RADIATIVELY-DRIVEN WINDS: MODEL IMPROVEMENTS, IONIZATION BALANCE AND THE INFRARED SPECTRUM

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ABSTRACT

Recent improvements to theoretical stellar wind models and the results of empirical modelling of the ionization balance and the infrared continuum are discussed. The model of a wind driven by radiation pressure in spectral lines is improved by accounting for overlap of the driving lines, dependence of ionization balance on density, and stellar rotation. These effects produce a softer velocity law than that given by Castor, Abbott and Klein (1975). The ionization balance in ζ Puppis is shown to agree with that estimated for an optically thick wind at a gas temperature of 60,000 K. The ionization model is not unique. The infrared continuum of ζ Pup measured by Barlow and Cohen is fitted to a cool model with a linear rise of velocity with radius; this fit is also not unique. It is concluded we should try to find a model that fits several kinds of evidence simultaneously.

I. INTRODUCTION

As I understand the charge to the members of this panel, each of us is to present his favorite model of the temperature structure of the wind of an O star, and show how it fares with all the various observational requirements; thereby will the correct theory of stellar wind dynamics be found. Interesting as this may be, I am afraid it will not lead to the hoped-for result, for two reasons. The first is that present observations are not sufficient to select a single model; there are simply not enough observables that can be related to the temperature of the gas in a simple and sensitive way. The second reason is that the temperature may not be closely coupled to the processes that make the stellar wind go. Other forces than gas pressure may be more important in the overall dynamical scheme. Another problem that arises in comparing the alternative models is that some are what I would call "theoretical models," in which there are no arbitrary functions but everything is derived (approximately!) from first principles, and others are "empirical models" in which temperature and velocity are

P. S. Conti and C. W. H. de Loore (eds.), Mass Loss and Evolution of O-Type Stars, 175–190.

arbitrary functions. As always happens, "theoretical" models never fit the data, and "empirical" models always do.

My own work on stellar wind models (all of it in collaboration with David Abbott, now at U. Wisconsin, and Richard Klein, now at Kitt Peak) began with a "theoretical" model -- outflow driven by radiation pressure in a large number of spectral lines á la Lucy and Solomon. This model was quite simplified in terms of its physical assumptions, such as radiative equilibrium, but it did make quite definite predictions of the velocity and temperature structure of the wind as well as of the rate of mass loss. These predictions were quickly discredited by observations of the ionization structure of the stellar wind and indirectly of the velocity field. However, no other "theoretical" model has been advanced. Since I, too, want to agree with observation as much as possible, I have dabbled with "empirical" models. This has taken the form of investigating how the ionization balance in the wind relates to the gas temperature in the perilous regime of large EUV optical depth, and also of trying to analyze the infrared observations of ζ Puppis.

My talk today is going to range over a wider area than just the temperature of the wind. On the "theoretical" side, I would like to talk about improvements in the basic model which I hope make it more realistic. These improvements address the complications of (1) overlap in frequency of the wind-driving resonance lines, (2) the effect on the driving force of the variation of ionization balance with density, and (3) stellar rotation. With regard to "empirical" models I offer comments and cautions regarding the interpretation of ionization balance data in terms of a temperature. To underscore the caution I present a model of ζ Pup that explains the ionization balance in terms of photoionization in an optically thick stellar wind (a realistic case). In this model there is a moderate elevation of the wind temperature from radiative equilibrium. My second "empirical" comment is that the infrared fluxes of ζ Puppis, observed by Barlow and Cohen, are fitted exceedingly well by a cool isothermal wind model having the observed rate of mass loss and a slowly rising velocity law, as indicated by line-profile studies.

II. IMPROVEMENTS TO THE RADIATION-DRIVEN WIND MODEL

This model is developed in its simplest form by Castor, Abbott and Klein (1975), based on the idea of Milne (1926) refined by Lucy and Solomon (1970). The identity of the driving spectral lines was investigated more carefully by Abbott (1977); see also Castor, Abbott and Klein (1976). For this model the following assumptions were made: (1) spherical symmetry; (2) steady radial flow; (3) only the forces of gravity, radiation scattered by free electrons and radiation scattered by ions are significant; (4) energy balance with no mechanical heating. This model is not advanced as a perfect representation of reality, but rather as a calculable one that accounts for what appear to be the dominant processes. Assumption (3) could be modified to include the thermal pressure of the gas (in the model we made, it was), but this has very little effect on the dynamics for any reasonable temperature since the wind is so highly supersonic. (This is supported by the calculations of MacGregor [1978], for a model with a temperature rise.)

