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On elevation models as input for mass-balance calculations of the Greenland ice sheet

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ABSTRACT. In this paper the elevation model for the Greenland ice sheet based upon radio-echo-sounding flights of the Technical University of Denmark (TUD) (Letréguilly and others, 1991) are compared with the satellite-altimetry model (Tscherning and others, 1993) improved with airborne-laser and radar altimetry (IA model). Although the general hypsometry of both data sets is rather similar, differences seem to be large at individual points along the ice margin. Over the entire ice sheet, the difference between the IA model and the TUD model is 33 m with a root-mean-square error of 112 m. Differential GPS measurements collected in the ice-marginal zone near Søndre Strømfjord show that the IA model is more accurate than the TUD model. The latter data set underestimates the elevation by approximately 150 m in the ice-marginal zone near Søndre Strømfjord.

Calculation of the ablation with an energy-balance model and with a degree-day model points to a 20% decrease in the ablation if the IA model is used. Not only does this show the sensitivity of ablation calculations to the orographic input but it also indicates that the ablation calculated by the models used nowadays is relatively overestimated.

INTRODUCTION

Dynamical modelling of the Greenland ice sheet is seriously hampered by the limited number of measurements that have been made of meteorological and glaciological variables. Often, data have only been collected in restricted coastal areas or during limited time periods, whereas there is a need for long-term measurements in varying climatological areas. To obtain data sets for larger areas, statistical relations are normally derived from the limited measurements available. An example of this approach is the parameterization of the temperature distribution in terms of *latitude* and *elevation* by Reeh (1991) and Huybrechts and others (1991), which was based upon 18 stations with mean July temperature measurements and six stations with annual mean temperature measurements (Ohmura, 1987). This is more or less the only thing one can do when data sets are so limited and, as long as good statistical correlations are obtained, this approach is acceptable. It should, however, be noted that the use of this procedure introduces an uncertainty in the calculations, if the elevation distribution is poorly known.

For this reason, the best elevation distribution should be used for model studies. Comparing results of different model studies which are not based upon the same input data (for instance elevation) does not provide insight into the various physical processes involved in the model formulations. Most mass-balance modelling work of the Greenland ice sheet done in recent years (Huybrechts and others, 1991; Reeh, 1991; Wal and Oerlemans, 1994) has been made possible by the digital elevation model given by Letréguilly and others (1991) as a grid of 20 km × 20 km resolution. The surface elevations for the ice sheet have been computed from data obtained by radio-echo-sounding flights undertaken by the Electromagnetic Institute (EMI) of the Technical University of Denmark in the late 1970s, further abbreviated as TUD model. The aircraft altitude, and hence the elevations, were derived from a pressure altimeter. To evaluate the accuracy of the data, Letréguilly and others (1991) compared these measurements with terrestrial altimetry measurements at the ice-drilling sites of Dye 3 and Camp Century to examine ice thickness as well as elevation. Letréguilly and others (1991) concluded that the elevation model compared reasonably well with these two point measurements as well as with a published map by Ohmura (1987) and a satellite-altimetry map (south of

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72° N) by Bindschadler and others (1989). It should be noted that the sites Dye 3 and Camp Century are relatively well covered by the measurements.

The elevation in the ice-marginal zone is very important (a) for mass-balance calculations, because the ablation is concentrated in these areas, and (b) for ice-dynamical studies, because surface slope near the margin is an important parameter for validating ice-dynamical model performance. Fortunately, new and more accurate elevation measurements are now available. Satellite-altimetry data from GEOSAT (south of 72.1° N and ERS 1 (north of 72.1° N), supplemented with airborne altimetry from the so-called Greenland Aerogeophysical Project (GAP) (Brozena, 1991), airborne laser altimetry from the Airborne Ice Mapping project (AIM) (Krabill and others, 1995), and a local terrestrial survey performed on the summit (Ekholm and Keller, 1993), have made it possible to construct a new elevation model for the Greenland ice sheet. The model used in this paper is that of Tscherning and others (1993) improved with airborne laser and radar altimetry (S. Ekholm; a full-coverage, high-resolution, topographic model of Greenland, computed from a variety of digital elevation data, submitted to *Journal of Geophysical Research–Solid Earth*). Here, this model is referred to as the integrated-altimetry (IA) model. The model is given as a grid in geographical coordinates with a resolution of 5′ and 10′ in latitudal and longitudal directions, respectively, which corresponds to a grid resolution of approximately 10 km in latitudinal direction and 3–9 km in longitudinal direction. A slightly pessimistic estimate of the general accuracy is obtained by omitting the AIM observations from the modelling process and regarding them as ground truth instead. In this manner, an overall accuracy level of 13 m is found. The AIM survey data are available south of 72° N, so the true model accuracy is possibly slightly better on the southern ice sheet, within the AIM area of coverage. Satellite-altimetry data are highly unreliable in more steeply sloping areas and the model accuracy near the margin of the ice sheet, with slopes greater than 1°, is probably as limited as 75–100 m.

