COMPARISON OF POLAR MOTION RESULTS USING LUNAR LASER RANGING

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ABSTRACT. A comparison of polar motion results from three sources [Bureau International de l'Heure (BIH), Defense Mapping Agency Hydrographic/Topographic Center (DMAHTC-Doppler), the International Polar Motion Service (IPMS)] was performed using lunar laser ranging (LLR) data. The rms errors, both of the LLR data and of the determinations of polar motion by the three services, decreased in recent times. The BIH and Doppler polar motion are comparable in quality (12 ± 6 cm for BIH values taken from August 1976 through May 1980, 14 ± 7 cm for Doppler results taken from June 1977 through May 1980). The IPMS errors were substantially larger (33 ± 7 cm for data taken from August 1976 through May 1980). All three analyses give 13 ± 3 cm as an estimate for the combined LLR modeling, fitting, and instrumental error (noise) for the last four years.

O. Calame (ed.), High-Precision Earth Rotation and Earth-Moon Dynamics, 125–137.

I. INTRODUCTION

With observations from a single observatory, lunar laser ranging data, like classical astrometric data, is only sensitive to two components of Earth rotation, UTO and variation of latitude. The UTO information is the easier of the two to recover (Fliegel et al., 1981; Langley et al., 1981b) since it imposes nearly diurnal (25 hour) signatures. The variation of latitude is more challenging since the dominant sensitivity occurs at periods of a month or more, which are time scales which overlap the periods associated with the lunar orbit and physical librations. Langley et al. (1981a,b) have attempted to determine corrections to polar motion at approximately monthly intervals. In this paper we set ourselves the more modest goal of comparing the rms residuals of fits of 9 1/2 years of lunar laser data using smoothed polar motion from the three sources: Bureau International de l'Heure (BIH), Defense Mapping Agency Hydrographic/Topographic Center (DMAHTC) Doppler data and the International Polar Motion Service (IPMS). We apply and extend the technique of Harris and Williams (1977), which utilized the fact that polar motion errors cause the rms range error to increase with zenith angle, while instrumental errors and errors in the parameters of the orbit, librations, and lunar coordinates do not. While the determination of accurate corrections to polar motion is clearly a much desired product of lunar ranging, the procedure of this paper does not require these corrections in order to assess the rms polar motion error; it separates polar motion error from other sources of noise.

II. THEORY

In a companion paper in this volume, Fliegel, Dickey, and Williams (1981) have discussed how the residuals of the lunar laser data through May 1980 have been decomposed into daily components A_j and B_j, given nominal values of UT1 and polar motion and a general fit (August 1969 to May 1980) of the orbit, librations, and coordinates with these nominal values. In fitting the residuals of a single day (with index j) the A_j is a constant and the B_j is a coefficient of sin H, where H is the hour angle. That paper considered the UTO correction derived from the B_j. This paper discusses the daily constant coefficient A_j. The set of A_j of this paper was created from general fits using UT1 already corrected from previous Fourier smoothing (see Fliegel *et al.*).

Within one day, the A_j will contain any systematic errors which change slowly with respect to the observing period, typically no more than 6 hours. Thus the A_j will contain any errors in fitting or modeling the parameters of the lunar orbit, physical librations, or reflector and observatory coordinates, plus errors in the nominal polar motion, plus systematic and random instrumental errors in the range measurements. In a statistical sense the rms polar motion error can be distinguished from most other errors because its contribution to A_j depends on the zenith angle of the moon at transit.

When the moon is on the meridian with zenith angle z_j, we will assume that the error in range can be represented by two components

$$A_j = N_j + P_j S_j \qquad (1)$$

where the noise N_j includes instrumental, modeling, and fitting errors which do not depend on zenith angle, P_j is the component of polar motion error parallel to the meridian of the station (variation of latitude), and

$$S_j = \sin z_j. \tag{2}$$

In terms of errors in the two conventional polar motion components, ΔX_j and ΔY_j (in length units here)

$$P_j = \Delta X_j \cos\lambda - \Delta Y_j \sin\lambda \tag{3}$$

where λ is the east longitude of the observatory (McDonald).