The quantities that can be predicted with this model are the rate of mass loss, and the dependences on radius of the velocity and the gas temperature. The mass loss agrees within a factor 2 with observation for the supergiants for which the rate of mass loss can be found from the radio flux or from the infrared excess. The predicted velocity law gives quite reasonable terminal velocities and a qualitatively correct shape, but seems too steep at small radii. The gas temperature turns out to be about 0.9 times the effective temperature of the star, consistent with radiative equilibrium. Sadly, this is inconsistent with the observed presence of O VI and perhaps also with N V.

The problem with the gas temperature points to the necessity of some additional heating mechanism, the nature of which is certainly not known at the present. The problem with the steepness of the velocity law suggests that we have oversimplified the dynamics. Two aspects of the dynamics that are suspect are, first, that we have assumed that each spectral line acts independently of the others, whereas in reality they overlap in frequency and therefore modify one another; and, second, we have ignored the effect of ionization changes with radius on the radiation force. Furthermore, if the star is rotating there are additional large terms in the equations of motion that we have not considered.

A. Overlapping Lines. The difficulty we face when the rest frequencies of the strong lines are spaced by less than the Doppler shift corresponding to the terminal velocity of the wind is that the photons originating in the stellar continuum that a particular line would normally scatter may have been, in some sense, "used up" by another line. Of course, the photons are not actually "used up." Collisional destruction of the photons is very unlikely, so interaction with one line can only alter the frequency and direction of a photon, after which it is still present to scatter in another line. Perhaps it is helpful to focus our attention on a single photon from its origin at the photosphere until it leaves the stellar envelope going outward, or flies back into the photosphere again. In the meanwhile it may have interacted with several different lines. Each such interaction consists of several, perhaps a very large number, of scatterings, but owing to the large velocity gradient in the gas, all the scatterings occur in a small volume. Therefore we can find the impulse given to the material at that point from the photon's initial and final directions; it is as if the photon scattered only once.

Now comes a tricky part. Suppose, as an idealization, that the strong spectral lines are distributed at random in frequency, within some fairly large band. Then our photon, having scattered many times in one line, will fly across the envelope a random distance until its frequency matches that of the next strong line. (Distances convert to frequencies owing to the Doppler effect in the expanding envelope.) Then the photon scatters in this line some number of times before proceeding in a new direction, and so on. The whole process is a kind of diffusion, in which the number of particles is conserved. The mean free path for this diffusion is the distance in space that corresponds to a Doppler shift equal to the mean interval in frequency between strong lines. This is not a hard diffusion problem to solve, and the boundary condition is the known net flux of photons at the photosphere. The result is the angular distribution of the radiation averaged over the large frequency band in question. How does this help us compute the total force on the material? In order to find the force due to a particular line we need to know the intensity of the radiation available for scattering in this line, which therefore includes the effects of scattering in all the lines at higher frequency. In general this is a nasty quantity, but if the lines are randomly distributed in frequency then the average of this intensity over several lines is the same as the frequency average we find from our diffusion problem.

The picture outlined in the preceding paragraph leads to a quantitative result for the line force in the following way. The expression for the force on a unit mass of material due to an individual line is (Castor 1974)

$$f_L = \frac{2\pi\nu}{\rho c^2} \int_{-1}^{1} \mu I(\mu) \left[\mu^2 \frac{dv}{dr} + (1-\mu^2) \frac{v}{r}\right] \left\{1-\exp[-\tau(\mu)]\right\} d\mu \quad . \quad (1)$$

