Nuclear Activity in Galaxies Across Cosmic Time
Proceedings IAU Symposium No. 356, 2019
M. Pović, P. Marziani, J. Masegosa, H. Netzer, S. H. Negu & S. B. Tessema, eds.
doi:10.1017/S1743921320003506

Mass-radius relation of compact objects

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Abstract. Compact objects are of great interest in astrophysical research. There are active research interests in understanding better various aspects of formation and evolution of these objects. In this paper we addressed some problems related to the compact objects mass limit. We employed Einstein field equations (EFEs) to derive the equation of state (EoS). With the assumption of high densities and low temperature of compact sources, the derived equation of state is reduced to polytropic kind. Studying the polytropic equations we obtained similar physical implications, in agreement to previous works. Using the latest version of Mathematica-11 in our numerical analysis, we also obtained similar results except slight differences in accuracy.

Keywords. Compact objects, mass limit, equation of state, mass-radius relation.

1. Introduction

Compact objects (COs) represent the final stage in the evolution of ordinary stars. They are formed when stars cease their nuclear fuel to support themselves against gravity. They are generally categorized as black holes, neutron stars, or white dwarfs depending upon the masses of their progenitors (Shapiro and Teukolsky 1986). Today, the COs are amongst important objects used to understand better the structure, formation, and evolution of stellar sources. Yet, there are issues concerning the structure and size of the COs by themselves (Cardoso and Pani 2019). Thus, there is an active research for better understanding and application of the COs. So, in this paper we worked out EFEs to study the COs mass limit in relation to their radii. The detailed method is given as in the upcoming section.

2. Methodology

EFEs were used to derive EoS with simplifying boundary conditions such as variational techniques (Weinberg 1972). With the assumption of high densities and low temperature characteristics of the COs, the state equation is further reduced to polytrope kind (Chandrasekhar 1967). Finally, the obtained polytrope is being worked out for analysis both analytically and numerically where the results are being compared with previous results.

3. Results and discussion

Working out the EFEs, (as outlined in the methodology) we obtained the Lane-Emden equation given as in the standard books (Chandrasekhar 1967; Weinberg 1972). The analytical solutions exist only for polytropes of indices n = 0, 1 and 5 as shown earlier (Chandrasekhar 1967). For the other values of n, the Lane-Emden equation is to be integrated numerically. Thus, we used the latest version of *Mathematica-11* to integrate the Lane-Emden equation numerically for polytropes of integral indices of $n = 0 - 6$. The results are depicted as in Figure 1. The stable solutions of the COs are determined from

Table 1. Numerical solutions of Lane-Emden equation for mass-radius relationship of compact objects with integral polytropic indices, $n = 0 - 5$. Authors' solutions vs Chandrasekhar (1967).

	ξ_1		$\left(-\xi^2 \frac{d^2\theta(\xi)}{d\xi^2}\right)\Big|_{\xi=\xi_1}$	
n	**Seman & Tolu**	**Chandrasekhar (1967)**	**Seman & Tolu**	**Chandrasekhar (1967)**
0	2.449489	2.4494	4.89898	4.8988
1	3.141581	3.14159	3.14159	3.14159
2	4.35287	4.35287	2.41104	2.41105
3	**6.89685**	**6.89685**	**2.01824**	**2.01824**
4	14.97153	14.97155	1.79723	1.79723
5	∞	∞	1.73204	1.73205

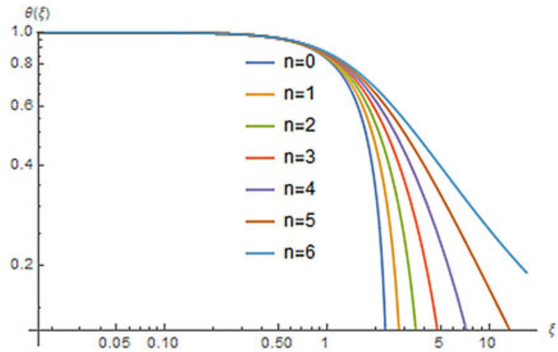

Figure 1. The LogLogPlot of polytropic solutions of Lane-Emden equation for Mass-Radius relationship of compact objects with integral polytropic indices, $n = 0 - 6$.

the intersections of θ-ξ curves with the ξ-axis. Where, θ and ξ are the parameterized dimensionless density and radius of the COs respectively. So, in our sample plots the solutions exist for integral polytropic indices $n = 0 - 5$, while $n = 6$ is non-existent. On the other hand, we compare our numerical results of the parameterized mass-radius relations of the COs with that given by (Chandrasekhar 1967) as in table 1. As we observe from the table, our numerical results are also similar except slight differences in accuracy.

4. Conclusions

The polytropic equation we have obtained has similar physical implications, in agreement to the earlier works by others. Furthermore, our numerical results where the latest version of Mathematica-11 used is similar except slight differences in accuracy.

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https://doi.org/10.1017/S1743921320003506 Published online by Cambridge University Press