz^2 - \eta yz^2 = h \quad (1)$$

By noting H_0 the sum of the first four terms of the Hamiltonian at every timestep t_i, one constructs the "quasi-integral" timeseries, $H_0(t_i)$, which characterizes a trajectory, in analogy to a canonical coordinate timeseries.

For every trajectory we calculate $\chi(t)$ along with $H_0(t_i)$. After defining an appropriate number of segments, we calculate the value of the correlation dimension, $D^{(2)}$, for each of them. For this we calculate the correlation integral, $C_2(r)$, through a formula proposed by Isliker (1994), using state vectors generated from $H_0(t_i)$ via the time-delay reconstruction scheme (Takens 1981). Practical details concerning this subject can be found in Isliker (1994).

During our numerical experiments we have found trajectories exhibiting different types of behaviour; two of them are shown in Fig. (1). The ELN and $D^{(2)}$ values for all the segments studied are shown in the bottom-right panel of Fig. (1), from which a positive correlation between the ELN's and the $D^{(2)}$'s can be deduced. This fact indicates that the statistical properties of transport are not the same throughout the phase space (see also Zaslavsky 1994) but depend on the measure of surviving invariant sets within this region. In this respect, $D^{(2)}$ can be used as a comparative measure for the stochasticity level of different regions.

The two trajectories shown here demonstrate a behaviour similar to that of *stable chaos* (Milani & Nobili 1992). The ELN of Trajectory II (bottom of Fig. 1) has a larger value than any of the first three segments ($t \leq 5 \times 10^5$) of Trajectory I

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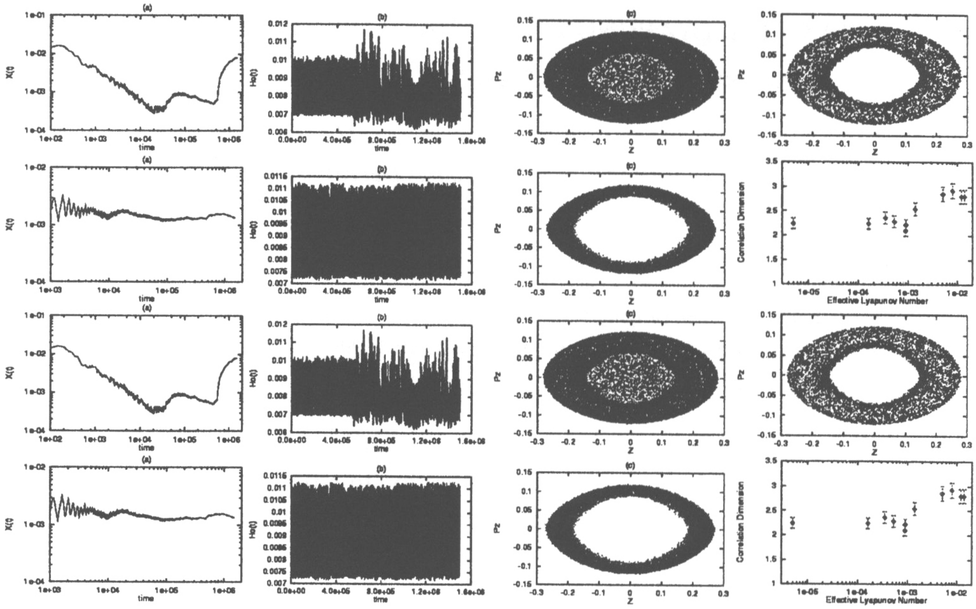

Fig. 1. Top: $\chi(t)$, $H_0(t)$ and $z - p_z$ projections for trajectory I. The last figure is the projection up to $t = 5 \times 10^5$. Bottom: The same plots for trajectory II. Bottom right: graphic representation of the results found the relation between ELN and $D^{(2)}$ for several trajectory segments.

(top) and, yet, the latter is the one which manages to escape towards a more chaotic region. This indicates that T_E must depend also on some quantity other than the Lyapunov number. If we assume a dynamical system whose action space is a "box" divided into two regions by an imperfect barrier, then T_E should be inversely proportional to the "collision frequency" (or λ) as well as to the measure of the "holes" (or the Hausdorff codimension) of the barrier (see also Gaspard & Baras 1995). Accordingly, in our model, $D^{(2)}$ changes when motion takes place close to a barrier (low ELN values). We believe that this quantity may prove important for a statistical description of chaotic motion in problems of astronomical interest.

References

Contopoulos G. and Barbanis B., 1989, A& A 222, 329
Gaspard P. and Baras F., 1995, Phys. Rev. E 51, 6, 5332
Isliker H., 1994, Ph.D. Thesis, ETH: No 10495, Swiss Federal Institute of Technology, Zürich
Konishi T., 1989, Prog. Theor. Phys. Suppl. 98, 19
Milani A. and Nobili A. M., 1992, Nature 357, 569
Takens F., 1981, Lecture Notes in Mathematics, vol. 898 (Springer, New York, 1981)
Varvoglis H. and Anastasiadis A., 1996, AJ 111, 1718
Zaslavsky G., (1994), Phys. D 76, 110