

# DYNAMO THEORY AND THE SOLAR CYCLE

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**Abstract.** In this paper solutions of the mean field induction equation in a spherical geometry are discussed. In particular, the 22-year solar magnetic cycle is considered to be governed by an axisymmetric, periodic solution which is antisymmetric with respect to the equatorial plane. This solution essentially describes flux tubes travelling as waves from mid-latitudes towards the equator. In a layer of infinite extent the period of such dynamo waves solely depends on the strength of the two induction effects, differential rotation and  $\alpha$ -effect (cyclonic turbulence). In a spherical shell, however, mean flux must be destroyed by turbulent diffusion, so the latter process might actually control the time scale of the solar cycle.

A special discussion is devoted to the question of whether the angular velocity *increases* with increasing depth, as the dynamo waves seem to require, or whether it *decreases*, as many theoretical models concerned with the Sun's differential rotation predict. Finally, theories for the sector structure of the large scale photospheric field are reviewed. These describe magnetic sectors as a consequence of the sectoral pattern in the underlying large scale convection, as non-axisymmetric solutions of the mean field induction equation, or as hydromagnetic waves, modified by rotational effects.

## 1. Introduction

The subject of 'mean field electrodynamics', which was discussed during this morning's session, has led us to a mean-field induction equation. In the present lecture I intend to demonstrate and discuss solutions of this mean-field induction equation. In a spherical geometry, in particular, there is an axisymmetric solution, which is periodic in time and antisymmetric with respect to the equatorial plane; the cyclic behaviour of the large scale magnetic field of the Sun usually is ascribed to this solution. I shall discuss mean fields of this type in Section 2. Before doing so, however, let us recall some of the conditions which should be satisfied for the mean field induction equation to be a good approximation.

One of these conditions has to do with the neglect of all terms in the induction equation for the *fluctuating* part of the magnetic field which contain products of fluctuations. As compared to the linear terms, these second order terms are small if either the magnetic Reynolds number is small or if the fluctuating velocity is 'slow' in the sense that

$$v\tau \ll l, \tag{1}$$

where  $v$ ,  $\tau$  and  $l$  are rms velocity, correlation time and correlation length of the turbulent convection. Both these conditions are *not* satisfied in the solar convection zone. It is usually assumed that  $v\tau \approx \bar{l}$  (and observations of granules and supergranules confirm this assumption); and the magnetic Reynolds number is very large indeed. As shown in Figure 1, it increases from  $\approx 10^3$  in the photosphere to  $\approx 10^{10}$  at a depth of  $10^5$  km. Now the magnetic Reynolds number is the ratio of the free decay time,  $\mu\sigma l^2$ , of a magnetic field fluctuation to the lifetime of the convective eddy which

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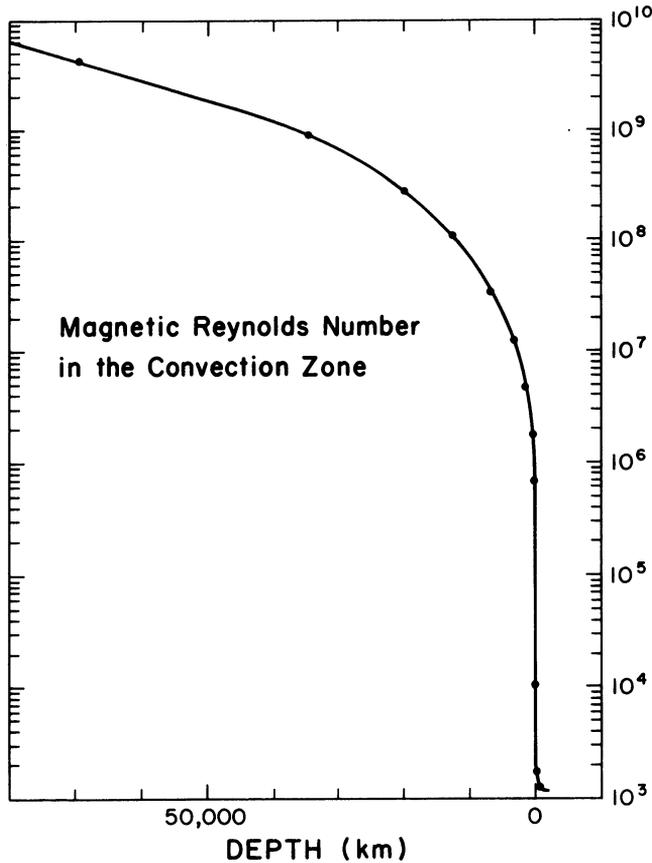


Fig. 1. The magnetic Reynolds number,  $\mu\sigma vl$ , as a function of depth,  $z$ , in the solar convection zone. The conductivity,  $\sigma$ , is taken from Kopecný and Soytürk (1971) for the four values closest to the surface, and computed from  $\sigma = 0.003 \times T^{3/2}$  mho  $\text{m}^{-1}$  (Spitzer, 1962; Piddington, 1975), where the temperature,  $T$ , is taken from the table of Baker and Temesváry (1966). The convection velocity,  $v$ , is taken from the same table, and  $l = z/2$  is assumed. For the three photospheric values I used  $v = 1 \text{ km s}^{-1}$  and  $l = 100 \text{ km}$ .

created that fluctuation. Since this ratio is large, more magnetic field fluctuations will be created before the old ones can decay, so that, in an equilibrium state, the rms magnetic field will be large as compared to the mean field. It therefore appears that in the solar case the above-mentioned second-order terms should *not* be neglected. I think that this is essentially the point where the criticism of Piddington (1971, 1972, 1973, 1975) of the solar dynamo theory becomes relevant.

How is the 'mean field' defined? In the mean field electrodynamics, as it is developed by Steenbeck *et al.* (1966), Krause (1968), Rädler (1968a, b) and Krause and Rädler (1971), mean values originally are understood as ensemble averages. As the authors point out, they may be replaced by averages over space or time. Such averages are of course required for comparison of observations with predictions of the theory. But what is an appropriate volume or an appropriate time span of integration? On the Sun, there is no length scale which is large compared to *all*

convection cells but still small compared to the solar radius. This problem appears to be particularly intriguing since it is the *largest* convection cells which contribute most to the  $\alpha$ -effect which is so essential for the regeneration of the mean field (Stix, 1974).

After these somewhat pessimistic introductory remarks concerning but two of many problems related to the solar dynamo (see e.g. Stix, 1974) I shall nevertheless proceed to illustrate the properties of the mean-field induction equation. The capability of this equation to explain the solar cycle will be demonstrated in the following section, and some problems related to special forms of differential rotation will be discussed in Section 3. Section 4 will contain some remarks concerning the sector structure of the large scale solar field. Whenever it is convenient, I shall comment on the above-mentioned problems and, at least in some cases, speculate how they might eventually be solved.

## 2. The Axisymmetric Periodic Mean Field

The induction equation for the mean magnetic field,  $\mathbf{B}$  (with the overbar omitted since only mean fields will be discussed), was first, in a simplified form, given by Parker (1955), and subsequently derived under more general circumstances by Steenbeck *et al.* (1966), Krause (1968), Rädler (1968a, b; 1969), Iroshnikov (1970a), Parker (1970), Moffat (1970a, b), Yoshimura (1972), Gubbins (1974a), and Deinzer and Stix (1975). In its most often used form it is

$$\frac{\partial \mathbf{B}}{\partial t} = \text{curl}(\mathbf{v} \times \mathbf{B} + \alpha \mathbf{B}) - \eta \text{curl curl } \mathbf{B}, \quad (2)$$

where  $\mathbf{v}$  is the mean velocity field,  $\alpha \mathbf{B}$  the additional electric field caused by the helicity of the fluctuating velocity field, and  $\eta$  the turbulent electro-magnetic diffusivity. The 'semi-empirical' equation derived by Leighton (1969) can be written essentially in the same form (Yoshimura, 1972; Stix, 1974). The large magnetic Reynolds number dynamos of Braginskii (1964; see also Soward, 1971, 1972; Gubbins, 1973a) also lead to equations of the form (2).

Let us suppose that the mean velocity is a pure rotation, and is *symmetric* with respect to the equatorial plane, i.e., in spherical polar co-ordinates  $(r, \theta, \phi)$ ,

$$\mathbf{v} = (0, 0, r\omega(r, \theta) \sin \theta), \quad (3)$$

where  $\omega(r, \pi - \theta) = \omega(r, \theta)$ . Let us further suppose that  $\alpha(r, \theta)$  is *antisymmetric* with respect to the equatorial plane, i.e.  $\alpha(r, \pi - \theta) = -\alpha(r, \theta)$ . This is plausible since the helicity of the convection, which causes the  $\alpha$ -effect and is itself caused by the Coriolis force, is also antisymmetric; and all explicit expressions for  $\alpha$  indeed have this antisymmetry. Whenever the induction effects have these symmetries the solutions of Equation (2) can be divided into two uncoupled sets of solutions, namely the 'antisymmetric' (dipole-type, odd) and the 'symmetric' (quadrupole-type, even) sets. In general, both sets contain steady and time-dependent modes, which can be axially symmetric or  $\phi$ -dependent. Only the axisymmetric modes will be considered in this section.