In this formula all quantities are evaluated at the distance r from the center, μ is the direction cosine of a ray with respect to the radial direction, $I(\mu)$ is the intensity of the radiation available for scattering in the line, $v(r)$ is the outward flow speed, and $\tau(\mu)$ is the Sobolev optical depth of the line for the position and direction in question. It depends on direction as $\tau(\mu) = \tau_{rad}[\mu^2+(1-\mu^2)d\ell nr/d\ell nv]^{-1}$. According to the argument given above, we can identify I with the average intensity in a frequency band surrounding the line. In that case the average net flux F_ν in the band is related to I by

$$F_\nu = 2\pi \int_{-1}^{1} I(\mu)\mu d\mu \quad . \quad (2)$$

Since the net flux is conserved, we can calculate F_ν from the properties of the photosphere -- it is the continuum flux. We can now recast equation (1) in this way:

$$f_L = \frac{\nu F_\nu}{\rho c^2} \frac{dv}{dr} \left[1-\exp(-\tau_{rad})\right] \mathcal{F}_a \quad , \quad (3)$$

in which $\mathcal{F}_a$ is the ratio

$$F_a = \frac{\int_{-1}^{1} \mu I(\mu)\left[\mu^2+(1-\mu^2)\,d\ell nr/d\ell nv\right]\left\{1-\exp[-\tau(\mu)]\right\}d\mu}{\left[1-\exp(-\tau_{rad})\right]\int_{-1}^{1}\mu I(\mu)\,d\mu} \quad . \quad (4)$$

Apart from the factor F_a, equation (3) is the formula for the force we have used up to the present. Thus F_a is the factor required to account for overlapping lines and the correct angular distribution of the radiation. In the likely event that the outward intensity is larger than the inward intensity for every ray, we can see that F_a is smaller than unity where $d\ell nv/d\ell nr$ is larger than unity, namely the inner part of the envelope, and larger than unity in the outer region where $d\ell nv/d\ell nr$ is smaller than unity. This is illustrated in Figure 1, in which the overall F_a is shown for a distribution of lines with different strengths and for which overlap is a large effect. It may seem counterintuitive that in the high velocity region, where line overlap should be more serious, the force turns out to be larger than in the inner region. The explanation of the paradox lies, first of all, in noting that the photons are not "used up." The effect is in fact due to saturation of the line. Increasing optical depth of the line reduces the force. The assumption that the photons all travel radially, our earlier model, underestimates the optical depth in the inner region and overestimates it in the outer region. Interaction with several lines broadens the angular distribution and increases the effect.

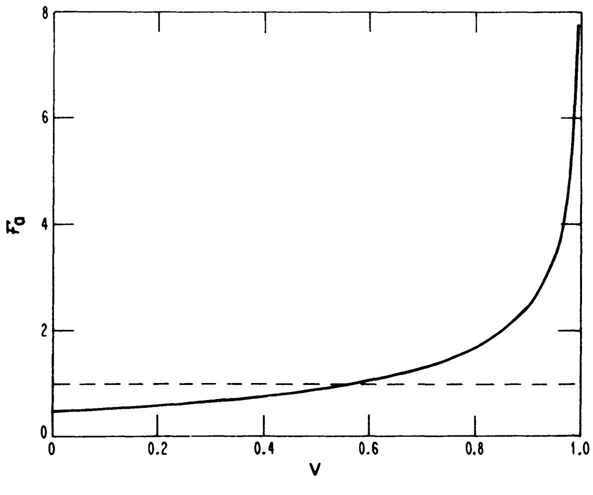

Figure 1. Angular-distribution factor in the radiation force as a function of outflow velocity. The model has $v = v_\infty(1-R/r)^{1/2}$ and three strong lines per frequency interval $\nu_0 v_\infty/c$.

https://doi.org/10.1017/S0074180900013462 Published online by Cambridge University Press

Before investigating the dynamical effect of the new factor in the radiation force, we must note that there is a competing factor: the effect of ionization changes on the total force.

B. Ionization Changes. Except possibly for the highly ionized species like O VI and N V, the ions in the stellar wind seem to be produced by photoionization. In the simplest photoionization model the degree of ionization depends on the ratio of the diluted stellar flux to the electron density, F/N_e. Since the flux varies as $1/r^2$ and the density varies according to mass conservation as $1/vr^2$, the degree of ionization depends on the velocity. The stage of ionization that is one step lower than the most abundant one will vary in abundance as v^{-1}, while the one that is one step higher varies as v. How the force will vary with v on account of these ionization changes depends on which stage of ionization produces the most force. It happens that the higher stages of ionization have their resonance lines shifted away from the peak of the flux distribution toward the EUV, and thus contribute less. This is not an invariable rule. For example, the Li sequence ions behave in the opposite way. Nonetheless, most of the force is produced by the lower ions. The total force due to the lines of a particular ion varies with the ion abundance f_{ion} as $f_{ion}^{0.1-0.2}$. The final result is that the total force varies as $v^{-0.1}$. This result, as well as support for the general statements above, comes from the detailed force calculations of Abbott (1977).

