THE WIND-SOCK THEORY OF COMET TAILS

John C. Brandt and Edward D. Rothe

I. Introduction

Type I or ionic comet tails on the average make an angle of a few degrees at the nucleus with the prolonged radius vector in the direction opposite to the comet's orbital motion. This fact was explained by Biermann (1951) as the aberration angle caused by the comet's motion in the outflowing solar wind plasma, and, as is well known, led to the discovery of the solar wind itself. Mathematically, the direction of the tail $\vec{T}$ is given by the vector equation

$$\vec{T} = \vec{w} - \vec{V} \tag{1}$$

where $\vec{w}$ is the solar wind velocity and $\vec{V}$ is the comet's orbital velocity. Equation (1) or simplified forms of it have been used extensively to derive properties of the solar wind (Belton and Brandt 1966; Brandt 1967; Brandt, Harrington and Roosen 1973). The solar wind properties derived from ionic comet tails agree with directly determined properties in all cases where comparison is possible and, hence, the validity of equation (1) has been established. If the solar wind determines the gross shape of the entire plasma tail, what is this shape and how can it be calculated?

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There are at least three conceptually distinct approaches to calculating the shapes on ionic comet tails.

(1) The Mechanical or Bessel-Bredichin Theory. The details of this approach first treated by Bessel (1836) are widely known. A constant acceleration is assumed to act on the tail material, and in the calculations, this is included by using a reduced solar gravity. Bredichin defined Type I comet tails as syndynes with extra repulsive force $(1 - \mu) \approx 18$. Unfortunately, syndynes are tangent to the prolonged radius vector at the nucleus which is contrary to the observations. The tail curvature given by a syndyne with $(1 - \mu) \approx 18$ is probably not correct either and we return to this point below.

(2) The Smoke Theory. Here, the force on the tail material is given by a momentum transfer depending on the relative velocity of the solar wind with respect to the tail material (see Belton 1965, Appendix 1). Hence, if $\vec{r}'$ is the velocity of the tail material, we would need to include a force of the form

$$F\ (w - \vec{r}') \tag{2}$$

and calculate the speed of the tail material $\vec{r}'$ at all points. Our understanding of the solar wind interaction with plasma tails is insufficient to permit accurate calculation of the forces required on the smoke theory. This difficulty obviously applies to an entire class of theories requiring specific forces. Fortunately, knowledge of specific forces may not be necessary.

(3) Wind-Sock Theory. Here we do not need the forces accelerating the material along the tail. The magnetic field along the tail is used to channel the tail plasma and the location of the magnetic field lines in the tail is determined by the local momentum field in the solar wind. The magnetic field acts as a transparent wind sock. This viewpoint implies that the field lines are trapped in the cometary plasma around the nucleus long enough for them effectively to be fastened to the comet's head. The first explicit statement of this concept known to us was by Alfvén (1957) who wrote:

> "The tail should no more be considered as gas moving freely in space. Instead the tail is a real part of the comet, fastened to the head by magnetic field lines."

II. Theory

The gross shape of an ionic comet tail on the wind-sock theory assuming constant solar wind speed can be calculated by applying the equation $\vec{T} = \vec{w} - \vec{v}$ pointwise along the tail. The basic geometry in the plane of the comet's orbit is shown in Figure 1. By projecting the components of solar wind velocity into the cometocentric coordinate system, we obtain the basic equation for the wind-sock theory, viz.,

$$\frac{dy}{dx} = \frac{-V \sin \gamma - w_r \sin \alpha + w_\phi \cos \alpha \cos i'/\cos b}{w_r \cos \alpha - V \cos \gamma + w_\phi \sin \alpha \cos i'/\cos b}$$

Many of the quantities used are illustrated in Figure 1. In addition, w_r and w_ϕ are the radial and azimuthal components of the solar wind