In this paper, the IA model will be compared with the TUD model. Special attention is given to the ice-marginal zone near Søndre Strømfjord, since detailed differential GPS measurements are available for this area. Finally, the discrepancies in ablation of the Greenland ice sheet, as revealed by the two elevation models, will be discussed. The ablation is calculated both with an energy-balance model and a degree-day model.

COMPARISON OF THE HYPSOMETRY OF THE GREENLAND ICE SHEET

Various estimates of the volume (V) and surface area (A) of the Greenland ice sheet have been presented in the literature. Without claiming to be complete, Table 1 presents a compilation of the principal estimates. These estimates are based primarily upon seismic measurements made in the period 1948–53 (Holtzscherer and Bauer, 1955) and improved later on by various regional measurements or other interpolation techniques. The surface elevation for the TUD model is based upon data obtained by radio-echo-sounding flights undertaken by the Electromagnetic Institute (EMI) of the Technical University of Denmark. Satellite-altimetry data from ERS-1, together with GAP airborne altimetry and local terrestrial surveys on the summit, enabled the construction of the IA model. Roughly, we find that the overall estimates of volume, surface and ice thickness vary by approximately 5% from their mean values. Note that the estimates of volume, area, ice thickness and surface elevation in Table 1 are not entirely independent of each other, because they are based partly on the same data. The data in Table 1, for the IA model, have been interpolated to the 20 km × 20 km grid used for the TUD model.

If we ignore differences in surface area, we can compare the hypsometry of the estimates presented by Oerlemans and others (1993), the TUD model and the IA model. Oerlemans and others (1993) determined planimetrical elevation intervals from a map given by Weidick (1971). The mean surface elevation of this model is considerably lower than the mean surface elevation of the two other data sets, as can be seen in Table 1. In the distribution used by Oerlemans and others (1993), the surface area of the higher accumulation area is considerably smaller than for the two other data sets. The reason

Table 1. A compilation of various estimates of volume (V), surface area (A), ice thickness (H) and surface elevation (h_s) of the Greenland ice sheet

	V $\times 10^6$ km^3	A $\times 10^6$ km^2	Mean H m	Mean h_s m a.s.l.
Holtzscherer and Bauer, 1955	2.667	1.726	1545	
Radok and others, 1982	2.988	1.670	1790	
Weidick, 1985		1.701		
TUD model: Letréguilly and others, 1991	2.825	1.671	1691	2126
Oerlemans and others, 1993; Weidick, 1971				1892
IA model 20 km × 20 km	2.733	1.671	1636	2159

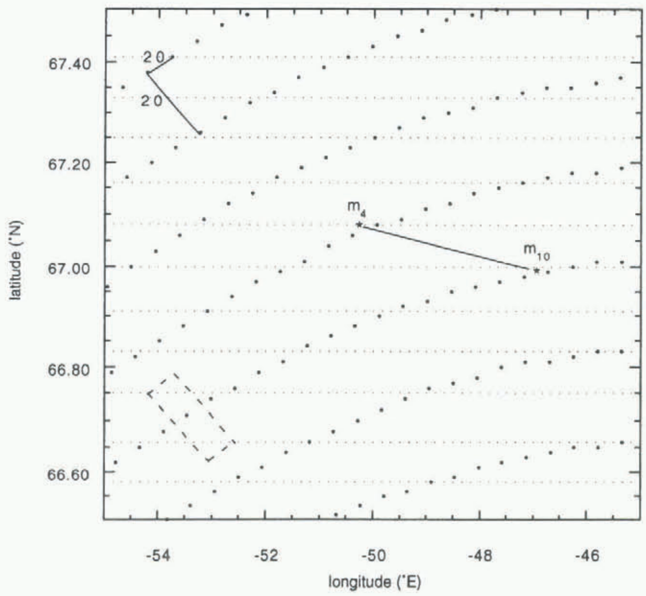

Fig. 1. A close-up of the IA model grid (small dots) and the TUD model grid (large dots). The dots indicate points at which the elevation is prescribed. The dashed rectangle, in the lower left corner is an example of an area used for interpolating the IA model to the 20 km × 20 km grid. About ten points are used for the interpolation. The line m_4–m_{10} indicates the location of the Gimex surface-elevation measurements. Note that the bars in the upper left corner show the latitudinal exaggeration of the projection in km.