When the effect of ionization changes is included with the line-overlap factor, discussed above, in the expression for the line radiation force, the momentum equation for steady flow (neglecting gas pressure) takes the form

$$r^2 v \frac{dv}{dr} = -GM(1-\Gamma) + Ck(v)\left[r^2 v \frac{dv}{dr}\right]^{\alpha} \quad , \qquad (5)$$

where the force constant k now depends on velocity according to

$$k(v) \propto v^{-0.1} F_a \quad . \qquad (6)$$

The function k(v) has a minimum at some value of v, and becomes large both when v is small and when v approaches the terminal velocity. The value of v at the minimum is about $0.1v_\infty$ for the function F_a shown in Figure 1. The correct velocity law and rate of mass loss are determined by finding the solution of equation (5) that passes through a regular singular point. That point is in fact the place where k attains its minimum. The rate of mass loss is given by the formula in CAK, except that the minimum k must be used. (Gas pressure is a 1-4% effect at the singular point, so it should not modify these results very much.)

The effect of the variation in k is that r^2vdv/dr is smaller interior to the singular point, and larger exterior to it, compared

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with the constant value for the CAK model. This produces a more slowly rising velocity law, which is the direction indicated by observation. In order to find the new velocity law quantitatively we have to find the angle factor F_a self-consistently with the velocity law, which has not yet been done. (Recall that F_a depends on the solution to a diffusion problem, one of the ingredients of which is the velocity law.)

C. Stellar Rotation. The effect of stellar rotation on a radiatively driven stellar wind is a particularly unpleasant problem. The three dimensional aspects of even the simpler problem of a gas-pressure-driven wind have not been fully explored (see Nerney and Suess 1975), and the tensorial character and explicit dependence on velocity gradients of the radiation force are not likely to make the problem easier. Marlborough and Zamir (1975) have investigated the effects of radiation force exerted in the continuum, but I think the effect of line force is crucial. I have had a crude stab at including rotation in the framework of the CAK model. I assumed a flow that had zero component of velocity in latitude, for which the angular momentum per unit mass was conserved. For this simple model the effect of rotation is only to add a known centrifugal term to the equation of motion, which can be solved in the usual way. (I have not added the complications discussed in the preceding sections.)

The results for this model are (1) the mass loss is nearly unchanged by rotation, and (2) the rise of velocity with radius is quite appreciably slower when rotation is included, and the terminal velocity is less. The lack of change in the rate of mass loss reflects the fact that for this model the singular point, where the mass loss is determined, falls at a very large radius where the centrifugal force has

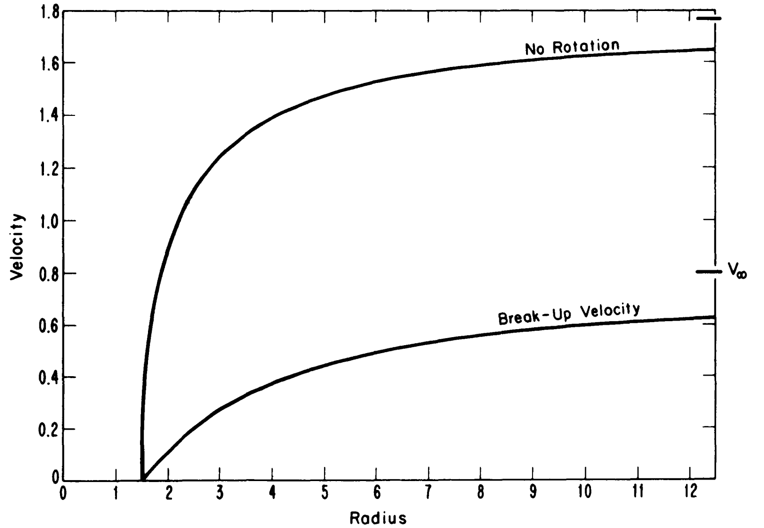

Figure 2. Velocity law for a rotating stellar wind. The upper curve is for no rotation, the lower curve is the equatorial run of velocity for rotation at the break-up speed.