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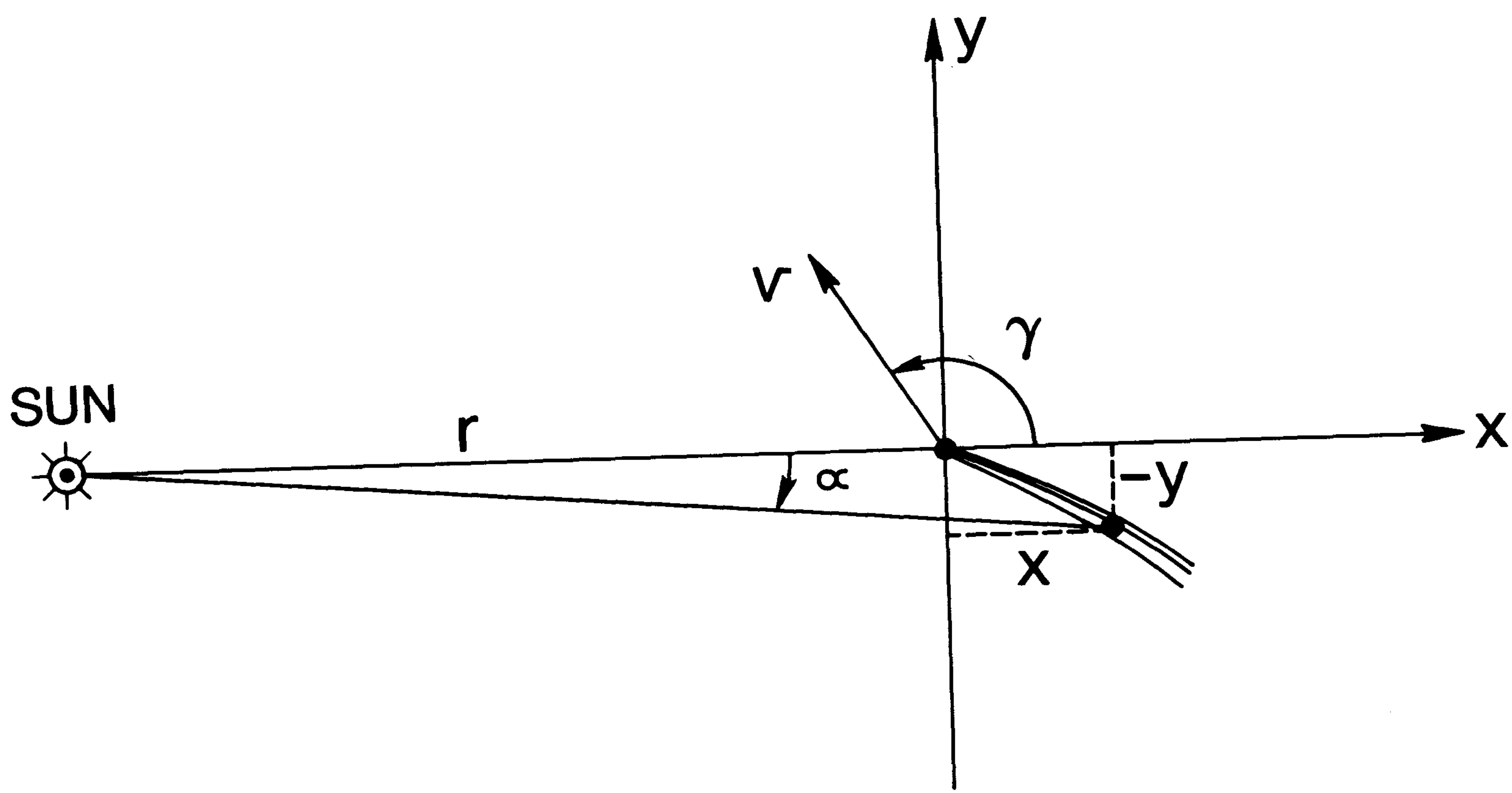

Figure 1. The geometry for the wind-sock theory of ionic comet tails in the plane of the comet's orbit.