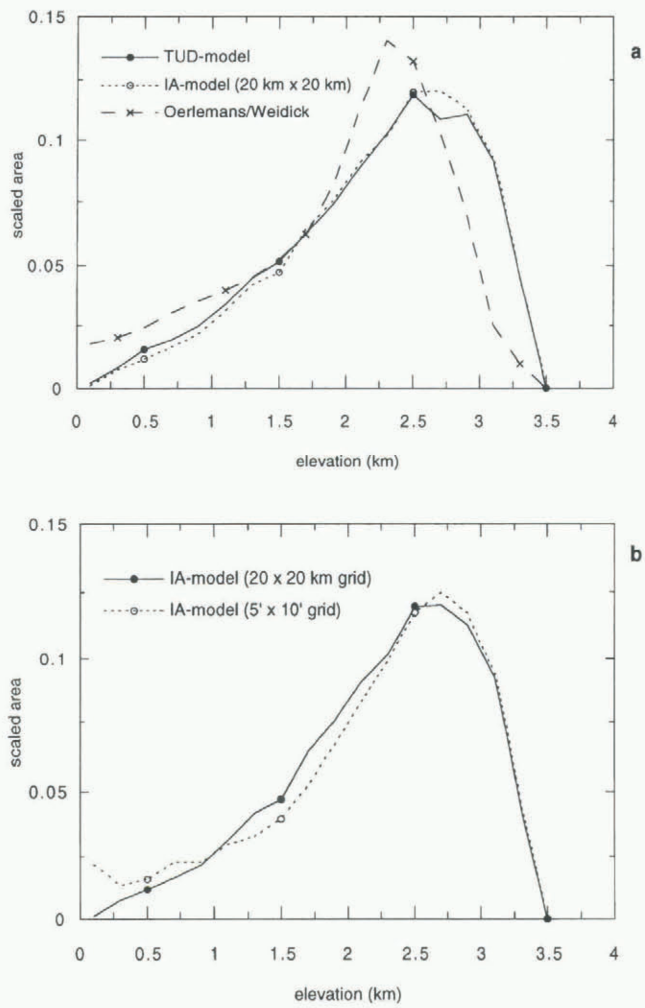

Fig. 2. The hypsometry of the ice sheet for three different data sets, the TUD model and the IA model both on the 20 km × 20 km grid (a). The hypsometry of the IA model on the 20 km × 20 km grid and on the 5′ × 10′ grid (b).

for this difference is unclear. On the other hand, the (ablation) area below 1000 m a.s.l. is significantly larger in this data set compared to the two other data sets, due to a better resolution of the outlet glaciers. A comparison of the hypsometry of the TUD model and the IA model produces remarkably similar results. To facilitate a comparison, we project the more detailed version of the IA model on the grid points of the TUD model. This projection is achieved by averaging all the elevations of the IA model available within a grid box, in which the sizes of the grid cells are provided by the TUD model (see Fig. 1). Averaging is performed with a scaling of one over the distance in kilometres from the IA coordinate to the TUD coordinate. In this way, approximately ten points from the IA model are used to calculate one elevation at the 20 km × 20 km grid. The results of this data transformation are presented in Figure 2a. Except around the dip at 2800 m a.s.l., the hypsometry of the two elevation models is rather similar. One can argue that this might be due to the arbitrary interpolation procedure. We therefore also show in Figure 2b the detailed version of the IA model with a resolution of 5′ and 10′ in latitude and longitude directions, respectively, as well as the interpolated version. Figure 2b also shows that the interpolation leads to an underestimation of the area below 500 m a.s.l., whereas the overall distribution is rather similar. To resolve the outlet glaciers which are typical small-scale features, one needs high resolution as taken into account by Oerlemans and others (1993). Nevertheless, Figure 2a and 2b suggests that the overall geometry is represented in a similar way in both elevation distributions. However, this does not necessarily mean that there are no differences from place to place.

