DYNAMIC DECISION PROBLEMS IN AN INSURANCE COMPANY

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1. INTRODUCTION

1.1. — In this paper we shall consider some of the decisions which have to be made in the normal course of business in an insurance company. We shall see that the "right" decisions can be found only when the problems are analysed in their proper dynamic context.

As examples of the decision problems which we shall study, we can mention the following:

(i) What premium rates should be quoted on the insurance contracts, which the company offers to the public?

(ii) How much should the company spend to promote the sale of its policies?

(iii) When should the company refuse to underwrite a proposed insurance contract?

(iv) How shall the company reinsure its portfolio of insurance contracts?

(v) What reserve funds should an insurance company keep?

(vi) How shall the company's funds be invested?

Any actuary will be familiar with such problems, and he will probably feel that these problems cannot be satisfactorily solved with the methods offered by the classical actuarial theory.

1.2. — In some earlier papers [1] and [2] it has been argued that such problems can best be solved in the frame work of *utility* theory. As an illustration we shall take Problem (iii) in the preceding paragraph, and consider an insurance company in the following situation:

(i) The company has a capital S.

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(ii) The company holds a portfolio of insurance contracts which will lead to a total payment of x to settle claims. $F_1(x)$ is the distribution of the variate x.

When all contracts in the portfolio have expired, the company will have a capital

$$z_1 = S - x$$

z_1 is a variate with the distribution

$$G_1(z_1) = 1 - F_1(S-z_1), \quad z_1 \leq S$$

Let us now assume that this company is offered an amount P, if it will accept an insurance contract (a reinsurance treaty) with claim distribution $F_2(y)$. If the company accepts, its capital when all contracts have expired will be:

$$z_2 = S + P - x - y$$

The distribution of this variate will be:

$$G_2(z_2) = 1 - H(S+P-z_2), \quad z_2 \leq S+P$$

If the variates x and y are stochastically independent, the distribution H will be the convolution of F_1 and F_2.

1.3. — If the company considered in the preceding paragraph, accepts the offer, it must in some sense find the distribution G_2 better than G_1. In order to compare two arbitrary distributions, and select the best, the company must have a *preference ordering* over the set of all probability distributions.

A preference ordering of this kind must obviously depend on "subjective" elements, such as the company's willingness to assume risks. The ordering can usually be described in several different ways. If the ordering is consistent in the precise sense, defined by von Neumann and Morgenstern [8], it can be described in a particularly simple way. In this case there exists a function $u(x)$, so that $G_2(x)$ is preferred to $G_1(x)$ if and only if

$$\int_{-\infty}^{+\infty} u(x)\, d\, G_2(x) > \int_{-\infty}^{+\infty} u(x)\, d\, G_1(x)$$

The function $u(x)$ is usually referred to as the *utility function*, because it can be interpreted as the utility associated with an

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amount of money equal to x. From our point of view it is, however, sufficient to consider $u(x)$ as a convenient way of describing a preference ordering.

1.4. — The utility theory of von Neumann and Morgenstern is mathematically elegant, and in many ways very attractive. It can, however, not be of much practical use, unless we know something about the shape of the utility function, which represents the preference ordering of our insurance company.

As an approach to this problem we can ask what is the utility of the capital, left with the company, when all contracts in the portfolio have expired. It seems that we can answer this question only if we know something about the future plans of the conpany, i.e. the kind of insurance business which the company expects to underwrite in later periods. This naturally leads us to consider the essentially static decision problem in a dynamic setting.

2. A simple Dynamic Model

2.1. — As a first approach to a dynamic formulation of the problem, we shall consider an insurance company which operates under the following conditions:

(i) The company has an initial capital S.

(ii) In each successive underwriting period the company makes a profit x, which is a variate with distribution $F(x)$. The profit in any period is stochastically independent of profits in other periods.

(iii) If the company's capital becomes negative at the end of an underwriting period, the company is ruined, and will go out of business.

(iv) If at the end of a period the company's capital exceeds Z, the excess will be paid out as dividend immediately.

This model is a *Random Walk* with an absorbing barrier at $S = 0$, and a reflecting barrier at $S = Z$.