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velocity. The angles i' and b are the inclination of the orbit with respect to the solar equator and the heliographic latitude, respectively. If necessary, this equation could easily be generalized to three dimensions; in this case, the direction cosines at each point (dx, dy, dz) would be similarly determined.

A simple analytical result can be obtained on the basis of some reasonable approximations. For a comet away from the sun (i.e., non-sungrazers) and near perihelion, $\alpha \ll 1$ and $w_r \gg \cos \gamma$, respectively, are good approximations. Then, equation (3) can be written

$$\frac{dy}{dx} = \frac{-V \sin \gamma - w_r \alpha + w_\phi \cos i'/\cos b}{w_r} \tag{4}$$

Equation (4) can be used to evaluate the coefficients in a Taylor's series. If we let

$$-y = A + Bx + Cx^2 + Dx^3 + \ldots , \tag{5}$$

we find

$$\begin{aligned} A &= 0 \\ B &= \left[\frac{V \sin \gamma - w_\phi \cos i'/\cos b}{w_r - V \cos \gamma}\right] \\ C &= +\frac{B}{2r} \\ D &= \frac{-B}{6r^2} \end{aligned} \tag{6}$$

If we write

$$X = x/r \qquad\qquad (7)$$
$$Y = y/r$$

where r is the comet's heliocentric distance, our final equation becomes

$$-Y = BX + \frac{B}{2} X^2 + \ldots \qquad (8)$$

for $X \le 0.3$, the cubic term is 1.3% or less compared to the sum of the first two terms.

The Taylor's series solution can be obtained without the approximations used to write equation (4) and is

$$-Y = BX + \left[\frac{w_r^{\,2} + \left(\frac{w_\phi \cos i'}{\cos b}\right)^2 - V\left(\frac{w_\phi \sin \gamma \cos i'}{\cos b} + w_r \cos \gamma\right)}{(w_r - V \cos \gamma)^2} \right] \frac{BX^2}{2} + \ldots \qquad (9)$$

For most cases, the term in brackets in equation (9) is close to 1 and equation (9) reduces to equation (8). In doubtful cases, equation (9) provides a check on the applicability of the simple solution.

Our approximate (but rather accurate solution) for steady solar wind conditions depends only on the quantity B which is the tangent of the aberration angle at the nucleus used in the earlier work. The calculated tails are nearly straight near the head, but show curvature well away from the head. The curvature arises from the geometrical divergence of the radial direction in a spherical coordinate system.

III. Applications

First, we briefly reexamine the historical question of Bredichin's identification of the Type I tails with syndynes from the mechanical theory for $(1 - \mu) \approx 18$. We have checked back to some of Bredichin's (1884) original work in which the observations (for the Great Comet of 1882) were given. They show little or no tail curvature and we have no difficulty fitting the wind-sock theory with modern solar wind parameters (Figure 2). Bredichin's fit with $(1 - \mu) \approx 18$ would not be bad in an rms sense, even though the aberration angle at the nucleus was in error on the average by $\approx 5^{\circ}$ and the curvature was too large. Visual observations of very bright comets in the 19th century may be a valuable untapped source of solar wind data.

The wind-sock theory can also be applied to the geomagnetic tail (Figure 3). Behannon's (1970) observations gave an aberration angle of $3\overset{\circ}{.}1$ and this is closely approximated by the solar wind parameters chosen. Figure 3 shows that observations would be required at tenths of A.U. from Earth to detect the effects of the tail curvature; such observations are unlikely in the near future. A comparison of an accurate computer integration of equation (3) with the Taylor's Series result of equation (8) is also shown.