In order to proceed further I introduce the two magnetic Reynolds numbers

$$R_\alpha = \frac{\alpha_0 r_\odot}{\eta}, \quad R_\omega = \frac{\Delta\omega r_\odot^2}{\eta}; \quad (4)$$

$\alpha_0$  is a typical value of  $\alpha$  in the northern hemisphere,  $r_\odot$  is the radius of the Sun and  $\Delta\omega$  is a characteristic value of differences in the angular velocity. The condition

$$|R_\alpha| \ll |R_\omega| \quad (5)$$

then defines the so-called  $\alpha\omega$ -dynamo, where, in the  $\phi$ -component of Equation (2), the  $\alpha$ -effect can be neglected in comparison to the effect of differential rotation. The ratio of the poloidal and toroidal field components is then  $\sim |R_\alpha/R_\omega|^{1/2}$  (Steenbeck and Krause, 1969). In the solar case we have  $B_{\text{pol}}/B_{\text{tor}} \approx 0.1$  (Yoshimura, 1975c)\*, and may therefore use the limit (5). In this limit, the frequencies and growth rates of the magnetic field solutions depend on the product,  $P$ , of the two magnetic Reynolds numbers defined by Equations (4);  $P$  is called the *dynamo number*.

In order to find solutions of Equation (2) in a spherical geometry a number of numerical models have been described in recent years. In these, Equation (2) is either reduced to an eigenvalue problem (Steenbeck and Krause, 1969; Deinzer and Stix, 1971; Roberts and Stix, 1972; Levy, 1972; Köhler, 1973; Stix, 1973; Deinzer *et al.*, 1974) or it is solved as an initial value problem (Leighton, 1969; Jepps, 1975; Yoshimura, 1975a). Although different functions  $\omega(r, \theta)$  and  $\alpha(r, \theta)$  have been employed in these models, some general properties are common to them. These are: *If  $\alpha \cdot \partial\omega/\partial r$  is negative in the northern and positive in the southern hemisphere (i.e.  $P < 0$ ) the most preferred mode has dipole-type symmetry and is oscillatory.* With most preferred I mean that the growth rate of the mode becomes positive at the smallest  $|P|$ . *Moreover, for the same sign of  $\alpha \cdot \partial\omega/\partial r$  the oscillatory modes of both symmetries, dipolar and quadrupolar, travel from higher latitudes toward the equatorial plane. For the opposite sign of  $\alpha \cdot \partial\omega/\partial r$  the travel direction is reversed and the quadrupolar oscillatory mode is preferred.* Exceptions to the first of these rules occur sometimes, in particular if the spatial distribution of the induction effects is such that high order harmonics play only a minor role in the magnetic field. The preferred modes would then be steady and quadrupolar for  $P < 0$  and steady and dipolar for  $P > 0$  (Stix, 1973; Deinzer *et al.*, 1974). However, these cases have a very large spatial separation of the inducing effects and are thus very probably irrelevant to the Sun where the dynamo is believed to be confined to the convection zone (see below, in part. Figure 5). Another possibility to make steady modes preferred is by means of a sufficiently strong large scale meridional circulation (Roberts, 1972), but this is not observed on the Sun.

The migration of the oscillatory magnetic fields can be understood in terms of a propagating wave ('dynamo wave', 'Parker wave'). This becomes particularly clear if local cartesian co-ordinates are introduced. In this way the latitude migration of the mean solar field was first explained by Parker (1955). More recently, Yoshimura (1975b) pointed out that *the dynamo waves generally propagate along the surfaces of*

\* This value is an upper limit; higher spatial resolution of the magnetograph would probably lead to a smaller ratio.

*isorotation*. The direction of propagation is given by the vector

$$\alpha \cdot \nabla \omega \times \mathbf{e}_\phi, \tag{6}$$

where  $\mathbf{e}_\phi$  is the unit vector in the azimuthal direction. Thus, if  $\alpha > 0$  in the northern hemisphere and the angular velocity increases with increasing depth, the migration is equatorwards, in agreement with the numerical results mentioned before.

A schematic illustration of an antisymmetric oscillatory  $\alpha\omega$ -dynamo is given in Figure 2. Consider an antisymmetric toroidal mean field as shown in Figure 2a. The  $\alpha$ -effect, with  $\alpha_{\text{north}} > 0$  and  $\alpha_{\text{south}} < 0$ , will cause a symmetric toroidal mean current,  $\sigma\alpha\mathbf{B}$ , and an associated poloidal field as indicated in Figure 2b. This poloidal field is subject to the differential rotation. For  $\partial\omega/\partial r < 0$  the toroidal field pattern shown in Figure 2c will result. Add this to the original field of Figure 2a; the result is an enhancement of the original toroidal flux tubes on their equatorial side, and a destruction on their polar side, with the net effect that the tubes are moved closer to the equatorial plane (Figure 2d). At the same time weak flux tubes of opposite polarity are formed at higher latitudes, indicating the advent of the next cycle. The argument is now repeated: symmetric currents are again caused by the  $\alpha$ -effect and again they generate a poloidal field (Figure 2e). From this, differential rotation winds up the toroidal field shown in Figure 2f, which resembles the original one, but is reversed in sign.

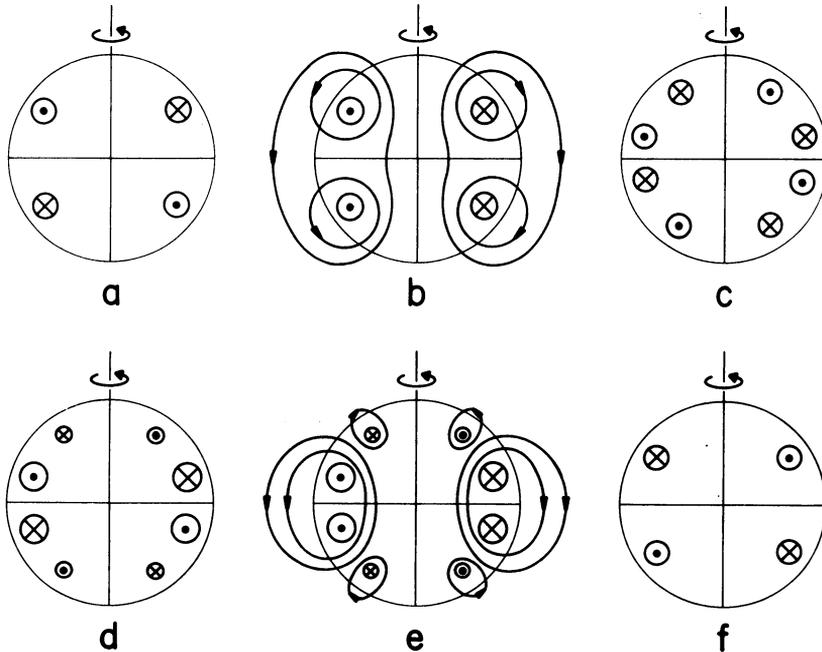


Fig. 2. Oscillatory dynamo action, with  $\alpha_{\text{north}} > 0$ ,  $\alpha_{\text{south}} < 0$ , and  $\partial\omega/\partial r < 0$ . The toroidal field (a), together with the  $\alpha$ -effect, sets up toroidal currents and so generates a poloidal field (b). This is wound up by differential rotation (c), with the result that the original field is moved equatorwards (d). Again, toroidal currents and associated poloidal fields are caused by the  $\alpha$ -effect (e), so that differential rotation finally can reverse the field (f). An encircled dot (cross) indicates a vector pointing out of (into) the plane of the figure.

The scheme outlined in Figure 2 differs from the schemes described by Steenbeck and Krause (1969, Figure 2) and Krause and Rädler (1971a, Figure 4). There the field reversal depended on a phase lag caused by diffusion and a spatial separation of the two induction effects,  $\alpha$ -effect and differential rotation. The numerical models show however that such a spatial separation is not necessary in order to obtain oscillatory modes (Roberts, 1972). On the contrary, as already mentioned, it seems to favour the steady modes (Deinzer *et al.*, 1974).