If we let Z go to infinity, i.e. if we assume that the company never will pay any dividends, we obtain the model which forms the basis of Lundberg's "Collective Theory of Risk" [7]. This assumption is not very realistic, and the resulting theory has not found many practical applications, although it may have stimulated further research. This has been pointed out i.a. by de Finetti [6], who first studied the far richer theory we obtain by adding a reflecting barrier to the model.

2.2. — To illustrate the possibilities of de Finetti's generalization, let us first consider the function

$D(S,Z)$ = the expected number of operating periods before ruin occurs.

From the conditions in para 2.1 it follows that

$$D(S,Z) = 0 \qquad \text{for } S < 0$$

$$D(S,Z) = D(Z,Z) \qquad \text{for } S > Z$$

For $0 \leq S \leq Z$ it is easy to see that $D(S,Z)$ must satisfy the integral equation:

$$D(S,Z) = 1 + \int_{-S}^{\infty} D(S+x, Z)\, dF(x)$$

De Finetti studied the special case where

$$F(x) = 0 \qquad \text{for } x < -1$$

$$F(x) = 1-p \qquad \text{for } -1 \leq x < 1$$

$$F(x) = 1 \qquad \text{for } 1 \leq x$$

In this case the integral equation reduces to the difference equation

$$D(S,Z) = 1 + p\, D(S+1,Z) + (1-p)\, D(S-1,Z)$$

This equation can be solved by elementary methods, and the nature of the solution has been discussed in some detail in another paper [4].

As a more general case, let us assume that $F(x)$ is continuous, and that a density function $f(x) = F'(x)$ exists.

The integral equation can then be written:

$$D(S,Z) = 1 + \{1 - F(Z-S)\}\, D(Z,Z) + \int_0^Z D(x,Z)\, f(x-S)\, dx$$

This is an equation of Fredholm's type, with the simple kernel $f(x-S)$, and it can be solved by different methods. We can, for instance, form the iterated kernels

$$f^{(1)}(x-S) = f(x-S)$$

$$f^{(n)}(x-S) = \int_0^Z f^{(n-1)}(x-t)\, f(t-S)\, dt$$

and obtain the Liouville-Neumann expansion

$$D(S,Z) = 1 + \{1-F(Z-S)\}\, D(Z,Z) + \sum_{n=1}^{\infty} \int_0^Z f^{(n)}(x-S)\, dx$$
$$+ D(Z,Z) \sum_{n=1}^{\infty} \int_0^Z \{1-F\,(Z-x)\}\, f^{(n)}(x-s)\, dx$$

We determine $D(Z,Z)$ by requiring the solution to be continuous at $S = Z$. This gives the equation

$$D(Z,Z) = 1 + \{1-F(0)\}\, D(Z,Z) + \sum_{n=1}^{\infty} \int_0^Z f^{(n)}(x-Z)\, dx$$
$$+ D(Z,Z) \sum_{n=1}^{\infty} \int_0^Z \{1-F\,(z-x)\}\, f^{(n)}(x-z)\, dx$$

2.3. — If at the end of underwriting period t the company's capital S_t exceeds Z, the excess $s_t = S_t - Z$ will be paid out as dividend—to share holders or policy holders, as the case may be. Hence the company will make a sequence $s_0, s_1, \ldots s_t \ldots$ of dividend payments. This sequence is a discrete stochastic process.

Let us now consider the expected discounted value of these payments, i.e.

$$E \left\{ \sum_{t=0}^{\infty} v^t\, s_t \right\}$$

where $0 < v < 1$ is a discount factor.

Since this obviously depends on the initial capital S, and on the reserve requirements represented by Z, we shall write:

$$V(S,Z) = E \left\{ \sum_{t=0}^{\infty} v^t\, s_t \right\}$$

From the conditions in para 2.1 it follows that

$$V(S,Z) = 0 \qquad \text{for } S < 0$$
$$V(S,Z) = S - Z + V(Z,Z) \qquad \text{for } S > Z$$

For $0 \leq S \leq Z$ the function $V(S,Z)$ must satisfy the integral equation

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$$V(S,Z) = v \int_{-S}^{Z-S} V(S+x,Z)\, dF(x) + v \int_{Z-S}^{\infty} \{V(Z,Z) + x + S - Z\}\, dF(x)$$

For the simple discrete case considered in the preceding paragraph, the integral equation reduces to the difference equation

$$V(S,Z) = vp\, V(S+1,Z) + v(1-p)\, V(S-1,Z)$$

This case has been discussed by de Finetti [6], and in more detail in some other papers [3] and [4].