To study the local differences, we subtracted both surface-elevation distributions from each other at the 20 km × 20 km grid resolution. Averaging the differences over the ice-sheet area gives on average an elevation difference of 33 m (see also Table 1). The mean-squared-error of the difference field, as presented in Figure 3, is 112 m. Figure 3 shows these differences over the entire domain. One can observe that in the higher areas of the ice sheet the differences are fairly small, whereas along the margin the discrepancies are considerable. Note that in Figure 3 the plot limits are taken to be 100 m but at isolated spots differences can be far larger in spite of the similar hypsometry.

In the next section, we compare the two elevation distributions with elevation measurements in an area near the margin, for which the two elevation models revealed large differences.

EXAMINATION OF THE ELEVATION MODELS FOR THE ICE MARGIN NEAR SØNDRE STRØMFJORD, WEST GREENLAND

Accurate ground-truth information on the ice-sheet elevation is limited, especially in the ablation zone of

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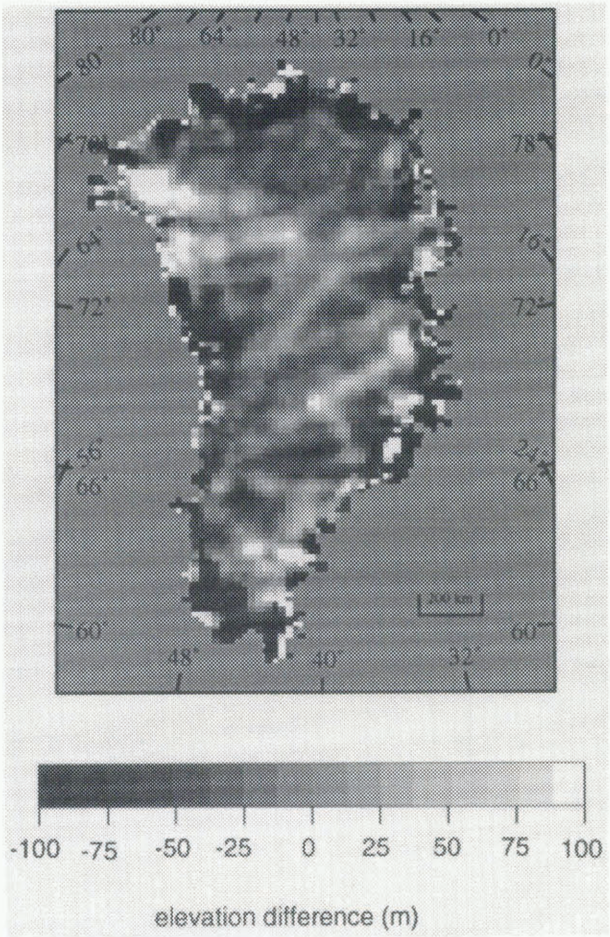

Fig. 3. The difference in elevation revealed by comparing the TUD model and the IA model on the 20 km × 20 km grid. The plot boundaries are arbitrarily chosen at 100 m; larger discrepancies occur at individual spots. The surrounding grey area indicates the area which is ice-free (tundra and sea).