It is illustrative to show oscillatory magnetic fields in form of a movie, and 12 frames of such a movie are shown in Figure 3. The field is a numerical solution of the

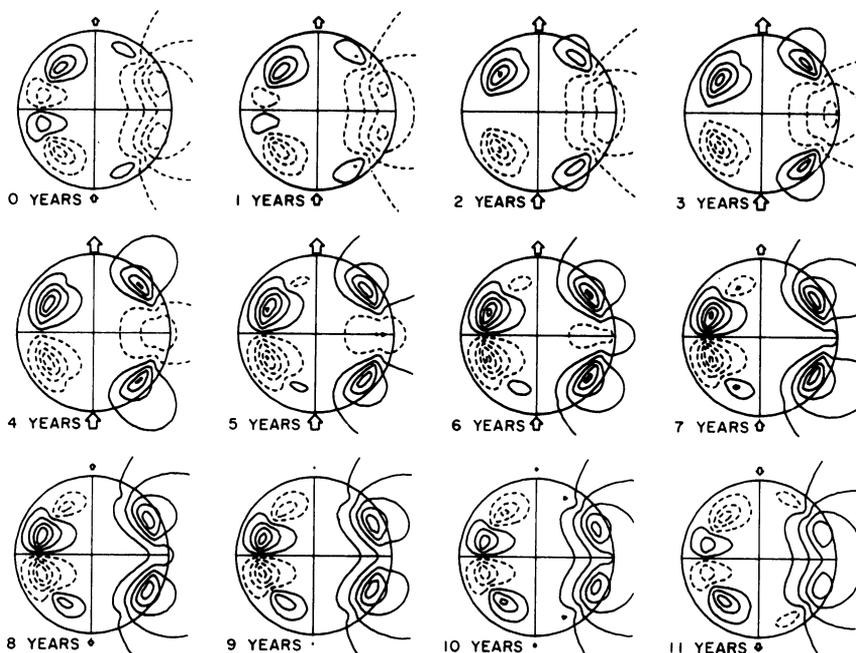


Fig. 3. A numerical solution of an antisymmetric, oscillatory  $\alpha\omega$ -dynamo. The shear is radial, with  $\partial\omega/\partial r < 0$ , and  $\alpha \sim \cos \theta$ . Each frame is a meridional cross-section through the solar model. The contours of constant toroidal field strength are on the left and the poloidal lines of force are on the right. The levels of the curves are  $\pm 0.1, \pm 0.3, \dots, \pm 0.9$  times the maximum values (over the whole cycle) of the toroidal field and the poloidal flux function respectively. Positive values (solid curves) indicate toroidal fields pointing out of the figure, and clockwise poloidal field lines; negative values (dashed curves) indicate the opposite. The direction and magnitude of the field at the poles is indicated by vertical arrows. The time scale is adjusted so that 11 years cover one half-cycle.

model described by Deinzer and Stix (1971). I would like to emphasize two features of this solution. First, the toroidal mean field, which is shown on the lefthand side of each frame in the form of isogauss contours, migrates towards the equatorial plane, and two flux tubes are visible during most of the time interval shown. This, of course, means that two consecutive cycles overlap. How much of this effect would be visible on the solar surface in form of overlapping butterfly wings cannot however be predicted by this model; the process of field eruption depends on the magnitude of

the field which cannot be obtained from a linear theory. The second feature to which I would like to draw your attention is the harmonic structure of the field, in particular of the poloidal field which is shown on the righthand side of each frame in form of its lines of force. The octupole harmonic is clearly discernible during the entire cycle so that the mean poloidal field never looks like a pure dipole. Moreover, the poloidal field, at least at low latitude, participates in the *equatorward* migration of the toroidal field, as it should do according to the scheme of Figure 2. The harmonic structure of the field seems to be essential for the oscillatory (or ‘migratory’) nature of the  $\alpha\omega$ -dynamo. This is confirmed by a model of Steenbeck and Krause (1966) which had steady solutions when the field expansion was truncated after the first poloidal and toroidal harmonics, but yielded oscillatory fields without such a truncation (Krause and Rädler, 1971b; Roberts, 1972). For the field shown in Figure 3 6 poloidal and as many toroidal harmonics were retained, and no changes occurred at higher truncation levels.

There is observational evidence that the solar mean poloidal field does migrate towards the equator. Figure 4a shows contours of the radial field component in a

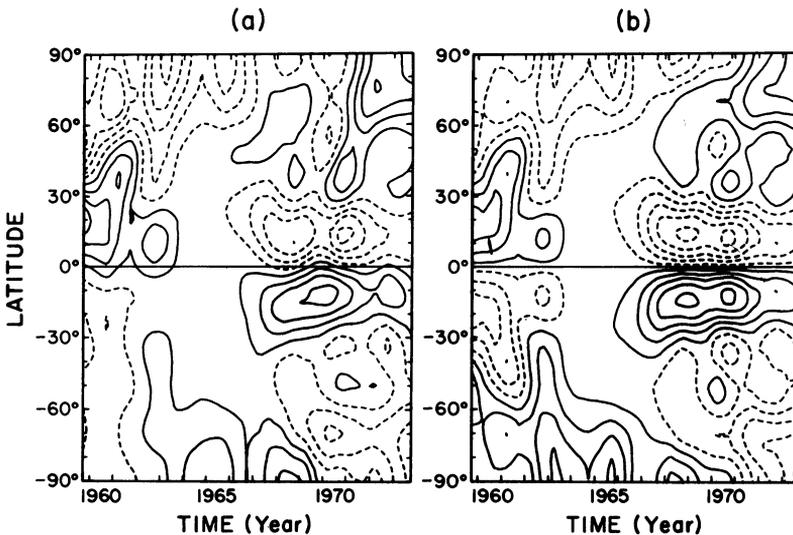


Fig. 4. Contours of the radial magnetic field component, 1959–1973. The total field (a) and the antisymmetric part (b) are shown. The contour levels are  $\pm 0.1, \pm 0.3, \dots \pm 0.9$  times the maximum values, which are 1.65 (a) and 1.19 (b) G. Solid curves indicate positive levels, dashed curves indicate negative levels. The smoothing constant in time is 1 yr, and the smoothing in latitude is determined by the truncation of the expansion (7) after the 9th harmonic.

time-latitude diagram, computed according to

$$B_r(t, \theta) = \sum_{n=1}^9 (n + 1)g_n^0(t)P_n(\theta), \tag{7}$$

where  $P_n(\theta)$  are zonal surface harmonics and the  $g_n^0$  are zonal expansion coefficients.

The latter were obtained by Altschuler *et al.* (1974) from the photospheric magnetic field data, measured by the Mt. Wilson magnetograph. Such a 'butterfly diagram' of the poloidal field was also computed by Stenflo (1972), but his data extended only through 1969. For this reason and because of too little smoothing in time his diagram did not show the features relevant to the solar cycle as clearly as Figure 4a. These features are even more pronounced in Figure 4b where only the odd harmonics of the sum (7) were retained; this figure is antisymmetric and therefore the exact observational counterpart of the antisymmetric theoretical poloidal field, as shown on the right of the frames of Figure 3. The butterfly wings in the latitude range  $\pm 35^\circ$  during 1965–1973 almost exactly coincide with the wings of the classical sunspot butterfly diagram of cycle 20 (Yoshimura, 1975c), which represents the subsurface toroidal field. As shown in Figure 3, this coincidence is also present in the theoretical model.

I would like to add here a remark concerning the averaging problem mentioned in the introduction: At least as long as we are not interested in  $\phi$ -dependent mean fields we may simply average over all longitudes and over a sufficiently long period of time; the time smoothing constant in Figure 4 is 1 yr.

Several features of the solar cycle are not well resembled by the field shown in Figure 3. For example, the field penetrates too far into the interior of the Sun. This is, of course, a consequence of the constant magnetic diffusivity used in the model. Since this diffusivity is the *turbulent* diffusivity, it should be replaced by a much smaller value in the radiative core. The skin effect would then prevent the oscillatory field from entering the core. A model of this type was computed by Roberts and Stix (1972), and Figure 5 shows four phases of the resulting field. The field expulsion from the solar core can alternatively be simulated by an inner boundary condition, as in the models of Köhler (1973) and Yoshimura (1975a).

Another difference of the field shown in Figure 3 and the real solar cycle is that the field is, on the average, located at too high latitudes. Also, the poloidal field at high latitudes virtually does not migrate towards the poles, as it should do in order to resemble the observed poloidal field (Figure 4a) and to reproduce the poleward rush of the high latitude prominence zones. Both these deficiencies do not however occur in more sophisticated models. For example, a concentration of the  $\alpha$ -effect and shear regions to lower latitudes brings the butterfly wings closer to the equator and at the same time allows the polar field to diffuse freely towards the poles (Stix, 1974; Yoshimura 1975a). Figure 6 shows how the neutral line of the radial field takes longer to drift towards the poles as the shear is more concentrated to low latitudes. This result is obtained from the model of Köhler (1973; see also Stix, 1974); the radial shear is proportional to  $\sin^n \theta$ . A very similar result was found earlier by Leighton (1969, Figure 4).