If $F(x)$ is continuous, and a density function exists, the integral equation can be written:

$$V(S,Z) = v \int_{0}^{Z} V(x,Z)\, f(x-S)\, dx + v \int_{Z}^{\infty} \{V(Z,Z) + x - Z\}\, f(x-S)\, dx$$

This is again an equation of Fredholm's type. It can be solved by forming the iterated kernels and taking the Liouville-Neumann expansion:

$$V(S,Z) = v\{1-F(Z-S)\}\, V(Z,Z) + v \int_{Z}^{\infty} xf(x+Z-S)dx + \sum_{n=1}^{\infty} v^n \int_{0}^{Z} f^{(n)}(x-S)dx + V(Z,Z) \sum_{n=1}^{\infty} \int_{0}^{Z} \{1-F(z-x)\}\, f^{(n)}(x-s)\, dx$$

To determine $V(Z,Z)$ we require the solution to be continuous at $S = Z$, and obtain:

$$V(Z,Z) = v\{1-F(0)\}\, V(Z,Z) + v \int_{0}^{\infty} xf(x)dx + \sum_{n=1}^{\infty} v^n \int_{0}^{Z} f^{(n)}(x-Z)\, dx + V(Z,Z) \sum_{n=1}^{\infty} \int_{0}^{Z} \{1-F(z-x)\}\, f^{(n)}(x-z)\, dx$$

2.4. — It is clear that the two functions $D(S,Z)$ and $V(S,Z)$ are relevant to a number of decisions which have to be made in an insurance company.

For instance:

(i) If the objective of the company is to maximize the expected

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discounted value of its dividend payment, we are led to seek the value of Z, which maximizes $V(S,Z)$, for given S.

(ii) If the required reserve Z is given, and the objective of the company is to survive as long as possible, we have some information about how the company will make its reinsurance decisions.

To illustrate this, let us assume that the company receives an offer of the type we considered in para 1.2. If the offer is accepted, the expected duration of life of the company will be

$$\int_0^\infty D(S+P-y,\ Z)\ dF_2(y)$$

If the company pursues its overall objective in a consistent manner, it will accept the offer only if this increases the expected life, i.e. if

$$\int_0^\infty D(S+P-y,\ Z)\ dF_2(y) > D(S,Z)$$

This means, however, that the company makes its decision as if its preference ordering over probability distributions is represented by the utility function $D(S,Z)$. Hence it appears that the static decision problem considered in Section 1, is solved almost automatically when the problem is placed in its natural dynamic context.

It is possible to discuss such decision problems in full generality. To bring out the main features of the problems, it is, however, sufficient to discuss a special case. In the following we shall do this, and we shall indicate when the results derived from the special case have general validity.

3. A Special Case

3.1. — In general Fredholm's integral equation has no simple explicit solution. We are therefore led to seek a case where the basic distribution $F(x)$ has a form giving a solution which can be discussed in detail by fairly elementary methods.

As a reasonably realistic example, we could consider the case:

$$f(x) = e^{x-P} \qquad x \leq P$$
$$f(x) = 0 \qquad x > P$$

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We can interpret this to mean that our company in each operating period receives an amount of premiums P, and accepts a portfolio with the claim distribution $F(x) = 1 - e^{-x}$. It is natural to assume that $P > 1$, so that the game is favorable to the company.

It has been shown in another paper [5] that the integral equation in this case reduces to a differential-difference equation, which has a solution given by a finite expression. This expression is, however, far from simple, and is not very suitable for detailed discussion.

3.2. — As another example, let us consider

$$f(x) = k\alpha e^{-\alpha x} \qquad \text{for } x > 0$$

$$f(x) = (1-k)\,\alpha e^{\alpha x} \qquad \text{for } x < 0$$

The value of $f(x)$ for $x = 0$ does not matter. We shall assume that $1/2 < k < 1$, i.e. that the underwriting is favorable to the company.

The obvious objection to this probability distribution is that it does not put any upper limit to the gain, which the company can make in a single underwriting period. We can justify our choice of $f(x)$ simply by its mathematical convenience.

We can also assume that the company invests its funds in very speculative shares, which may give a very high yield.