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become negligible. This conclusion could change when the overlapping-line and ionization effects are put in, since then the singular point will be much closer in to the center. The effect on the velocity-radius relation is shown in Figure 2 for a star rotating at the break-up velocity. (By the way, nothing dramatic happens to the stellar wind as this limit is approached.) The fact that the velocities at a given radius are smaller when rotation is included means that the densities are larger, which will produce stronger emission in the Balmer and other lines, just as if the rate of mass loss had been increased. This model could be tested by looking for an anti-correlation between Vsini and the limiting velocity for the UV P Cygni lines.

III. EMPIRICAL STELLAR WIND MODELS

A. Ionization Balance. What we are most interested in for today's discussion is the temperature of the wind. Most of the efforts that have been made toward determining the temperature directly from observations have focused on the observed ionization balance, and in particular on the presence of the highly ionized species O VI and N V in the ultraviolet spectra of the O stars. These efforts are courageous, because we normally expect the ionization balance to be nearly independent of the local temperature at the outside of the star. The temperature affects the ionization directly through collisional ionization by electrons, but this process is very weak. There is also an indirect effect through thermalization of the EUV radiation field; however, the stellar wind is not sufficiently optically thick for this to occur. That would be the end of the story had not the O VI absorption lines shown up in the Copernicus spectra. One can calculate the rate of photoionization of O V by the emergent flux from an appropriate model atmosphere, and it is orders of magnitude too small to explain the observed line strengths; hence we discard photoionization. Collisional ionization being weak, we crank the gas temperature way up to give the desired ionization rate. This model works fine, and Henny Lamers will describe it in more detail. However, it may not be the only possible model, even among those with a roughly uniform temperature in the wind. Perhaps photoionization was discarded a little too hastily.

Let us first compare the rates of collisional ionization and photoionization for an ion, when the ionizing radiation is optically thick. Let R_{1K} be the rate coefficient for photoionizing the ground state of the ion, given by

$$R_{1K} = \int_{\chi/h}^{\infty} \frac{4\pi}{h\nu} J_\nu a_\nu d\nu \quad , \tag{7}$$

where χ is the ionization potential, J_ν is the local continuum intensity, and a_ν is the photoionization cross section. The quantity R_{K1} is defined by a similar integral, with J_ν replaced by the local Planck function B_ν. The significance of R_{K1} is that $N_1{}^*R_{K1}$ is the rate of radiative recombinations to the ground state, also written as $N_eN_+\alpha_1$ in terms of the recombination coefficient α_1. Here N_1^* is the ground-state population given by LTE. The departure coefficient b_1 is defined as N_1/N_1^*. Now, if it happens that the stellar envelope is optically thick for the radiation that can ionize this ion, then to a reasonable approximation J_ν can be replaced by the continuum source function, which is given by $B_\nu/\overline{b_1}$. Here $\overline{b_1}$ is the mean coefficient for all ions that absorb this radiation. The result is that $R_{1K} = R_{K1}/\overline{b_1}$. The rate coefficient for ionization by electron impact is defined to be N_eC_{1K}, where C_{1K} is the result of folding the impact ionization cross section with the Maxwellian distribution of electron energies. If we look in Allen (1973) for reasonable estimates of these cross sections, we come up with the formula

$$\frac{N_eC_{1K}}{R_{K1}} = 4 \times 10^{-15} \frac{n_*^7}{z^6} T_e^{-1/2} N_e \quad , \tag{8}$$

in which n_* is the effective principal quantum number for the target valence electron and z is the charge of the higher ion. If we put R_{K1} in terms of R_{1K} and take reasonable numbers for n_*, z, and T_e, we get

$$\frac{N_eC_{1K}}{R_{1K}} \approx \frac{b_1N_e}{2 \times 10^{18}} \quad . \tag{9}$$

We see that b_1N_e is the governing parameter. If it is larger than 10^{18} or so, then collisional ionization dominates (coronal approximation); if it is less, then photoionization is more important. 10^{18} may seem a little large for an electron density, but remember that if the electron temperature is large but the ionization is moderate then b_1 must be very large.