Figure 4 shows a photograph of Comet Kohoutek taken at the Joint Observatory for Cometary Research (JOCR) on January 19, 1974. We would anticipate no difficulty in explaining the gross shape of the main ion tail on the basis of steady solar wind conditions. However, it is important to note that our model may require comparison with averages

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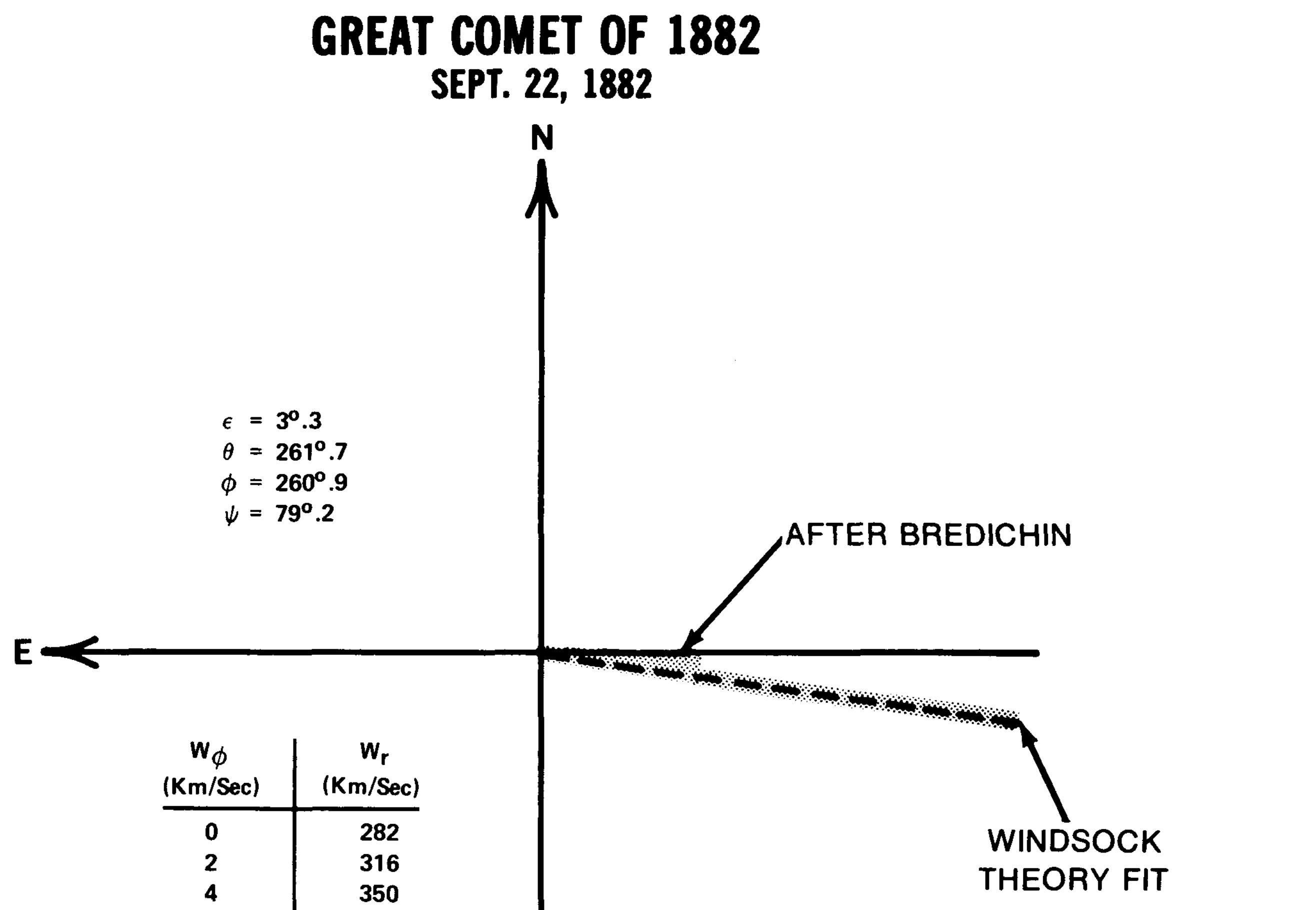

W_ϕ (Km/Sec)	W_r (Km/Sec)
0	282
2	316
4	350
6	385
8	420
10	450

Figure 2. Schematic showing the observations (shaded) of the Type I and Type II tails of the Great Comet of 1882. Solar wind speeds of $w_r \approx 400$ km/sec and $w_\phi \approx 7$ km/sec are consistent with both present knowledge and the orientation of this ion tail.