The integral equation from para 2.3 can now be written as follows:

$$V(S) = v(1-k)\,\alpha e^{-\alpha S} \int_0^S V(x)\, e^{\alpha x}\, dx + vk\alpha e^{\alpha S} \int_0^S V(x)\, e^{-\alpha x}\, dx$$

$$+ vk\, V(Z)\, e^{\alpha(S-Z)} + \frac{vk}{\alpha} e^{\alpha(S-Z)}$$

For simplicity we have written $V(S)$ for $V(S,Z)$, since there should be no risk of misunderstanding.

3.3. — Differentiating the integral equation twice with respect to S, we obtain:

$$V'(S) = v(1-k)\,\alpha\, V(S) - vk\alpha\, V(S)$$

$$- v(1-k)\alpha^2 e^{-\alpha S} \int_0^S V(x) e^{\alpha x} dx + vk\alpha^2 e^{\alpha S} \int_S^Z V(x) e^{\alpha x}\, dx$$

$$+ vk\,\alpha\, V(Z)\, e^{\alpha(S-Z)} + vk\, e^{\alpha(S-Z)}$$

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and
$$V''(S) = v(1-2k)\,\alpha\,V'(S) - v\alpha^2 V(S)$$
$$+ v(1-k)\alpha^3 e^{-\alpha S}\int_0^S V(x)e^{\alpha x}dx + vk\alpha^3 e^{\alpha S}\int_S^Z V(x)e^{-\alpha x}dx$$
$$+ vk\,\alpha^2\,V(Z)\,e^{\alpha(S-Z)} + vk\,\alpha\,e^{\alpha(S-Z)}$$

From these expressions it is easy to see that $V(S)$ must satisfy the differential equation:

$$V''(S) - \alpha^2 V(S) = v(1-2k)\,\alpha\,V'(S) - v\,\alpha^2\,V(S)$$

or

$$(1-v)\,\alpha^2\,V(S) + v(1-2k)\,\alpha\,V'(S) - V''(S) = 0$$

Hence our integral equation is reduced to a homogeneous differential equation of the second order with constant coefficients. The general solution of this equation is:

$$V(S) = C_1\,e^{r_1 S} + C_2\,e^{r_2 S}$$

Here C_1 and C_2 are arbitrary constants, and r_1 and r_2 are the roots of the characteristic equation

$$r^2 - v(1-2k)\,\alpha\,r - (1-v)\,\alpha^2 = 0$$

We find

$$r_1 = \frac{\alpha}{2}\{v(1-2k) + (v^2(1-2k)^2 + 4 - 4v)^{1/2}\}$$

$$r_2 = \frac{\alpha}{2}\{v(1-2k) - (v^2(1-2k)^2 + 4 - 4v)^{1/2}\}$$

It is easy to verify that both roots are real, and that $r_1 > 0$, $r_2 < 0$.

3.4. — The constants C_1 and C_2 must be determined so that the general solution of the differential equation also is a solution of the integral equation. Substituting the general solution in the integral equation, we find:

$$C_1\,e^{r_1 S} + C_2 e^{r_1 S} = \frac{v(1-k)\,\alpha}{r_1 + \alpha}\,C_1\,\{e^{r_1 S} - e^{-\alpha S}\}$$
$$+ \frac{v(1-k)\alpha}{r_2 + \alpha}\,C_2\{e^{r_2 S} - e^{-\alpha S}\} + \frac{vk\,\alpha}{r_1 - \alpha}\,C_1\,\{e^{(r_1-\alpha)Z+\alpha S} - e^{r_1 S}\}$$

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$$+ \frac{vk\,\alpha}{r_2 - \alpha} C_2 \{e^{(r_2-\alpha)Z+\alpha S} - e^{r_2 S}\} + vk\, C_1\, e^{(r_1-\alpha)Z+\alpha S}$$
$$+ vk\, C_2\, e^{(r_2-\alpha)Z+\alpha S} + vk\,\alpha\, e^{\alpha(S-Z)}$$