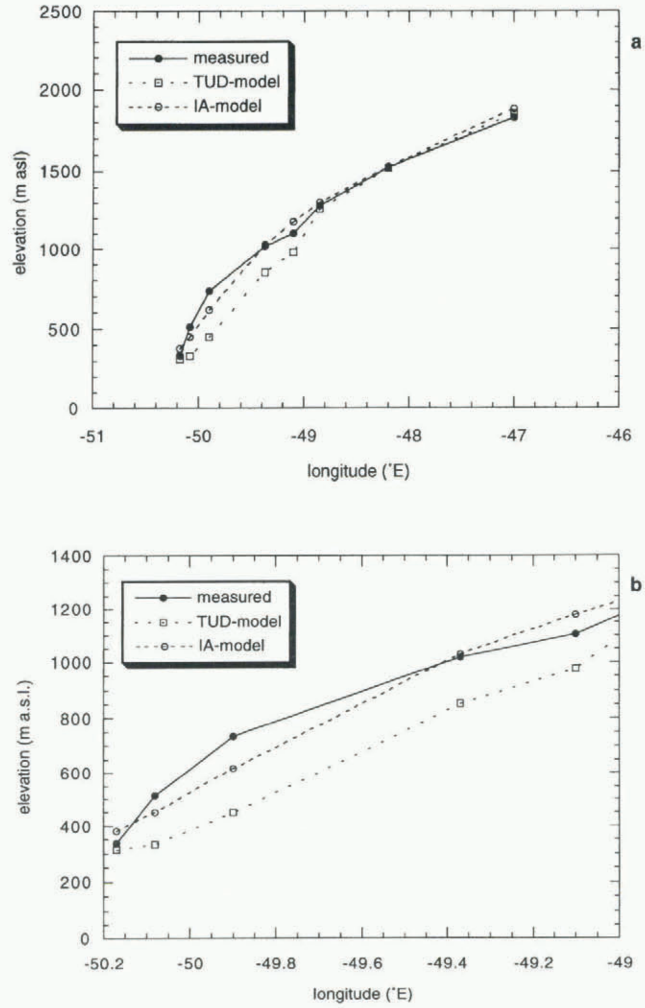

Fig. 4. A comparison of the elevation revealed by the TUD-model and IA-model data with GPS measurements carried out near Søndre Strømfjord in the framework of the Gimex expeditions in the period 1990–95 (a). A close-up of the area below 1300 m a.s.l. (b).

the ice sheet where surface slopes are relatively large. In the framework of the Gimex micrometeorological experiments, GPS measurements were collected along a transect of about 90 km (m_4–m_{10} in Fig. 1) perpendicular to the ice margin near Søndre Strømfjord in the period 1990–94. Two Magellan NAV-100 receivers were used for positioning. The instruments were used in differential mode and the accuracy of the positions is estimated to be 10 m in horizontal and 20 m in vertical direction, respectively. Comparing point measurements with a discrete gridded model is always a somewhat awkward exercise, because of the resolution difference. Figure 1 shows the scale difference of the two grids and the transect, m_4–m_{10}, with the measurements. From Figure 1, it can be seen that the transect is covered by about eight grid points in the east–west direction from the TUD model grid and by about 20 from the IA model.

Figure 4a shows the measured elevation together with the bilinearly interpolated elevations of the two models. It can be observed that the differences increase towards the ice margin. The TUD model grid underestimates the elevation west of −49° E, typically by 150 m as can be observed in Figure 4b. In terms of surface slope, this means that the 20 km × 20 km grid overestimates the slope by approximately 20%. Higher up on the ice sheet, differences between the two elevation models and the measurements are typically 20 m.

The satellite-altimetry observations are corrected for slope-induced errors according to the so-called "direct" method given in Ekholm and others (1995), resulting in mean errors of 10–35 m for slopes of 0.3–0.6°. This means that the observed differences between elevation model and GPS measurements (accuracy 20 m) are in agreement with the errors in the measurements, in the area above approximately 1300 m a.s.l., where surface slope is about 0.5°. Lower down on the ice sheet, the mean error of the slope-corrected satellite altimetry increases to approximately 40 m, corresponding to a 1° slope, the upper part of the allowable surface slope for satellite altimetry (Ekholm and others, 1995). The actual observed surface slope based on the GPS measurements is 2° at the margin and the standard deviation of the difference between the IA model and the GPS measurement is 72 m ($N = 6$). In this area, satellite altimetry is unreliable and the estimated accuracy of the elevation model is 75–100 m, which is in line with the observed

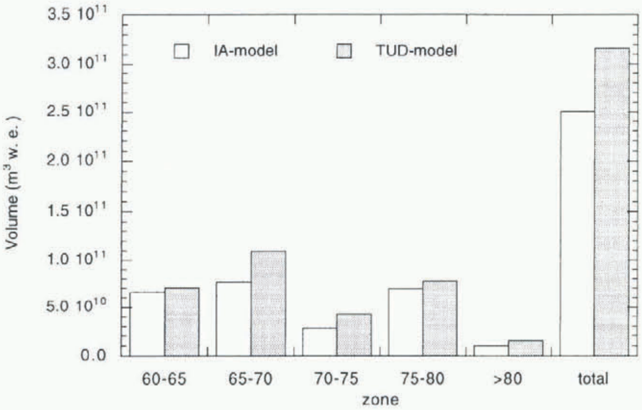

Fig. 5. The ablation on the Greenland ice sheet for different latitudinal zones calculated with an energy-balance model (Wal and Oerlemans, 1994). The white bars show the ablation with the IA model as input for the elevation and the grey bars show the ablation with the TUD model as input for the elevation. Both calculations use the 20 km × 20 km grid.

differences between GPS measurements and the IA model. In spite of the considerable standard deviation the mean difference is only 4 m.