Let us finally discuss the most important question concerning the oscillatory  $\alpha\omega$ -dynamo, namely the question how the period is defined. According to the scheme outlined in Figure 2 non-uniform rotation and  $\alpha$ -effect are responsible for the destruction and amplification of mean toroidal flux tubes. The strength of these two induction effects should therefore decide how long it would take these flux tubes to travel from midlatitudes toward the equator. And, indeed, the dynamo waves discussed in rectangular co-ordinates by Parker (1955), Iroshnikov (1970b), and Yoshimura (1975b) have periods which depend only on this strength (and the wave

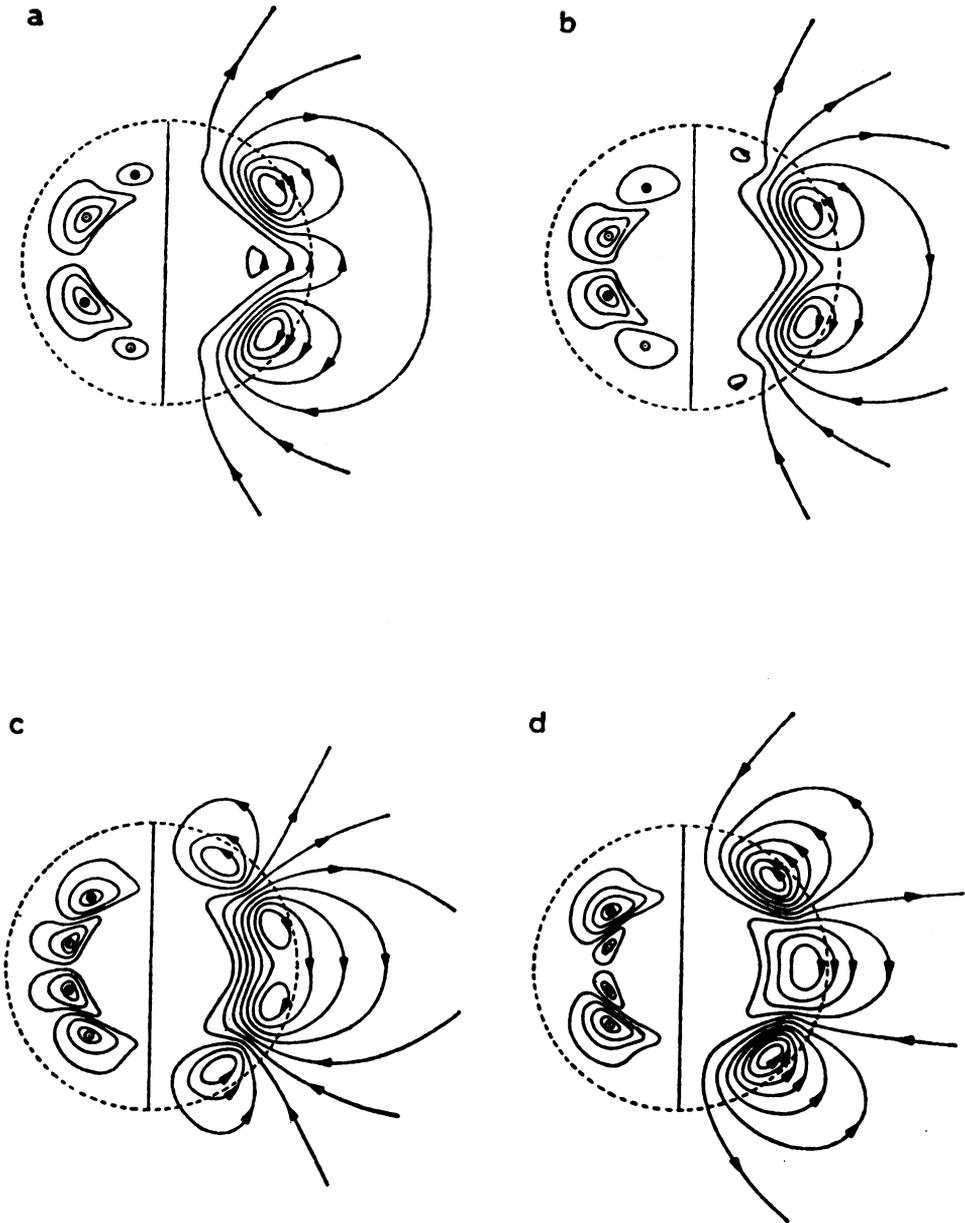


Fig. 5. Field expansion from the radiative solar interior. The diffusivity decreases from its turbulent value in the convection zone to the very small molecular diffusivity in the core, where the field cannot penetrate due to the skin effect (after Roberts and Stix, 1972).

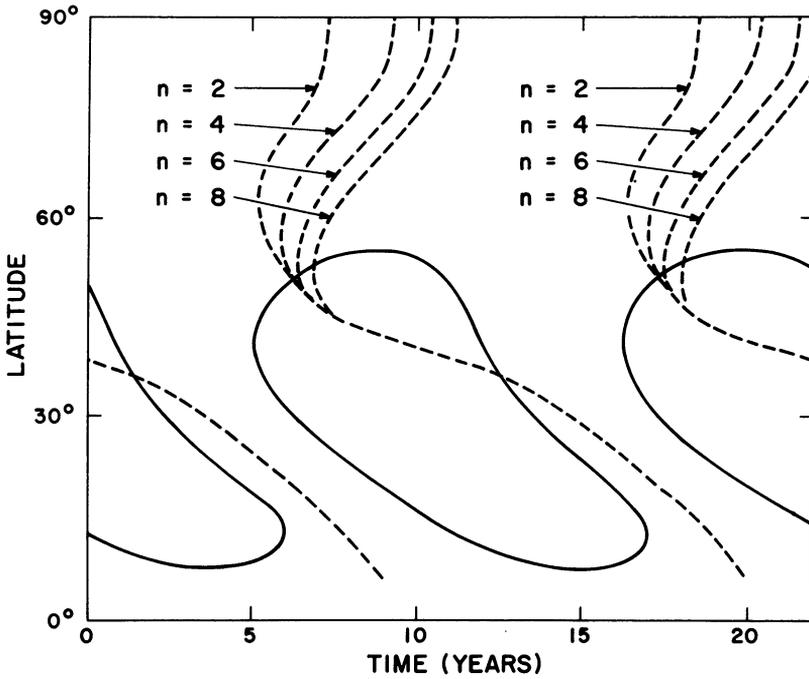


Fig. 6. The poleward drift of the neutral line of the radial field component (dashed curves) and the 50% level of the toroidal fields (solid curve). The latter is drawn only for the case  $n = 6$  in order to avoid confusion. The radial shear is  $\sim \sin^n \theta$ .

number,  $k$ ). The frequency is

$$\Omega = \left( \frac{|kH\Gamma|}{2} \right)^{1/2}, \tag{8}$$

(Parker, 1955), where  $H$  is the shear and  $\Gamma (= \alpha)$  measures the strength of the cyclones causing the  $\alpha$ -effect. Using  $k = 2/r_\odot$  (i.e. one flux tube in each hemisphere) and  $H \approx r_\odot \partial\omega/\partial r$  we see that (8) is equivalent to

$$\Omega = \left| \alpha \frac{\partial\omega}{\partial r} \right|^{1/2} \tag{9}$$

in a spherical geometry with radial shear. Unfortunately, it is difficult to estimate the values of  $\alpha$  and  $\partial\omega/\partial r$  in the solar convection zone. If we assume that the total variation of the angular velocity in depth is about the same as its total variation in latitude, and if we assume  $2 \times 10^8$  m as the depth of the convection zone we obtain  $|\partial\omega/\partial r| \approx 5 \times 10^{-15} \text{ m}^{-1} \text{ s}^{-1}$ . For the  $\alpha$ -effect Steenbeck and Krause (1969) made an estimate of  $0.26 \text{ m s}^{-1}$ ; the values of Yoshimura (1972) and Leighton (1969) are  $0.5$  and  $0.7 \text{ m s}^{-1}$  respectively (see Yoshimura, 1972). With  $\alpha = 0.5 \text{ m s}^{-1}$  we obtain a frequency of  $5 \times 10^{-8} \text{ s}^{-1}$  which is somewhat larger than the frequency of the real solar cycle ( $9 \times 10^{-9} \text{ s}^{-1}$ ). The frequency becomes still larger if we directly apply Krause's (1968) formula for  $\alpha$  and data obtained from the mixing-length theory

(Köhler, 1973), even if we correct for the effect of rotation angles larger than  $\pi/2$  which occur in convection cells in the case of a small Rossby number (Stix, 1974). We obtain  $\alpha \approx 50 \text{ m s}^{-1}$  (see Figure 7) and, accordingly, a frequency of  $5 \times 10^{-7} \text{ s}^{-1}$ . On the other hand we may use an *a-posteriori* argument, namely that the ratio of the poloidal and toroidal field strengths should match the observed ratio of, say, 0.01 and so obtain an  $\alpha$  between 0.01 and 0.05  $\text{m s}^{-1}$  (Köhler, 1973). The frequency (9) would then agree with the observed one.