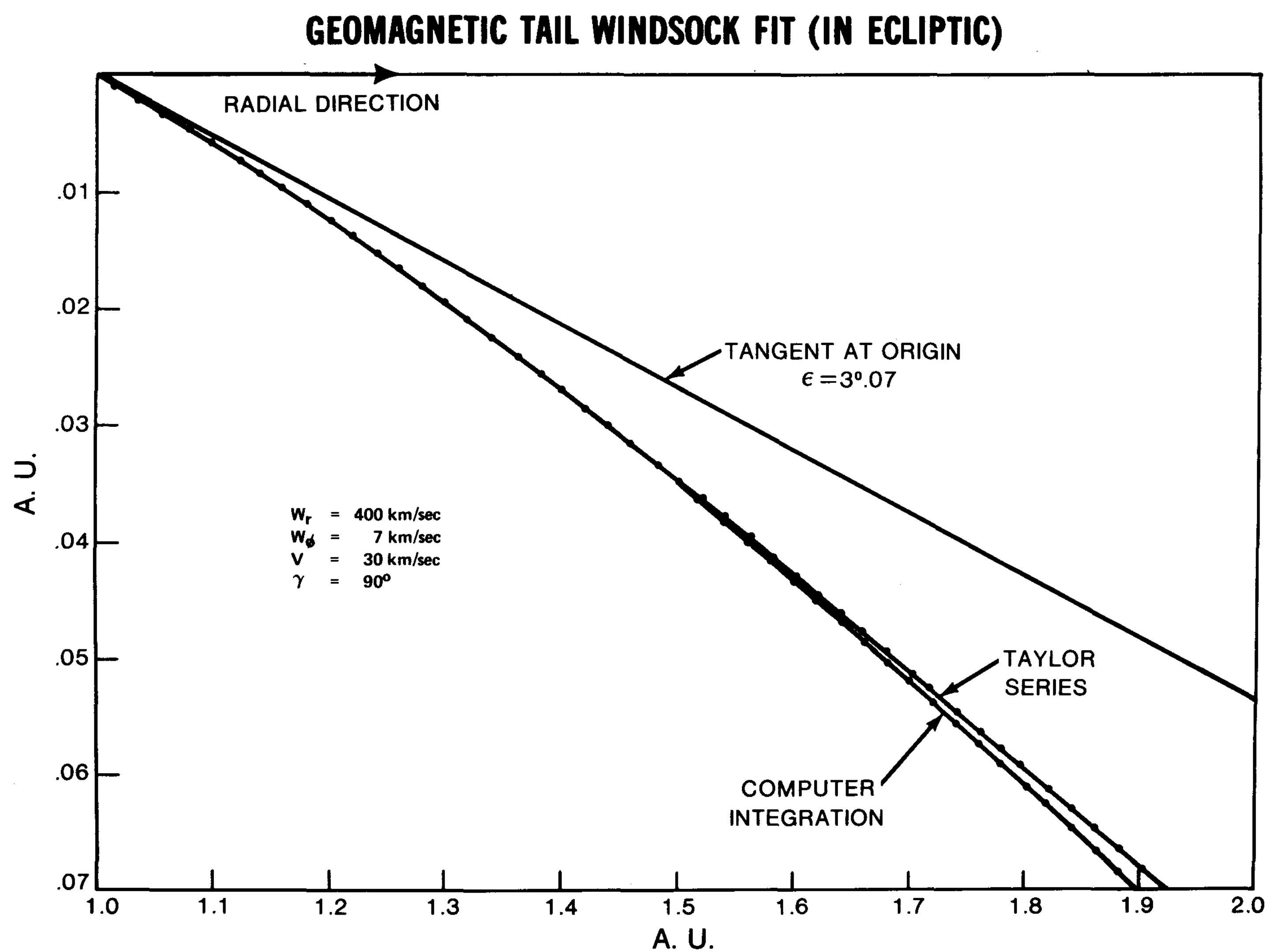

Figure 3. Application of the wind-sock theory to the geomagnetic tail as discussed in the text.