We shall write this expression as follows:

$$\left\{1 - \frac{v(1-k)\alpha}{r_1+\alpha} + \frac{vk\,\alpha}{r_1-\alpha}\right\} C_1\, e^{r_1 S} +$$
$$\left\{1 - \frac{v(1-k)\alpha}{r_2+\alpha} + \frac{vk\,\alpha}{r_2-\alpha}\right\} C_2\, e^{r_2 S} -$$
$$\left\{\frac{vk\, r_1}{r_1-\alpha} e^{(r_1-\alpha)Z} C_1 + \frac{vk\, r_2}{r_2-\alpha} e^{(r_1-\alpha)Z} C_2 + \frac{vk}{\alpha} e^{-\alpha Z}\right\} e^{\alpha S} +$$
$$\left\{\frac{v(1-k)\alpha}{r_1+\alpha} C_1 + \frac{v(1-k)\alpha}{r_2+\alpha} C_2\right\} e^{-\alpha S} = 0$$

This equation must hold for all values of S. Hence the four expressions in brackets must be zero.

It is easy to verify that the two first of these expressions, i.e. the coefficients of $e^{r_1 S}$ and $e^{r_2 S}$ are zero when r_1 and r_2 are the roots of the characteristic equation.

We then obtain the following two equations for the determination of C_1 and C_2

$$\frac{r_1\, e^{r_1 Z}}{r_1-\alpha} C_1 + \frac{r_2\, e^{r_2 Z}}{r_2-\alpha} C_2 = -\frac{1}{\alpha}$$
$$\frac{1}{r_1+\alpha} C_1 + \frac{1}{r_2+\alpha} C_2 = 0$$

The determinant of these equations is

$$D = \frac{r_1 e^{r_1 Z}}{(r_1-\alpha)\ (r_2+\alpha)} - \frac{r_2 e^{r_2 Z}}{(r_1+\alpha)\ (r_2-\alpha)}$$

and we find

$$C_1 = \frac{-1}{\alpha(r_2+\alpha)\ D},\quad C_2 = \frac{1}{\alpha(r_1+\alpha)\ D}$$

This gives us the following explicit expression for the expected discounted value of the dividend payments

$$V(S,Z) = \frac{1}{\alpha\, D}\left\{\frac{e^{r_2 S}}{r_1+\alpha} - \frac{e^{r_1 S}}{r_2+\alpha}\right\}$$

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This expression is maximized for the value of Z which minimizes the absolute value of D, i.e. the value determined by the equation

$$\frac{dD}{dZ} = \frac{r_1^2 e^{r_1 Z}}{(r_1-\alpha)(r_2+\alpha)} - \frac{r_2^2 e^{r_2 Z}}{(r_1+\alpha)(r_2-\alpha)} = 0$$

or

$$e^{(r_1-r_2)Z} = \frac{r_2^2(r_1-\alpha)(r_2+\alpha)}{r_1^2(r_1+\alpha)(r_2-\alpha)}$$

This value of Z is clearly unique, and independent of S, i.e. there exists a unique optimal level for the company's reserves. The result does, however, not seem to hold in the general case.

3.5. — By similar considerations we find that $D(S) = D(S,Z)$ must satisfy the integral equation

$$D(S) = 1 + (1-k)\alpha\, e^{-\alpha S} \int_0^S D(x)\, e^{\alpha x}\, dx$$

$$+ k\alpha e^{\alpha S} \int_Z^S D(x)\, e^{-\alpha X}\, dx + k\, e^{\alpha(S-Z)}\, D(Z)$$

Differentiating twice we find that the integral equation can be reduced to the differential equation

$$(2k-1)\,\alpha\, D'(S) + D''(S) + \alpha^2 = 0$$

The general solution of this equation is

$$D(S) = C_1\, e^{-(2k-1)\alpha S} - \frac{\alpha}{2k-1}\, S + C_2$$

where C_1 and C_2 are constants which must be determined so that the solution also satisfies the integral equation.

By a procedure similar to the one used in para 3.3, we find:

$$C_1 = -\frac{2k}{(2k-1)^2}\, e^{(2k-1)\alpha Z}$$

$$C_2 = \frac{k}{(2k-1)^2(1-k)}\, e^{(2k-1)\alpha Z} - \frac{1}{2k-1}$$

and

$$D(S,Z) = \frac{k}{(2k-1)(1-k)}\, e^{(2k-1)\alpha Z} - \frac{2k}{(2k-1)^2}\, e^{(2k-1)\alpha(Z-S)}$$

$$- \frac{1}{2k-1}(1+\alpha S)$$

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3.6. — To illustrate these results with a numerical example, let us take $\alpha = 1$, $r_1 = 0.1$ and $r_2 = -0.3$.