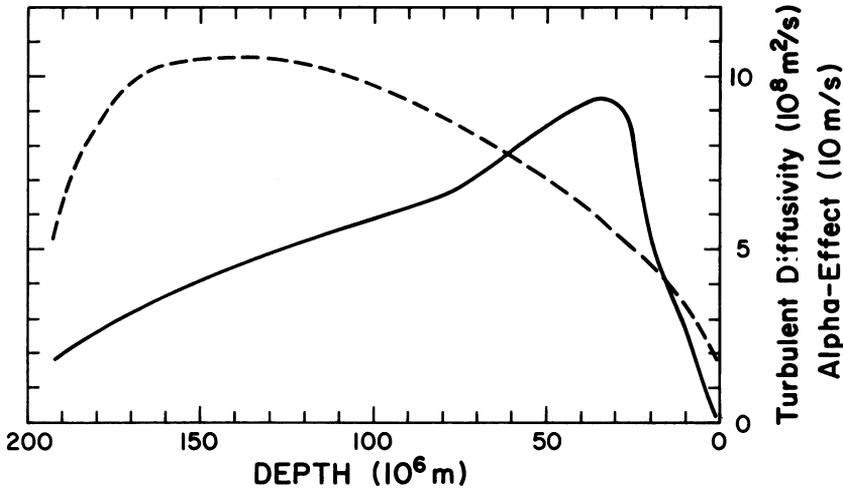


Fig. 7. Alpha-effect ( $\alpha$ , solid) and turbulent diffusivity ( $\eta_t$ , dashed) in the solar convection zone.  $\alpha$  is computed according to an expression derived by Krause (1968; see Steenbeck and Krause, 1969, Equation (23)), but is multiplied by the Rossby number,  $Ro$ , in the case  $Ro < 1$ . For the diffusivity I used  $\eta_t = \frac{1}{3}vl$ , where  $v$  and  $l$  are velocity and size of convection cells. Data of Baker and Temesváry (1966) were used to compute  $\alpha$  and  $\eta_t$ .

Turbulent diffusion plays no role in the foregoing interpretation of the solar cycle period. Only the growth rate depends on the diffusivity. This interpretation, which has been strongly emphasized by Yoshimura (1975a, b), contrasts with earlier work where the period of the cycle had been identified essentially with the time the mean field needs to diffuse over a characteristic distance,  $d$ , in latitude (Leighton, 1969) or depth (Steenbeck and Krause, 1969). According to this picture, the frequency is

$$\Omega = 2\pi / \frac{2d^2}{\eta_t} \tag{10}$$

where  $\eta_t$  is the turbulent magnetic diffusivity and the factor 2 appears in the diffusion time since fields of both polarities must diffuse in order to complete a full cycle.

Using  $\eta_t = 10^9 \text{ m}^2 \text{ s}^{-1}$  (Figure 7) and  $d = 2 \times 10^8 \text{ m}$  we find  $\Omega = 8 \times 10^{-8} \text{ s}^{-1}$ , which again is somewhat larger than the frequency of the real solar cycle. The value of  $\eta_t$  used here might however be too large by an order of magnitude (Stix, 1974).

Is the period of the solar cycle a wave period or a diffusion time? We have already seen (Figure 2) that it is the *wave* character of the  $\alpha\omega$ -dynamo which causes the

latitude migration of the mean toroidal fields. And we have also seen that this is confirmed by the predicted (Figure 3) and observed (Figure 4) participation of the poloidal field in this latitude migration. But on the other hand, there is also no doubt that turbulent diffusion plays an important role in the numerical models mentioned earlier in this section: without diffusivity, such models fail altogether to produce reasonable results (e.g. Yoshimura, 1975a, b). At least partially, this might be a consequence of the geometry of the models. In a slab of infinite extent the dynamo waves may easily propagate, but in a spherical geometry tubes of mean flux must dissipate as they reach the boundaries or the equatorial plane. This process of turbulent dissipation might well be so slow that it dictates the time behaviour of the entire dynamo.

I should like to emphasize once more that the diffusion discussed here is *turbulent* diffusion; it is relevant to the *mean* field, or the small wave number end of the magnetic spectrum (Krause, 1968; Rädler, 1968b; Parker, 1971). Thus, since I consider the solar cycle to be governed by a periodic mean field, I disagree with Dr Piddington who argued that turbulent diffusion is irrelevant to the solar cycle (Piddington, 1971, 1972, 1973, 1975). The difficult question, then, is of course what happens at small scales, i.e. at the large wave number end of the spectrum. Does the drift across the spectrum proceed to sufficiently large wave numbers so that ohmic dissipation can destroy the magnetic fluctuations? The large magnetic Reynolds number mentioned in the introduction seems to indicate that this is not the case. Possible solutions of this problem are that convective motions of smaller, so far unresolved, scale exist on the Sun, or (and) that *dynamic* mechanisms accelerate the dissipation of magnetic flux (e.g. Sweet, 1958; Petschek, 1964; Parker, 1972), or that the small scale magnetic flux is simply lost by eruption through the solar surface (e.g. Parker, 1973, 1975; Stix, 1974). The ‘ephemeral active regions’ recently described by Harvey *et al.* (1975) make the latter idea particularly attractive.

### 3. The Role of the Angular Velocity

One of the main facts discussed in the preceding section was that the dynamo waves propagate along surfaces of isorotation. Now some of the theoretical work on solar differential rotation – for a review see Gilman (1974) – suggests that in the convection zone these surfaces are cylinders, and that the angular velocity increases with increasing distance from the axis of rotation. With  $\alpha_{\text{north}} > 0$  and  $\alpha_{\text{south}} < 0$  the direction of wave propagation (expression (6)) is then such as depicted in Figure 8a, away from the equatorial plane. Of course, mean toroidal flux propagating in this way can never produce the observed butterfly diagram as it erupts to the solar surface.

We may consider three possible answers to this puzzle. Firstly, the Sun might not be an  $\alpha\omega$ -dynamo, so that the toroidal flux would not necessarily propagate in the direction given by the vector product (6). Secondly, we may accept the concept of a solar  $\alpha\omega$ -dynamo but question the use of a positive  $\alpha$  in the northern and negative  $\alpha$  in the southern hemisphere. The propagation of the dynamo waves would then be opposite to the arrows of Figure 8a, and butterfly wings like the observed ones would

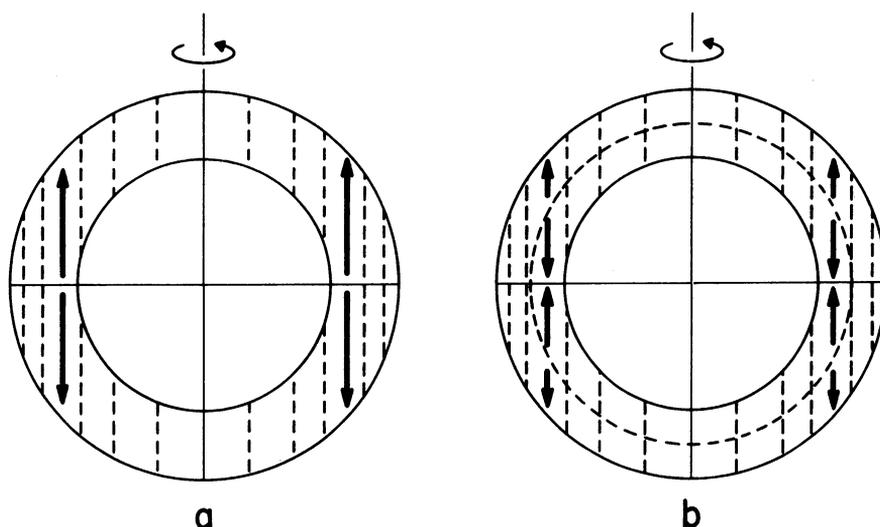


Fig. 8. Propagation of dynamo waves (heavy arrows) in the solar convection zone. The cylindrical surfaces of isorotation are indicated by the vertical dashed lines.  $\alpha_{\text{north}} > 0$  and  $\alpha_{\text{south}} < 0$  in (a) and outside the dashed circle of (b). Inside the dashed circle of (b) the sign of  $\alpha$  is opposite.

result. Thirdly, the theory of the Sun's differential rotation might be insufficient. The puzzle would then simply not exist.