There is an upper limit to b_1N_e, and hence to the importance of collision ionization, set by the condition that the rate of collisional ionization can not exceed the total rate of radiative recombination (three-body recombinations being negligible). The requirement is that

$$N_1N_eC_{1K} < N_eN_+\alpha \quad , \tag{10}$$

where α is the total recombination coefficient, including excited states and dielectronic recombination. Since $N_1^*R_{K1} = N_1R_{1K} = N_eN_+\alpha_1$ the condition is

$$\frac{N_eC_{1K}}{R_{1K}} \approx \frac{b_1N_e}{2 \times 10^{18}} < \frac{\alpha}{\alpha_1} \quad . \tag{11}$$

Equality in equations (10) and (11) defines the coronal approximation. It appears from equation (11) that collisional ionization can never be very large compared with photoionization, but we must remember that dielectronic recombination can cause α to be much larger than α_1. Ratios of 100 or more are not unusual, if T_e is over 10^5 K. Thus the typical value of $b_1 N_e$ for the coronal approximation is about $10^{20} - 10^{21}$ and photoionization, even for the optically thick gas, is small.

The very large ratio of α to α_1 indicates that recombination to the ground state is negligible -- excited states and particularly autoionizing states are more important. This raises the question: what about the inverse of those processes? Photoexcitation can create appreciable populations in the excited states, from which photoionization (or autoionization) can occur. In addition, the bulk of dielectronic recombination occurs through states of very large principal quantum number for which, at electron densities of order 10^{11} cm^{-3}, collisional effects become important, quenching the process. A detailed statistical equilibrium calculation for the stellar wind ions, which could answer these questions, has not yet been done. This is a high priority item for future work, but until it is done we can only worry that α/α_1 has been overestimated.

What is the ionization balance we predict if optically thick photoionization and collisional ionization are both taken into account? We add $N_1 R_{1K} = N_1 R_{K1}/\overline{b_1}$ to the left side of inequality (10), making it an equality. Now we must distinguish between the departure coefficient b_1 of the ion in question and the mean $\overline{b_1}$ of the absorbing ions. We find

$$N_1\left(\frac{R_{K1}}{\overline{b_1}} + N_e C_{1K}\right) = N_1{}^* R_{K1}\,\frac{\alpha}{\alpha_1} \qquad (12)$$

hence

$$\frac{1}{N_e b_1} = \frac{\alpha_1}{\alpha}\left(\frac{1}{N_e \overline{b_1}} + \frac{C_{1K}}{R_{K1}}\right) \approx \frac{\alpha_1}{\alpha}\left(\frac{1}{N_e \overline{b_1}} + \frac{1}{2\times 10^{18}}\right) \quad . \qquad (13)$$

If $N_e \overline{b_1}$ is larger than 2×10^{18}, the coronal approximation applies, and $N_e b_1$ is around 10^{21}. Otherwise $N_e b_1$ is larger than $N_e \overline{b_1}$ by a factor α/α_1. How do we decide on a value for $N_e \overline{b_1}$? One way is to use equation (13) for all ions, in each case using the appropriate average for $\overline{b_1}$; the result is a set of coupled nonlinear equations for the departure coefficients. However, equation (13) may not be adequate for the dominant absorbing ions in the gas; for these, effects like excited-state photoionization must be included. The difficulty that arises in treating these ions was discussed, in the case of He II, by Klein and Castor (1978). Instead of solving for all the ions, we might treat $\overline{b_1}$ for the dominant absorber as a free parameter. If we make the further (naive!) assumption that α/α_1 is the same for every ion, we find that all ions with an ionization potential as large or larger than the dominant one are described by the same departure coefficient,