We cannot solve Eq. 1 to evaluate N_j and P_j independently, but the equation can be squared so that mean square values of N_j and P_j can be considered statistically.

$$A_j^2 = N_j^2 + P_j^2 S_j^2 + 2 P_j S_j N_j \tag{4}$$

The individual range points are accompanied by estimates of the instrumental range errors, and the derived values of A_j have the associated uncertainties s_j which would result if those range errors were random. If the N_j were strictly from instrumental errors we would expect the N_j/s_j to have constant variance. We shall make this imperfect assumption and set

$$n^2 = E(N_j^2/s_j^2) \qquad (5)$$

where n is a normalized (dimensionless) factor which measures the rms ratio of the real noise to the assigned instrumental noise. E denotes the expected value. It is convenient to rewrite Eq. 4 as

$$A_j^2 = (N_j/s_j)^2 s_j^2 + P_j^2 S_j^2 + 2P_j S_j (N_j/s_j) s_j \qquad (6)$$

so that it may be fit in a least squares sense with the two variables n^2 and P^2 according to

$$y_j = n^2 s_j^2 + P^2 S_j^2 \qquad (7)$$

where P is the rms value of the P_j

$$P^2 = E(P_j^2). \qquad (8)$$

Examination of Eqs. 4 and 6 shows that A_j^2 depends on the product $2P_j S_j N_j$. This term has an unknown sign that can only be treated as noise when fitting Eq. 7. Consequently, a realistic weight for each A_j^2 depends on more than just the observational error. A realistic weighting scheme will permit the uncertainties in the least squares estimated values of n^2 and P^2 to be calculated. To assess the weighting consider the error expected from fitting y_j to A_j^2

$$A_j^2-y_j = (N_j^2/s_j^2-n^2)s_j^2 + (P_j^2-P^2)S_j^2 + 2P_jS_j(N_j/s_j)s_j. \quad (9)$$

The expected value of the square of this expression is to be taken. Assuming the P_j to be independent of the S_j, N_j, and s_j and using Eq. 8 one obtains

$$E[(A_j^2-y_j)^2] = E[(N_j^2/s_j^2-n^2)^2 s_j^4] + E[(P_j^2-P^2)^2 S_j^4)]$$
$$+4P^2E[S_j^2(N_j/s_j)^2 s_j^2]. \quad (10)$$

It is assumed that both P_j and N_j/s_j have Gaussian distributions with zero mean so that their fourth moments are given by

$$E(N_j^4/s_j^4) = 3n^4$$
$$(11)$$
$$E(P_j^4) = 3P^4.$$

The assumption that N_j/s_j is normally distributed means that it is independent of s_j. The powers of s_j and S_j are known and may be moved outside of the expected value operation yielding

$$E[(A_j^2-y_j)^2)] = 2n^4s_j^4 + 2P^4S_j^4 + 4P^2n^2S_j^2s_j^2. \quad (12)$$

The observed values of A_j^2 may be fit to the linear function (7) using a two parameter weighted least squares technique, the weights being the inverse of Eq. 12. Since Eq. 12 requires the unknowns n and P, we do this iteratively with the first iteration using n = 1 and P = 0. Convergence is good; a reasonable straight line can be fit with weighting functions simpler than that produced by Eq. 12. It is convenient to convert n^2 to a weighted rms noise N by multiplying by the weighted rms instrumental noise