Let me first discuss the second possibility. The only case where a negative  $\alpha$  in the northern hemisphere (and a positive  $\alpha$  in the southern) has been reported is the work of Yoshimura (1972). Using the Boussinesq approximation he solved the linearized equations of motion and induction explicitly and found that  $\alpha$  should be positive in the outer part of the convection zone in the northern hemisphere, and negative in the inner part, and should have the opposite signs in the southern hemisphere. Along cylindrical isorotation surfaces the dynamo waves would then travel in the directions indicated in Figure 8b. This possibility will also be discussed in a forthcoming paper by Durney (1976). The flux producing the butterfly diagram should, according to this model, erupt from the deeper part of the convection zone. This idea is consistent with recent estimates of Parker (1975) of the time of rise of the magnetic flux tubes. Only in the deeper part of the convection zone would these tubes stay long enough so that the dynamo could operate.

There are two arguments against this type of solar dynamo. Firstly, the  $\alpha$ -effect used here depends on the special type of averaging. If Yoshimura (1972) had computed his mean quantities not only as longitudinal averages, but also as depth averages, the result would have been  $\alpha \equiv 0$ . The contributions from the upper layer would have cancelled those from the lower layer. The only way to obtain a non-zero  $\alpha$ -effect when averaging over a volume is to make use of anisotropies in addition to the one introduced by rotation. Steenbeck *et al.* (1966) and Krause (1968) essentially used the density gradient and thus obtained their  $\alpha$  which is positive in the entire northern and negative in the entire southern hemisphere. Secondly, there is a definite phase relation between the poloidal and toroidal field components. Only the case

$\partial\omega/\partial r < 0$  and  $\alpha_{\text{north}} > 0$ ,  $\alpha_{\text{south}} < 0$  seems to be consistent with a negative (positive) radial field in the region of the northern (southern) butterfly wing of cycle 20 (see Figure 4 and Stix, 1976).

Since the  $\alpha$ -effect is caused by the helicity,  $\overline{\mathbf{v} \cdot \text{curl } \mathbf{v}}$ , of the convective flow,  $\mathbf{v}$ , we may wonder whether it is possible to obtain the helicity from numerical models of the convection zone and then compute  $\alpha$  according to

$$\alpha = -\frac{1}{3} \overline{\mathbf{v} \cdot \text{curl } \mathbf{v} \tau}, \quad (11)$$

where  $\tau$  is the correlation time of the velocity field (Steenbeck and Krause, 1969). For example, Gilman (1972) has simulated the convection zone in a rotating annulus, and Figure 9 shows the helicity which he obtained for Taylor numbers between  $10^2$  and  $10^6$ . We see that, for all Taylor numbers, left-handed helicity dominates in the northern hemisphere, i.e.  $\alpha_{\text{north}} > 0$ , according to Equation (11). If at all a change in sign occurs within a hemisphere, the helicity tends to be right-handed in the upper and left-handed in the lower part of the northern hemisphere, so that the structure of  $\alpha$  would be opposite to that obtained by Yoshimura (1972). This difference must be caused by the different approximations used by the two authors (although both used the Boussinesq approximation and longitudinal averaging, and considered linear modes characterized by a single azimuthal wave number): e.g., Yoshimura used a thin shell with hydrostatic equilibrium in the vertical direction, which Gilman did not, and his model was spherical whereas Gilman's was an annulus. Before we can use Equation (11) to compute  $\alpha$  we probably have to wait for more numerical models.\* These models most desirably should include such features as the nonlinear superposition of different modes and a variation of the boundary conditions. And, after all, they should be non-Boussinesq since, as we know, the density gradient plays such an important role in the determination of the  $\alpha$ -effect.

Let me now discuss the first of the possibilities mentioned above. Mean field dynamo action without helicity, i.e. without  $\alpha$ -effect, was first proposed by Rädler (1969a). He found that an additional mean electric field of the form

$$\beta \boldsymbol{\omega} \times \mathbf{j} \quad (12)$$

should exist in turbulent flows under the influence of rotation, where  $\boldsymbol{\omega}$  is the rotation vector and  $\mathbf{j}$  is the mean current density. However, numerical results of Rädler (1969b, 1970) and Roberts (1972) and an analytical treatment in cartesian coordinates by Gubbins (1974) indicate that only *steady* dynamo action can be obtained from the use of expression (12). For oscillatory modes the convergence was only 'suggestive, but not convincing' (Roberts, 1972). I have extended Roberts' search for oscillatory modes to higher truncation levels in the spherical harmonic expansion, but was still unable to find satisfactory convergence† (Stix, 1976). This was particularly so when cylindrical surfaces of isorotation were considered.

\* P. A. Gilman has recently informed me that his new spherical shell model essentially leads to the same helicity as the annulus model.

† As reported by Dr Rädler at this Symposium, oscillatory  $\boldsymbol{\omega} \times \mathbf{j}$ -dynamos can be obtained from a slightly different model.

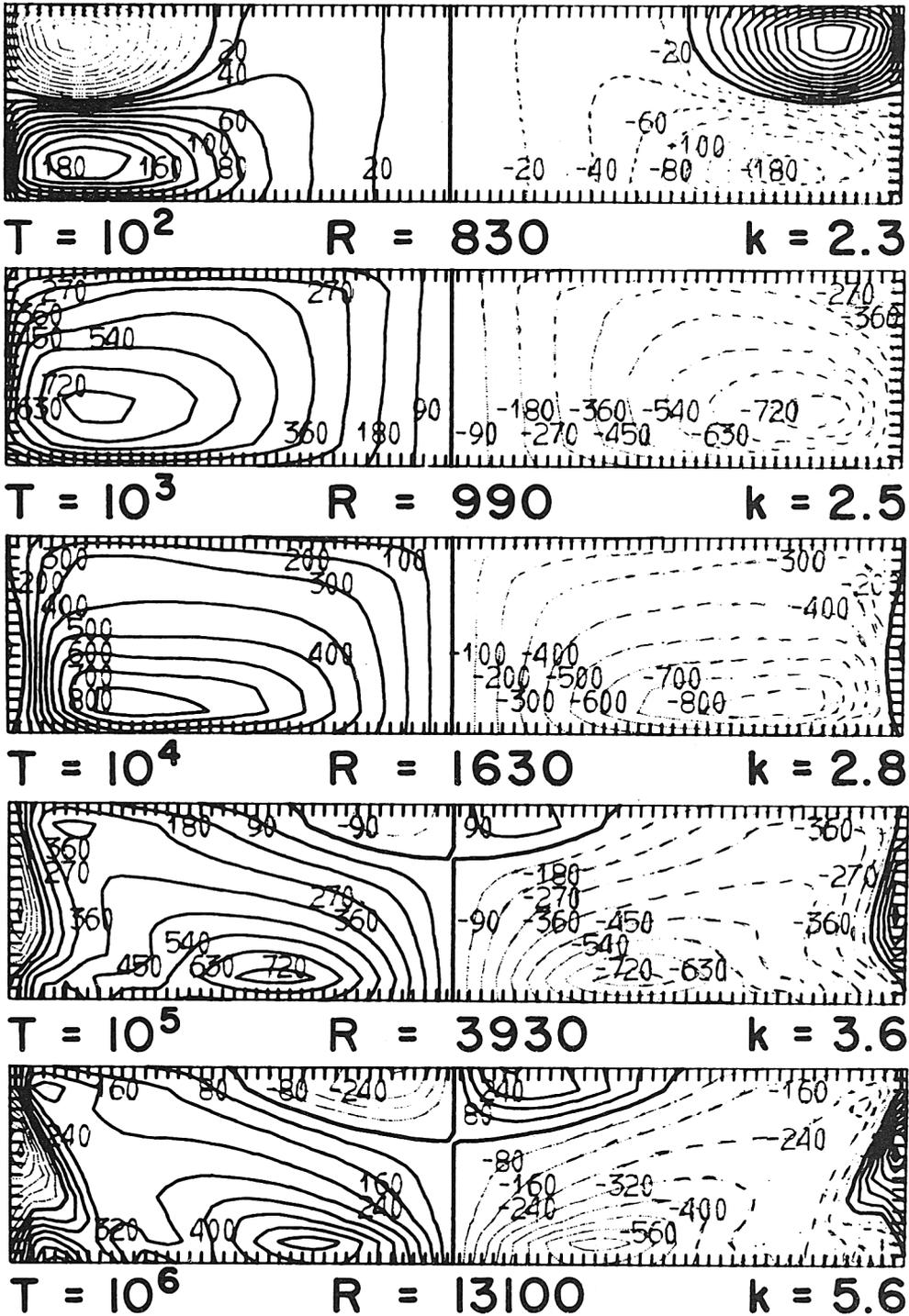


Fig. 9. Normalized helicity of the most unstable linear convection modes, according to the rotating annulus model of Gilman (1972).  $T$ ,  $R$  and  $k$  denote the Taylor number, the Rayleigh number, and the azimuthal wave number. Solid contours indicate positive (right-handed) helicity; dotted contours indicate negative (left-handed) helicity. The upper (lower) edge of each box is the top (bottom) of the convection zone, and north is to the right, south to the left (courtesy P. A. Gilman).