This corresponds to $v = 0.97$, and $k = 0.603$

We then find:

$$V(S,Z) = \frac{143\, e^{0.1S} - 91\, e^{-0.3S}}{16\, e^{0.1Z} + 21\, e^{-0.3Z}}$$

and $$D(S,Z) = 37.5\, e^{0.2Z} - 5\,(1+S) - 30\, e^{0.2Z\,(Z-S)}$$

Table 1 gives the value of the function $V(S,Z)$ for some selected values of S and Z. It is easy to verify that this function takes its maximal value for $Z = 3.45$.

Table 1

$V(S,Z)$ = Expected discounted value of dividend payments

S \ Z	0	1	2	3	4	5
0	1.41	1.57	1.68	1.74	1.73	1.68
1	2.41	2.74	2.93	3.02	3.02	2.94
2	3.41	3.74	4.03	4.16	4.16	4.04
3	4.41	4.74	5.03	5.21	5.20	5.14
4	5.41	5.74	6.03	6.21	6.19	6.02
5	6.41	6.74	7.03	7.21	7.19	6.98

Table 2 gives the value of the function $D(S,Z)$ for the same values of S and Z.

Table 2

$D(S,Z)$ = Expected duration of life of the company

S \ Z	0	1	2	3	4	5
0	2.5	4.2	6.2	8.7	11.7	15.4
1	2.5	5.8	9.6	13.3	19.0	25.2
2	2.5	5.8	11.2	16.7	23.6	32.4
3	2.5	5.8	11.2	18.3	27.0	37.0
4	2.5	5.8	11.2	18.3	28.6	40.4
5	2.5	5.8	11.2	18.3	28.6	42.0

4. The Decision Problems

4.1. — The example we have discussed in Section 3 brings out an obvious, but often overlooked truth: We cannot find the right

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decision unless we really know what we want. This may sound trivial, but our discussion indicates that it may not always be so easy to spell out what we want in an unambiguous way.

To illustrate the point, let us assume that we are the majority shareholders of an insurance company. We may then want to make the expected life of our company as long as possible. This implies, however, that the company should never pay any dividend, and this may not be quite what we want. Our second thought may then be to maximize the expected discounted value of the dividend payments which will be made over the lifetime of the company. However, is this really what we want?

4.2. — To throw some light on these questions, let us assume that at the end of an underwriting period our company has a capital $S = 4$. Let us further assume that the actuary of the company asks us to make one of the following four decisions:

(i) Set the reserve requirement at $Z = 3$, and pay a dividend $s = 1$. This will give:
Expected dividend payment $V(4,3) = 6.21$
Expected life $D(4.3) = 18.3$

(ii) Set the reserve requirement at $Z = 3.45$, and pay a dividend $s = 0.55$. This will give:
Expected dividend payment $V(4, 3.45) = 6.23$
Expected life $D(4, 3.45) = 22.7$

(iii) Set the reserve requirement at $Z = 4$, and pay no dividend. This will give:
Expected dividend payment $V(4,4) = 6.19$
Expected life $D(4,4) = 28.6$

(iv) Set the reserve requirement at $Z = 5$, and pay no dividend. This will give:
Expected dividend payment $V(4,5) = 6.02$
Expected life $D(4,5) = 40.4$

Is it obvious that we in this situation select Decision (ii)? Some people may well be willing to sacrifice some dividends in order to prolong the life of the company, and they may go in for Decision (iv).

4.3. — Our discussion indicates that we should be very careful in spelling out the *objectives* of our insurance company, before we get too excited over the advanced methods of operations research. These methods are powerful, and they will always give us the right solution, but this may be the solution to the wrong problem.

If the general manager of our insurance company wants to run the company strictly as a business enterprise, he will probably always seek out the decisions which maximize $V(S,Z)$. If, however, he is concerned with the social responsibility of the company, and the security which it offers to policy holders, he may also consider $D(S,Z)$ when making his decisions. He will probably try to balance the two elements, but it is not easy to specify how this should be done.

The general manager and his board must, however, solve this problem, and it seems that they must do it themselves, without much help from actuaries and other experts on operations research.

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