For the TUD model, a systematic difference is observed in Figure 4a and b for the lower part of the transect (< 1300 m a.s.l.), which is difficult to explain purely from the uncertainty in the measurements. Although this comparison is strictly local, it is likely that errors such as this occur more widely in the 20 km × 20 km grid. The data of the TUD model are based upon airborne-radar measurements, which rely upon pressure altimetry for the absolute elevation. In areas with rapidly changing elevation, a low frequency of measurements leads to large interpolation errors. This means that near the ice margin the largest errors might be expected for the 20 km × 20 km grid. Unfortunately, this area is very important, because ice-dynamical models can be tested in these areas in terms of, for instance, surface slope and ice thickness. The ice margin is even more important for ablation models, since ablation is almost entirely restricted to these areas.

DISCREPANCIES IN THE ABLATION OF THE ENTIRE ICE SHEET AS CALCULATED WITH TWO DIFFERENT ELEVATION MODELS AS INPUT PARAMETER

Two alternative approaches are used nowadays to calculate the ablation of the Greenland ice sheet: one is based on the energy balance of the surface, for instance, the model of Wal and Oerlemans (1994), and the other is based on a statistical correlation between temperature and ablation, the so-called degree-day models (Huybrechts and others, 1991; Reeh, 1991). In both approaches, it is assumed that the temperature field can be parameterized as a function of latitude and elevation, on the basis of a compilation of available data by Ohmura (1987). Although the two degree-day models are not identical, these models, as well as the energy-balance model of Wal and Oerlemans (1994), all use a grid with a spacing of 20 km and the elevation as a main input parameter, as it was digitized for this grid by Letréguilly and others (1991). Altogether, this gives 4219 positions on the ice sheet, representing an area of 1.69×10^6 km². A more thorough comparison of energy-balance and degree-day calculation of the ablation has been presented by Wal (1996). Changing the elevation from the TUD model to the IA model and keeping all other model parameters identical (including the resolution) provides insight into the sensitivity of the model to the elevation data. Figure 5 shows the ablation for different latitude zones for the two elevation distributions. Application of the IA model yields a reduction of 20% in the ablation compared to the application of the TUD model, as calculated by the energy-balance model of Wal and Oerlemans (1994). 50% of the difference between the two model runs is concentrated in the zone between 65–70° N and is primarily on the eastern side of Greenland. The same experiment yields similar results for the degree-day model of Reeh (1991). This large discrepancy between the two elevation models shows that our knowledge of the ablation distribution of the Greenland ice sheet is limited. A change in one of the most basic input parameters, the elevation, can easily yield changes of the order of 20%.

This reduced ablation will, in principal, also reduce the sensitivity of the ice sheet for climate change. Wal (1996) showed that the sensitivity increases for larger perturbations. This implies that a change to a lower-reference ablation distribution results in a smaller sensitivity. For a 1 K perturbation, the sensitivity of the energy-balance model of Wal and Oerlemans (1994) reduces about 3%, if the input is changed from the TUD model to the IA model, and keeping all other model parameters identical.

CONCLUSIONS

The paper has focused mainly on a comparison between two elevation models: on the one hand, the TUD model, which has been widely used in glaciological studies, and, on the other hand, the IA model. The comparison has revealed considerable discrepancies along the ice margin on individual grid points and 112 m r.m.s. over the entire ice sheet, although the overall hypsometry is rather similar. Validating the elevation models in an absolute sense is difficult but a comparison with GPS measurements in the area around Søndre Strømfjord shows that the IA model corresponds much better to the ground-truth observations. The observed standard deviation for the discrepancies between the IA model and the GPS measurements is in line with the conclusions of Ekholm and others (1995) for surface slopes smaller than 1°. For slopes between 1° and 2°, the standard deviation of the difference between the GPS measurements and the elevation model is found to be 72 m. Because a height difference of this order of magnitude has considerable consequences for the calculation of the ablation, it is necessary that more terrestrial measurements in ice-marginal zones are carried out. A second improvement for future work on mass-balance modelling is the increase in resolution to resolve outlet glaciers better (Fig. 2).

In spite of the rather similar overall geometry of the two elevation models, we observe a 20% reduction in the ablation when the IA model is used instead of the TUD model, irrespective of the way in which the ablation is calculated. Hence, we conclude that the contribution of the ablation was relatively overestimated previously.

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