Thus, the prospects of a solar  $\omega \times \mathbf{j}$ -dynamo are not very promising, and we must conclude that either a completely different (i.e. not mean-field-electrodynamics type) theory of the solar cycle has to be developed or we have to revert to the  $\alpha\omega$ -dynamo. I prefer the latter; after all, the Sun *does* rotate and there *is* therefore helicity in the solar convection zone, giving rise to an  $\alpha$ -effect.

The unsuccessful attempts to solve our puzzle by means of the first two possibilities leave us with the third. Are the surfaces of isorotation really cylinders? I think we should very carefully reconsider all assumptions and models which lead to such a law of rotation. I will however not do so in this lecture which is on solar dynamo theory and not on solar differential rotation.

#### 4. Magnetic Sectors

In a recent paper (Stix, 1974) I have discussed possible theories of the sector structure of the large scale photospheric magnetic field. I will, therefore, spend only a small portion of this lecture on solar sectors. In particular, I shall not review all the observational evidence. I shall, however, discuss one observational fact in some detail, namely the fact that large-scale features on the Sun apparently do not, or at least not completely, participate in the differential rotation of the solar plasma. This is true for the photospheric magnetic field itself (Wilcox and Howard, 1970) but also for related features such as the electron corona (Hansen *et al.*, 1969), the coronal green line emission (Antonucci and Svalgaard, 1974) and coronal holes (Wagner, 1975; Timothy *et al.*, 1975).

A behaviour like this can probably most easily be explained as a *wave* phenomenon, and the three theoretical approaches which I will consider in this section have this interpretation in common.

There are first the models which ascribe the magnetic sectors to a corresponding pattern in the large scale velocity field. For example, models of the solar convection zone by Busse (1970, 1973), Durney (1970, 1971), Yoshimura and Kato (1971), Gilman (1972, 1975) and Yoshimura (1974) predict large scale motions in the form of rolls aligned parallel to the axis of rotation. These rolls are not destroyed by the mean differential rotation. In fact, they *produce* the differential rotation as they transport angular momentum towards the equatorial plane. At the same time, these rolls are damped on one of their sides (in longitude) and built up on the other, which is typical for a wave phenomenon. How such a velocity field can cause a similar, i.e. sectorial, structure in the magnetic field has been discussed e.g., by Yoshimura (1971, 1972). The longitudinal wave number,  $m$ , obtained from this theory is comparatively large: For a convection zone thickness of  $0.2 r_{\odot}$ , Durney (1970) found that  $m = 10$  marked the most unstable mode, and Gilman (1975) obtained  $9 \leq m \leq 21$  for Taylor numbers between 0 and  $10^6$ . The resulting magnetic features should, therefore, be narrow and elongated. Perhaps the elongated coronal hole reported by Timothy *et al.* (1975) is a prominent example for such a feature (Figure 10).\*

\* Magnetic neutral lines derived from  $H\alpha$  observations indicate however that the magnetic sector underlying this coronal hole covered a longitude interval of approx  $45^\circ$ , at least twice as wide as the hole itself (McIntosh *et al.*, 1975). The longitudinal wave number,  $m$ , was therefore only 4.

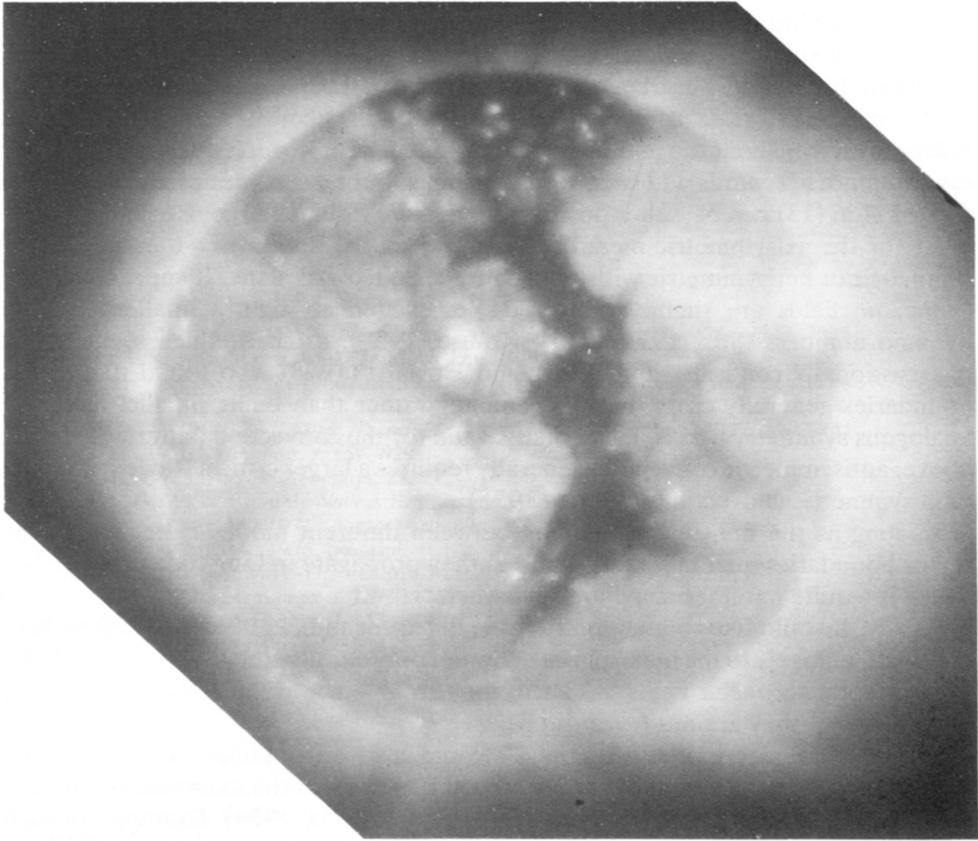


Fig. 10. An elongated coronal hole. This coronal image covers the wave-length intervals 3–32 Å and 44–54 Å; it was obtained on 1 June 1973 by the AS&E X-ray telescope on the Apollo Telescope Mount (courtesy A. S. Krieger, American Science and Engineering, Inc.).

In the observed photospheric magnetic field the dominant longitudinal wave number is usually much smaller than 10. This is particularly conspicuous in the spherical harmonic analysis of the Mt. Wilson data of Altschuler *et al.* (1974), who found that harmonics with  $m = 1$  and  $m = 2$  were dominant during most of the period from 1959 to 1972. The theoretical approach leading to such small longitudinal wave numbers is to consider the magnetic sectors as  $\phi$ -dependent solutions of the mean field induction equation. These solutions are proportional to

$$\exp(i(\Omega t + m\phi)), \quad (13)$$

and have been described by Stix (1971, 1974), Krause (1971) and Roberts and Stix (1972). Again, they are essentially waves, and their longitudinal propagation velocity, as computed by Stix (1974), is of the order 10 deg of longitude per year. This is about as fast as the sector structure inferred by Svalgaard and Wilcox (1975) propagates. According to these authors, a four-sector pattern persisted during the past five sunspot cycles, drifting slowly westwards during the first half and eastwards

during the second half of each cycle. Since only *linear* modes have been considered so far in theory, such a dependence of the drift on the phase of the cycle has not yet been predicted. It could only be obtained from a non-linear coupling of the sectoral modes (13) with the axisymmetric periodic mean field described in Section 2. Little is however known about the nature of this non-linear coupling. At this time I can only say that both eastwards and westwards propagating linear modes exist, i.e. the real part of  $\Omega$  in (13) can be either positive or negative (Stix, 1974).

As for the axisymmetric mean fields, the mean fields with  $m > 0$  can be either symmetric or antisymmetric with respect to the equatorial plane. It appears that the *symmetric* fields are slightly preferred, i.e. excited at slightly smaller absolute dynamo numbers (Stix, 1971, 1974; Krause, 1971), a result which seems to be observationally confirmed by Wilcox and Howard (1968) who found that sector boundaries generally cross the solar equator rather than being parallel to it. An analogous symmetry rule is, incidentally, valid for the convection pattern described above: antisymmetric convection generally requires a larger critical Rayleigh number than symmetric convection (Gilman, 1975).

