THE ENVELOPES OF SPHERICAL GALAXIES

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ABSTRACT. The light distribution in the envelopes of spherical galaxies seems to be caused by the existence of a break in the energy distribution, N(E) at E = 0. This, in turn is probably caused by the escape of positive energy stars.

Elliptical galaxies, with some exceptions like cDs, show a universal light distribution than can be represented in space as:

$$\varepsilon \propto r^{-2}(r_c + r)^{-2} \quad \text{(Jaffe, 1981).}$$

This suggests a very general mechanism of formation. Here I show that the r^{-4} behavior in the envelope is the result of a sharp break in the energy distribution of particles, N(E), near E = 0, the escape energy. This break, in turn, would be the result of any energy scattering processes whose cross section doesn't vary rapidly near E = 0.

To prove the first point, we write the relation between N(E) and f(E), the phase space density, to be (c.f. Binney, 1982):

$$N(E) = f(E)\ A(E)\ , \text{ where,}$$

$$A(E) \propto \int_{r=0}^{\phi(r)=E} (E - \phi(r))^{1/2}\, r^2 dr \quad .$$

In the envelope the potential will be essentially Keplerian, $\phi(r) \sim GM/r$ which, with the above integration, yields $A(E) \propto (-E)^{5/2}$.

Thus if N(E) has a sharp break at E = 0, for example:

$$N(E) = 0 \text{ for } E > 0, \text{ and}$$
$$N(E) = 1 \text{ for } E < 0, \text{ then}$$

$$f(E) \propto E^{5/2} \text{ near } E{=}0 \text{ , so, using the standard formula,}$$

$$\rho(r) = \int f(E)\ (E - \phi(r))^{1/2} dE \quad \propto (-\phi)^4 \quad \propto \quad (GM/r)^4$$

in the envelope, which shows the desired behavior.

T. de Zeeuw (ed.), Structure and Dynamics of Elliptical Galaxies, 511–512.

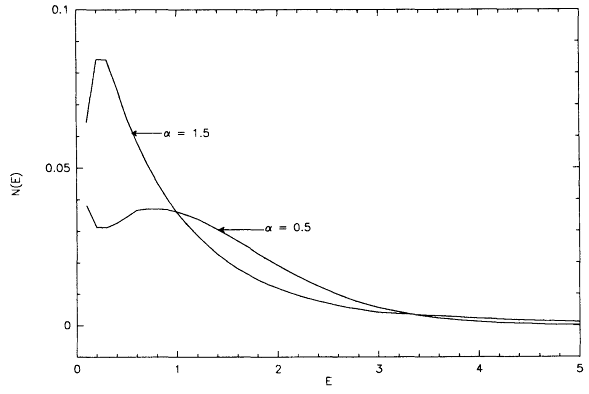
The only requirement is that $N(E) \propto E^0$ as $E \to 0$, i.e. that there is a sharp break in N at the escape energy. Such a break would be the natural result of any scattering process in energy space that doesn't vary radically near E=0; stars scattered to small negative energies stay there while those at small positive energies leave the system.

For example: if energy scattering only occurs near the nucleus, then the typical scattering length, ΔE, will be primarily a function of the velocity, $v = (2(E - \phi))^{1/2}$. For large values of ϕ and E near zero, v is only a slow function of E.

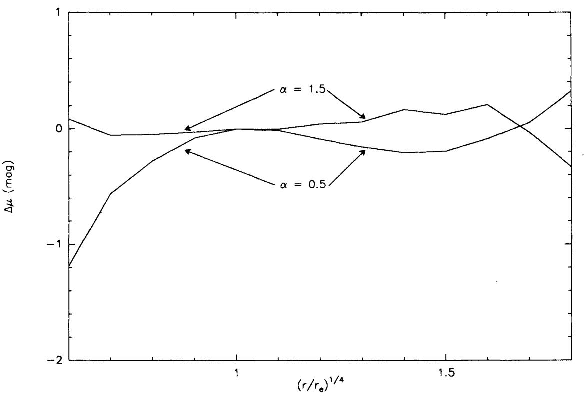
We have numerically calculated N(E) for a case where the scattering rate is:

$S(E \to E') \propto E^{-\alpha} \exp(-(E-E')^2/2\sigma^2)$ and $N(E, t=0) = \delta(E - 1.0)$.

Here are the resultant N(E) curves for $\sigma = 0.4$ and $\alpha = 0.5$ and 1.5:



At large radius the residuals of the corresponding surface brightness from the de Vaucouleur's law are, for either α at most a few tenths of a magnitude.



REFERENCES

Binney, J., 1982. Mon. Not. R. Astron. Soc. **200**, 951
Jaffe, W., 1982. Mon. Not. R. Astron. Soc. **202**, 995