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Figure 4. JOCR photograph of Comet Kohoutek on January 19, 1974.

of observations if sufficiently steady solar wind conditions are not found. For example, Comet Kohoutek on January 20, 1974 (Figure 5) showed a large disturbance in the shape of the main tail. Our solution with $B = 0.36$ would be a reasonable fit to the tail shape except for the disturbance as is shown schematically in Figure 6. The disturbance could be caused by a high-speed solar wind stream. Note that the quiet conditions for $B = 0.36$ are somewhat unusual; a large negative value of w_ϕ is necessary to produce a reasonable w_r value. We suspect that changes in solar wind conditions produce changes in gross plasma tail shapes and that study of tail shapes may provide information on velocity structures in the solar wind.

IV. Conclusions

We have presented a simple version of the wind-sock theory of ionic comet tails. The simple model is consistent with all facts known to us. There are straightforward improvements that can be made for the case of steady solar wind conditions (e.g., inclusion of effects due to the tail's magnetic field). Consideration of the non-steady case are also of considerable interest.

Figure 5. JOCR photograph of Comet Kohoutek on January 20, 1974.

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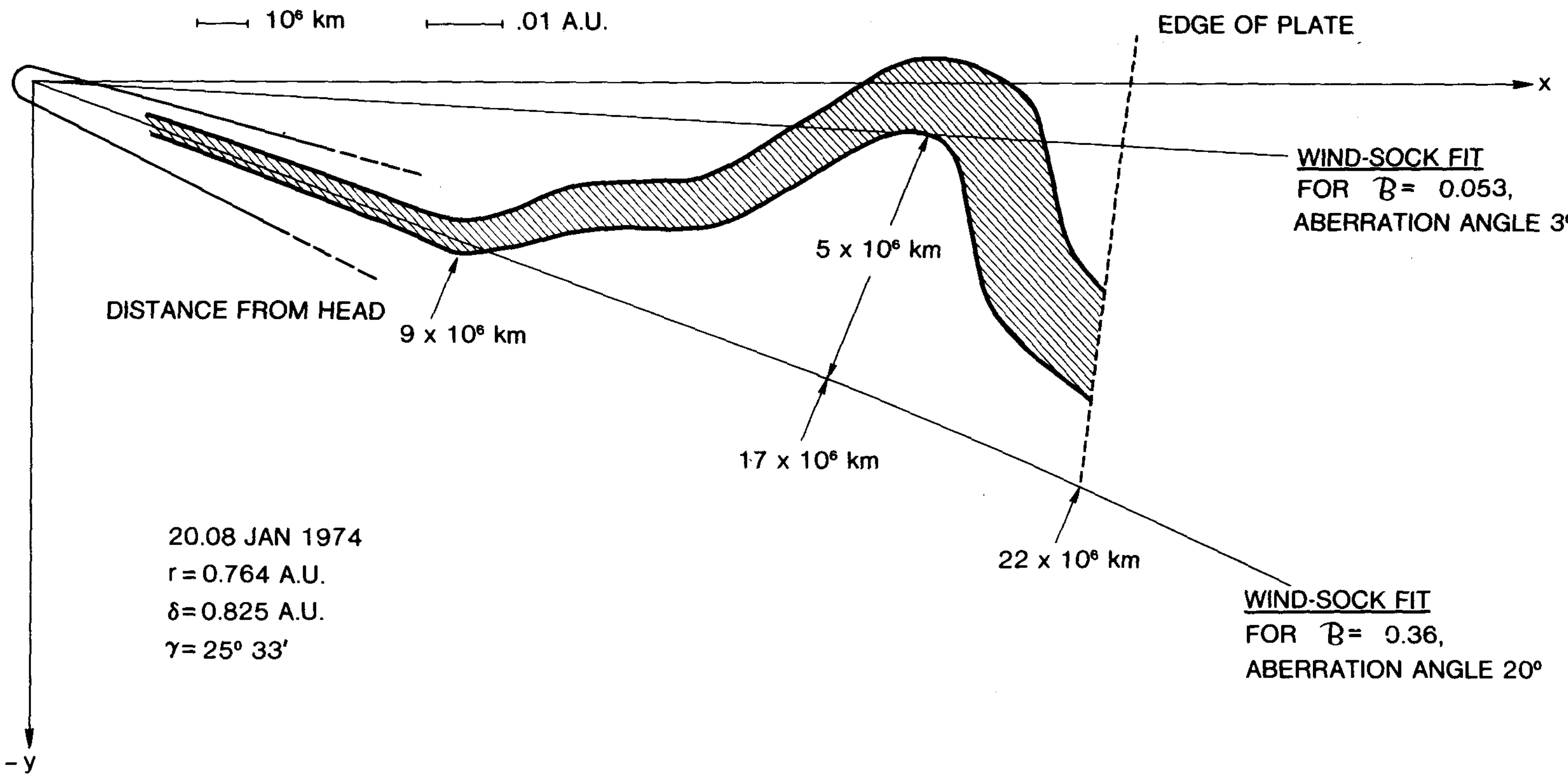

