In this book Arnold proves that in the sequence of caustics of a point on a convex surface close to a sphere each curve has at least 4 cusps. The proof requires an increase of closeness as the number of the caustic increases. For the first caustic this is the classical result of calculus of variations, which does not require any smallness condition at all. The author conjectures that such a condition is also unnecessary in the general setting.

Arnold discusses various generalisations and versions of the main problem he is considering. These are elegant statements of symplectic and contact geometry. Many of them concern the magic number 4. Some of them he proves: for example, the existence of at least four inflection points on a curve dividing the sphere into two parts of equal area. Some of them are left as conjectures: for example, the estimate of the number of bifurcations in a generic homotopy reversing a circular front on the plane. Although easy to formulate, the latter are apparently not so easy to prove.

The book is an excellent introduction to the area of low-dimensional geometry in which a mathematician of any level from an MSc student to a skilled professor would be able to find a source of interesting problems to solve. As in many previous cases the author opens up a new subject and encourages the reader to make his or her own contributions. Arnold suggests that investigations around the Jacobi Theorem constitute an area which is waiting for revolutionary methods like those which were used to solve another famous problem of his, the estimation of the number of fixed points of a symplectomorphism, and led, for example, to the Floer homology construction.

The book is extremely readable and contains many illustrations.

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PETROV, V. V. *Limit theorems of probability theory: sequences of independent random variables* (Oxford Studies in Probability, Vol. 4, Clarendon Press, Oxford, 1995), x+292pp., 0 19 853499 X, £50.

The results of limit theory for sums of independent random variables play a central role in probability theory, mathematical statistics and their applications. Much attention is paid to works of the Russian probabilistic school headed by A. N. Kolmogorov (Moscow) and Yu. V. Linnik (St Petersburg). In 1954 Gnedenko and Kolmogorov wrote a classic book on this subject. The present work is in the same spirit and contains many recent as well as classical limit theorems and probability inequalities for sums of independent random variables. A useful feature of the book is that every chapter has a section of addenda with precise formulations of many (mostly recent) results which are usually inaccessible in book form. It is a well-written scholarly work based on 487 references. Whilst reading the book one is amply rewarded by theory, but occasionally one feels a lack of concrete applications.

The first chapter contains a summary of some basic concepts and theorems of probability theory that are needed for the later discussion. In Chapter 2 useful inequalities for the maximum, for moments and for the concentration functions of sums of independent random variables are given. Besides, a brief treatment of exponential bounds is presented. The third chapter starts with the condition of infinite smallness and after infinitely divisible distributions have been characterized as limit laws—the fundamental result due to Khintchine (1937)—various weak limit theorems within the structure of such distributions are given. The next two chapters treat the central limit theorems and the weak law of large numbers with their rates of convergence in the sense of weak limit concept. The last two chapters deal with strong limit theorems, the strong law of large numbers and the law of the iterated logarithm.

There is a wealth of useful material in Chapters 3–7. But this is not reflected in the given subject index. A finer and more detailed indexing of these results would have enhanced the utility of this book both for researchers and students. On the other hand, the author index at the end is a useful guide and so are the bibliographical notes and addenda in each chapter. The St

Petersburg and Moscow schools of probability theory have indeed an impressive past contribution—to mention just a few illustrious names, P. L. Chebyshev, A. A. Markov, A. M. Lyapunov, A. Khintchine, Yu. V. Linnik and A. N. Kolmogorov. While I was reviewing this book it became once again apparent that this grand old tradition is still very much alive and one hopes it continues so in the future. For the next edition of this book one would expect a fuller coverage of Chinese and Japanese researchers' contributions in this area. In any case this text should be a useful reference for a long time to come.

To conclude this review I would like to highlight a few points.

(i) The nearness of the distributions of sums of independent random variables to the class of infinite divisible laws is well-surveyed and brings to date the work started by Lévy, Khintchine, Gnedenko, Kolmogorov, Prohorov and others.

(ii) It is quite reasonable to accompany every limit theorem by estimates of the speed of convergence. There is a vast literature devoted to the rates of convergence in the central limit theorem. Here, a good representative sample from this literature is given.

(iii) Earlier results on necessary and sufficient conditions for the law of the iterated logarithm without assumptions about the existence of moments are expressed in terms of the distribution of the summands.

As a small sampler of (iii) here is a generalization of Esseen's inequality without any assumptions about the existence of moments. Let $X_1,\dots,X_n$ be random variables with the distribution functions $V_1(x),\dots,V_n(x)$ and let $t_1,\dots,t_n$ be positive numbers. We define the truncated random variables $\overline{X}_j$ to be X_j if $|X_j|<t_j$ and 0 if $|X_j|\geqq t_j$, where $j=1,\dots,n$. We put

$$M_n=\sum_{j=1}^{n} E\overline{X}_j=\sum_{j=1}^{n}\int_{|x|<t_j} x\,dV_j(x),\quad N_n=\operatorname{Var}\sum_{j=1}^{n}\overline{X}_j,$$

$$\Delta_n=\sup_x\left|P\left(N_n^{-1/2}\sum_{j=1}^{n}(\overline{X}_j-E\overline{X}_j)<x\right)-\Phi(x)\right|,\quad \Gamma_n=\sum_{j=1}^{n}P(|X_j|\geqq t_j).$$

Theorem. *For all numbers $a>0$ and b we have*

$$\sup_x\left|P\left(\frac{1}{a}\sum_{j=1}^{n}X_j-b<x\right)-\Phi(x)\right|$$
$$\leqq\Delta_n+\Gamma_n+\frac{|ab-M_n|}{\sqrt{2\pi N_n}}+\frac{1}{2\sqrt{2\pi e}}\left|1-\frac{N_n}{a^2}\right|\max\left(1,\frac{a^2}{N_n}\right).$$

In the above theorem there are no assumptions about the independence or any type of dependence of the random variables $X_1,\dots,X_n$ or about the existence of any moments of these random variables. The condition $0<t_j<\infty\,(j=1,\dots,n)$ implies the existence of the moments of an arbitrary order of $\overline{X}_1,\dots,\overline{X}_n$. Under the additional assumption about the independence of the random variables $X_1,\dots,X_n$ the truncated variables $\overline{X}_1,\dots,\overline{X}_n$ are also independent and we can obtain estimates of Δ_n in the form

$$\Delta_n\leqq AN_n^{-3/2}\sum_{j=1}^{n}E|\overline{X}_j-E\overline{X}_j|^3. \qquad (*)$$

It is not hard to show that the inequality which follows from the theorem with the replacement of Δ_n by the right hand side of $(*)$ is a generalization of the various estimates in the literature.

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