$$N^2 = n^2/E(1/s_j^2). \qquad (13)$$

III. ANALYSIS

Three general solutions (LLR data from August 1969 to May 1980) were performed using the BIH, Doppler and IPMS polar motion results; their simple and weighted rms are summarized in Table I. These general solutions had incorporated the smoothed UT1 corrections from the laser data. The residuals for the 2954 ranges were then converted to 703 values of A_j and B_j by the daily decomposition procedure. For the BIH analysis, the circular D smoothed values of X and Y on the BIH 1979 system were utilized. The IPMS values used were taken from the Monthly Notes of the International Polar Motion Service prior to September 1973, and from U. S. Naval Observatory (USNO) Series 7 bulletins thereafter. These values as published are tabulated at intervals of .05 years, and were interpolated by us using Newton's interpolation formula to the third order. The Defense Mapping Agency Doppler solutions for the X and Y coordinates of the pole were also taken from USNO Series 7 bulletins, and smoothed using a Gaussian filter $\exp\{-^2/2\sigma^2\}$, with $\sigma = 15^d$. We tested these values by comparing the output of the same filter applied to the bi-daily solutions published in the DMAHTC polar motion reports, and found no differences as large as .001 arcseconds.

The IPMS results shown in the Figure indicate that the zenith angle effect is indeed present. The A_j have been grouped into weighted averages for every 0.05 interval of $\sin^2(z)$. Here, the intercept is the square of the noise term (N^2 in Eq.13) and the slope is the square of the polar motion term (P^2 in Eq. 7). The lines in the figures are the weighted least squares solutions to the daily values of A_j, not the averaged points in the graph. The three lines and their associated points show the dramatic improvement with time of both polar motion error and the noise error. Examining the residual plots as functions of time, a marked decrease in the residuals occurs in mid 1976 for both the IPMS and the BIH values. The Doppler polar motion gives a substantial drop in the residual in mid 1977. Consequently, the BIH and IPMS sets of A_j were divided for further analysis at JD 2443000 (August 26, 1976) and the Doppler set at JD 2443320 (June 10, 1977).

The fits for the three sources of polar motion are summarized in Table II, where the quoted errors are twice the formal errors. When all the data through May 1980 are considered, the lunar laser noise term, N, is the same for all three systems, 19±2 cm. This value drops to 13 cm if only recent data is considered. This decrease in the noise term in mid 1976 corresponds to instrumental improvements made at McDonald Observatory during 1976 and