Figure 6. Schematic representation of Comet Kohoutek on January 20, 1974 (Figure 5) as seen in the plane of the comet's orbit.

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References

Alfvén, H. 1957, Tellus, 9, 92.

Behannon, K. W. 1970, J. Geophys. Res., 75, 743.

Belton, M. J. S. 1965, A. J., 70, 451.

Belton, M. J. S., and Brandt, J. C. 1966, Ap. J. Suppl., 13, 125 (No. 117).

Bessel, W. 1836, A.N., 13, 185.

Biermann, L. 1951, Zs. f. Ap., 29, 274.

Brandt, J. C. 1967, Ap. J., 147, 201.

Brandt, J. C., Harrington, R. S., and Roosen, R. G. 1973, Ap. J., 184, 27.

Bredichin, Th. 1884, Ann. Moscou Obs., 10, 7.

B. Jambor: If I understand you correctly your Bredikhin approach to the problem fails because you do not get enough curvature in the tail; I would be curious to know what a more precise approach like the Finson-Probstein, not relying on approximations of series expansion, would yield.

J. C. Brandt: Yes. You know, I took that solely because for years that has been the definition of a Type 1 tail. These Type 1 and Type 2 appear in Bredikhin's papers, and I was curious as to how this got started.

D. J. Malaise: The windsock model has the nice feature that you can compute the shape of the tail. Has it not the drawback that you have to drop the assumption that the tail lies in the orbital plane of the comet. Even small departures from the orbital plane makes the computation of the true direction of the tail quite indeterminate in some projection situations.

J. C. Brandt: Now clearly, you can create such a comet. The Comet Mrkos was one such thing. It was at 90 degrees inclination, and that's going to be a problem. But with any care at all, it's not a problem.

K. Jockers: Your windsock model is the model of a tail which can withstand any tension along its axis but has to be in lateral momentum equilibrium. The small curvature of the tails is caused by a diverging but stationary solar wind flow field. How can this model be applied to an evidently non-stationary situation as on Jan. 20?

J. C. Brandt: I think, if you stop and think about it, that you can make qualitative statements about what happened.

K. Jockers: You know, that windsock has to respond to the changing non-stationary situation and that is completely different than that line you have calculated.

J. C. Brandt: It is not necessarily completely different from the line, but it may be. But if you know how to calculate that, why don't you let me know and we'll do it.