As long as the non-linear coupling between different modes is negligible, the sector boundaries maintain their shape as they propagate in longitude (Stix, 1974), which is a quite natural behaviour for a wave.

Finally, I would like to mention that the linear mean field modes described here may drift relatively to the mean plasma flow not only because they are waves, but also because they are *mean* fields. As such, they are subject to the *turbulent* magnetic diffusivity, i.e. they are *not* frozen-in fields.

Of course, linear modes with  $m \neq 0$  could also be obtained from the exact induction equation, i.e. without the  $\alpha$ -effect. Contrary to the axisymmetric modes they are not in conflict with Cowling's theorem (Cowling, 1934). Examples of such modes have been presented by G. O. Roberts (see P. H. Roberts, 1971) and Gubbins (1972, 1973b). However, the point here is that we need the mean field approach in order to explain the mean *axisymmetric* field, so I think it is only natural to use the same equation, namely the mean field induction equation, for *all* modes,  $m = 0$  and  $m > 0$ . In order to compute the  $m > 0$  modes described above I have indeed used the *same* forms of differential rotation and  $\alpha$ -effect which led to the axisymmetric solution of Section 2 (Figure 3).

The third theoretical explanation that has been proposed for the origin of solar magnetic sectors is that they are a manifestation of hydromagnetic waves travelling in the azimuthal direction along a subsurface toroidal field (Kato and Nakagawa, 1970; Suess, 1971, 1975; Roberts and Stix, 1972). The angular velocity of these waves is of the order  $Ro_m v / r_\odot$ , where  $v$  is the Alfvén velocity corresponding to the mean toroidal field and  $Ro_m$  is the magneto-hydrodynamic Rossby number,  $Ro_m \sim v / r_\odot \omega$  (Roberts and Stix, 1972; Suess, 1975). Reasonable propagation velocities ( $\sim 1$  deg of longitude per year) can be obtained in this way, but the estimate is very uncertain since  $v$  is uncertain; we do not know exactly the magnitude of the mean toroidal field, nor do we know the depth (i.e. density) we should use in order to compute  $v$ . Also, important effects, such as shear, stratification and the geometry of the spherical shell, have not been taken into account. We may therefore consider the hydromagnetic wave theory as the most speculative of the three theories offered in this section.

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\* Articles marked with an asterisk have been translated into English (Roberts and Stix, 1971).

## DISCUSSION

**Roxburgh:** Our picture of the formation of the Sun suggests that at an early stage it was completely convective and therefore probably a dynamo operated throughout the whole Sun producing a magnetic field in the central regions. What do you think, is the connection between the present central field and the present dynamo field produced in the solar convective zone?

**Stix:** I think if the central field exists it is not connected to the dynamo field because the time scale of the central field is much longer.

**Gilman:** Suppose that the bottom of the convection zone is rotating solidly at nearly the equatorial rate, as one interpretation of sector and coronal hole rotation indicates. Then the radial shear is much stronger near the pole than the equator. What sort of  $\alpha$ -effect dynamo would then occur, and would it be in agreement with the important observations?

**Stix:** I would expect toroidal fields generated at too high latitudes.

**Yoshimura:** I would like to comment about two questions. One is about the determination of the sign of  $\alpha$  and gradient of  $\omega$ . You said that it is possible to determine them by determining the phase shift between

the poloidal and toroidal field observed at the surface. However, this is possible only if the radial gradient of  $\omega$  is dominant. According to the diagrams of the fields of myself, there is virtually no phase shift observed. So that it is rather questionable that the determination is possible. Moreover, if the radial gradient of  $\omega$  is dominant, there is no branching of the two wings in each hemisphere in the diagram of the poloidal field which has been observed and presented in this symposium. The other question is about the interpretation of the sector structure of the magnetic field. You said the interpretation by the global-scale nonaxisymmetric velocity fields has some drawbacks, i.e. the theories predict wave numbers of 10 to 20. However, the theories so far studied have used many approximations. So, it is safe to say that the correct determination of a wave number has not yet been done. Moreover, I would like to make one more comment about Dr Busse's fluid mechanical convection experiment. He obtained some interesting results even in the experiment of plane parallel case. That is, besides the ordinary Bénard cell convection, he observed some coherent laminar flow cells whose scale is much larger than the Bénard cells and also larger than the depth of the system. In the case of the Sun also, it is possible that similar kind of larger scale coherent convective flow exists whose wave number may be as well as 2 or 3, or 5 or 6 besides the supergranular or granular convection. So we should be very careful about objecting to the existence of the global convection. If it exists, it surely can explain the sector structure of the magnetic field.

*Weiss:* It is difficult to predict the horizontal scale of plane form of convection. The azimuthal wavenumber of 10 or 11 was originally put forward on the assumption that the horizontal width would be comparable with the depth of the convecting layer in a Boussinesq fluid.

*Gilman:* Dr Yoshimura comments that current spherical shell convection models are not good enough to give reliable prediction of the longitudinal wave numbers. I am inclined to agree, and point out that strong enough shear in the differential rotation could significantly reduce the wave number predicted by current models, which are Boussinesq and based on initial states of solid rotation. Induced  $\mathbf{j} \times \mathbf{B}$  forces will also tend to reduce the longitudinal wave number. Compressibility should also contribute.

*Stix:* I am looking forward to seeing convection models with smaller longitudinal wave number.

*Stenflo:* I would like to comment on the sector structure of solar magnetic fields and the modes in longitude. Power spectra of the observed variations of magnetic fields with longitude show that the power is quite high out to wave numbers of 20 or even more.

*Wilcox:* The term 'magnetic sector' was first used in the description of a large-scale structure observed in the interplanetary magnetic field. These sectors were then found in the photospheric field observed with Babcock magnetographs, and are seen with particular clarity in observations of the mean solar magnetic field (the Sun seen as a star). If defined in this way there are nearly always either four or two sectors. It is these sectors that have been shown to have the rigid rotation properties discussed by Stix and other theorists.

*Deinzer:* It still seems that an angular velocity increasing inwards is required to produce the desired sort of butterfly diagrams. Imagine you drive the solar dynamo by differential rotation increasing inwards and being constant on spheres.

Is it entirely inconceivable that the electromagnetic (Lorentz) forces could drive the observed equatorial acceleration? Maybe this is more a question to Dr Gilman.

*Gilman:* Dr Deinzer asks whether the presence of  $\mathbf{j} \times \mathbf{B}$  forces could produce an equatorial acceleration when they are induced by a dynamo with rotation increasing with depth. This is conceivable, but I would suspect it would produce a fairly large dependence of the differential rotation on the magnetic cycle. Observations indicate this occurs weakly, if at all.

*Roxburgh:* If the central regions of the Sun are magnetically isolated from the surface convective zone then just beneath the convective zone there will be a region where the angular velocity increases inwards. As the surface regions are slowed down by the angular momentum loss in the solar wind there will be an angular velocity gradient between the more rapidly spinning core and the envelope. This region is probably weakly turbulent due to Goldreich-Schubert-Fricke instabilities and could be the site of the solar dynamo.

*Stix:* The dynamo could operate there only if this weak turbulence still provides sufficient turbulent diffusivity so that the mean field can diffuse down into the shear region. Perhaps convective overshooting would also help to do this.

*Mestel:* I would like to take up Dr Deinzer's query. If a dynamo does operate, then presumably the field amplifies until magnetic forces are strong enough to react back in at least one essential part of the motion. This morning we heard some ideas and preliminary results on magnetic back-reaction on the  $\alpha$ -term; maybe there is also an effect on the non-uniform rotation which drives the whole process. There is in fact one simple model – in which angular momentum is transported by magnetic stresses and large-scale meridional circulation only, but not by turbulence – which can give both equatorial acceleration and  $\Omega$  increasing inwards. This result is no more than a hint: one needs to include turbulence, and the circulation field should emerge from the theory, instead of being postulated. But perhaps the possibility should be borne in mind, especially if the purely hydrodynamic models of differential rotation continue to give trouble. Dr Gilman's point about a non-observed cycle-dependent  $\Omega$  is of course very important: to be

acceptable the theory would need to show that the time of redistribution of angular momentum when the field changes is long compared with the solar cycle.

*Durney:* I think that we all agree that the mixing length theory is logically highly unsatisfactory. However, it could well give results that are not too bad. It gives good results for the Sun because the parameters are adjusted to do so. Nevertheless it also predicts accurately the spectral type separating stars with fast and slow rotations, that is it predicts accurately the spectral type of stars having appreciable surface convection zones.