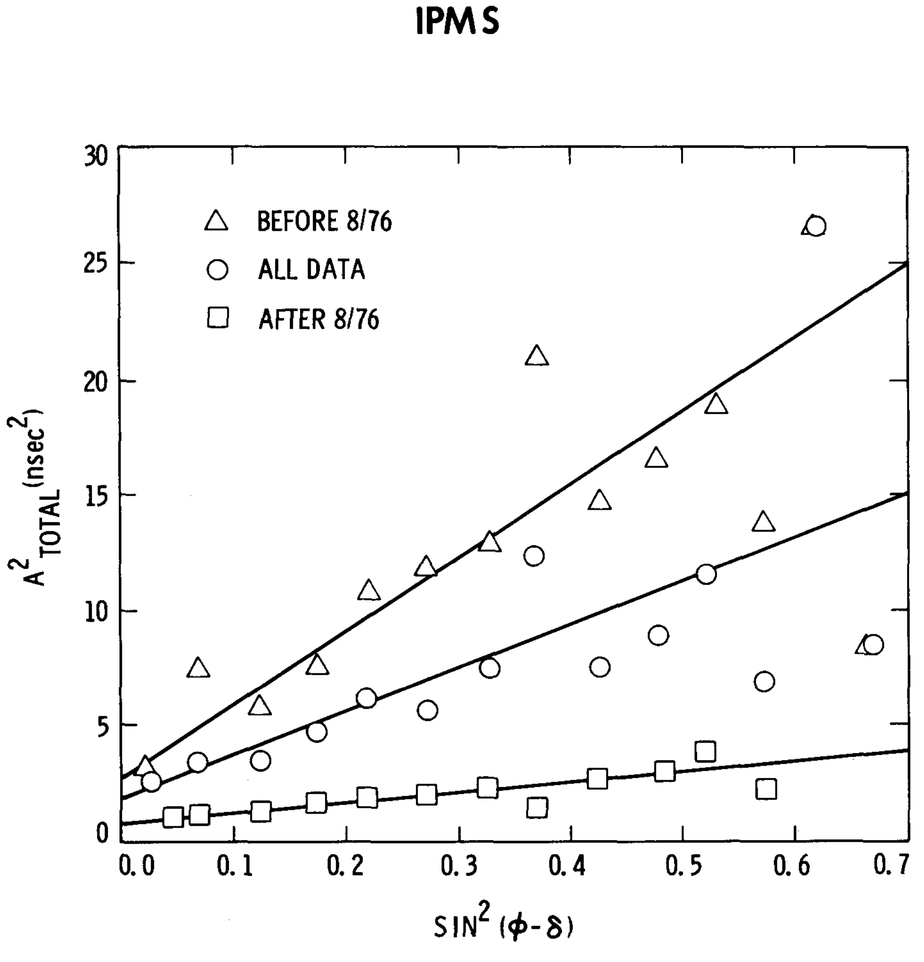

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TABLE I

POLAR MOTION	rms RESIDUAL (cm)	WEIGHTED rms RESIDUAL (cm)
BIH	31	27
DOPPLER	32	27
IPMS	44	41

TABLE II

POLAR MOTION		N (cm)	P (cm)
BIH	ALL DATA	19±2	22±6
	AFTER AUGUST 1976	13±2	12±6
DOPPLER	ALL DATA	19±2	24±5
(15d smoothing)	AFTER JUNE 1977	13±2	14±7
IPMS	ALL DATA	19±2	66±7
	AFTER AUGUST 1976	13±3	33±7

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1977. Therefore for the past four years, 13 cm is a fair estimate of modeling, fitting and instrumental error. As the weighted instrumental error is 8 cm during this period, approximately 10 cm of error is associated with the data analysis itself.

A decrease in the polar motion error is also seen in all analyses for recent times. The BIH and the Doppler values are comparable in quality, BIH having P = 22 ± 6 cm for all data (through May 1980), P = 12 ± 6 cm for data taken from 8/76 - 5/80 and Doppler having P = 24 ± 5 cm for all data, P = 14 ± 7 cm for data taken from 6/77-5/80. The IPMS errors (P = 66 ± 7 cm for all data and P = 33 ± 7 cm for data taken from 8/76-5/80) are substantially higher than either the BIH or Doppler errors.

The paper of Langley *et al*. (1981b) gives corrections to polar motion derived from lunar laser data. They then compare those results with various other sources of polar motion. For the time span 1976 to 1979, their rms difference from BIH (1979 system) values is 24 cm. For a test span of July 1976 to October 1978 their rms differences can be read from their figure as: BIH 20 cm, IPMS 36 cm, and Doppler (smoothing with σ = 15 days) 18 cm. They also give a best agreement of 16 cm with Doppler smoothed with σ = 28 days for this two year span. Comparing their results with our Table 2 shows that in all cases our estimate of the polar motion error for the recent data spans is smaller than their rms difference over two years. There are three major differences between the two studies: the data spans are not the same, the results of Langley *et al*. are averaged over several weeks, and their results are rms differences so that they include the error in the lunar laser derived variation of latitude. Presuming that errors add quadratically, the two results would be similar if there were 14 cm of noise in the lunar laser derived polar motion corrections of Langley et al. (1981a). This is in good agreement with their own error estimates. As can be seen from Table 2, this is quite competitive in accuracy with the Doppler and BIH sources, but as yet exists for only one component.

IV. SUMMARY

The major points of this analysis are:

1. We can estimate the quality of various sources of polar motion. The Doppler and the BIH values are substantially better than those from IPMS.

2. There is a marked improvement in accuracy in recent years, both in the LLR data and in the determinations of polar motion by the various agencies.

3. The LLR noise error, that is the modeling, fitting, and the instrumental error, is 13 cm for last four years.

4. The accuracy of polar motion sources can be tested for one component by analyzing lunar laser data. It is not necessary first to derive corrections to variation of latitude, or to average over more than 1/4 day.

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