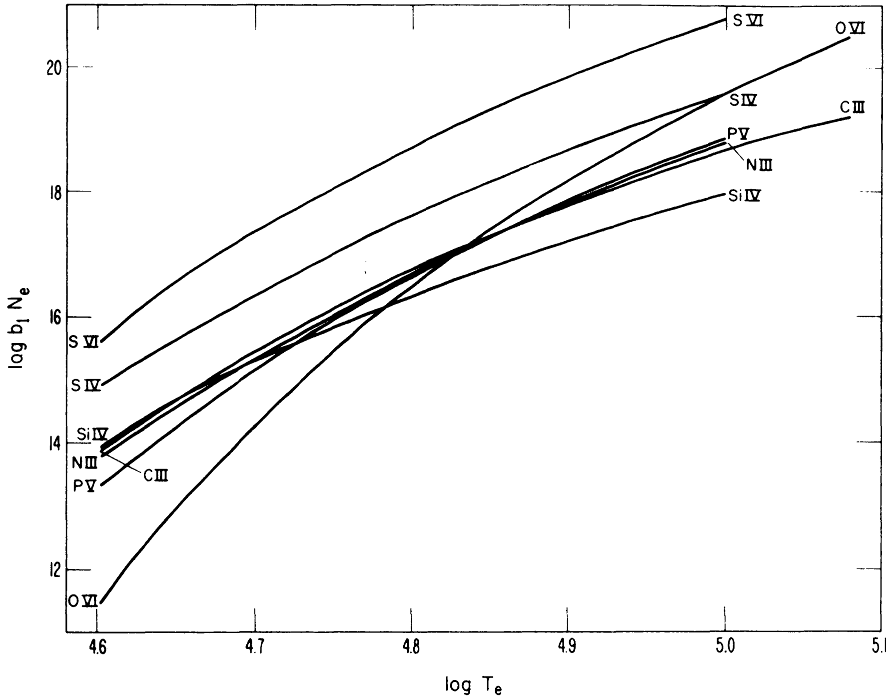

Figure 3. Loci in b_1N_e versus T_e on which the ions attain the abundances observed in ζ Puppis. b_1 is the departure coefficient from the Saha equation at the temperature T_e.

with the exception of the dominant one itself. We now have a <u>two</u>-parameter ionization balance: T_e and b_1N_e.

One does not really expect a single departure coefficient to fit every ion, but it is interesting to compare the ionization computed on this hypothesis with the observed ionization fractions for ζ Pup, the only star with a fairly complete set of data. In Figure 3 the observed ionization fractions for several ions have been converted to equal-ionization contours in the parameter diagram of b_1N_e versus T_e. Surprisingly, all the ions except sulfur agree within a factor 3 in b_1N_e provided the temperature is somewhere around 60,000 to 70,000 K. The best value b_1N_e is around 3×10^{16}, low enough for collisional ionization to be neglected. This is about 10 times larger than N_e times the departure coefficient of He II at this temperature, if in fact He II is ionized by dilute photospheric radiation at 35,000 K; this would be consistent with He II being the dominant ion and α/α_1 being 10.

What do we conclude from this? It does appear that it is possible to explain the observed ionization in ζ Pup with a model at a roughly uniform temperature of order 60,000 K by taking due account of the large optical depth in the far UV. This model is still a crude one at present because we have not found an accurate way of including the excited states in the ionization balance problem; this is a difficulty

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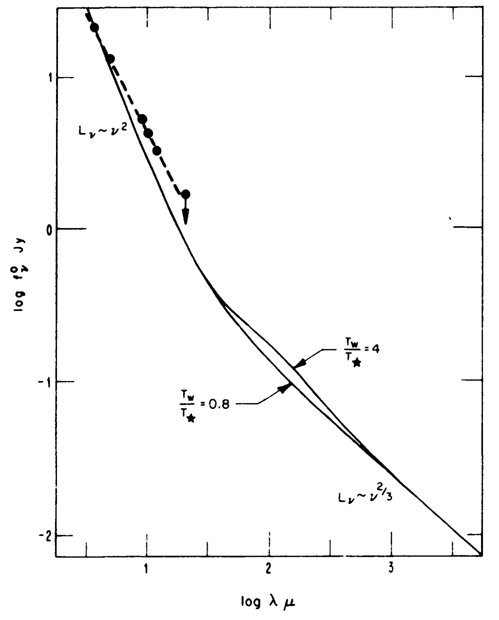
that is faced by all the ionization models. The present state of ionization modelling is a sad one. There are at least three models, with quite different temperature structures, that fit the observed ionization fairly well, and which are equally reasonable from a theoretical point of view. Clearly other kinds of data are needed if a choice is to be made.

B. Infrared Spectrum. The infrared spectrum provides data that can be translated into temperatures of the wind in a much more straightforward way than can the observed ionization balance. This is because the dominant absorption in the infrared is free-free, with a simple dependence on temperature and no complications due to departures from LTE. The price that is paid is that the dependence on temperature, while unambiguous, is not very strong. The infrared spectrum also depends on the density distribution, that is, on the velocity law, so the velocity and temperature must be found at the same time. The infrared spectrum alone is not sufficient for this, but in conjunction with other data, such as the $H\alpha$ profile, it should be possible to determine both variables. The best star for analysis is again ζ Pup. Morton and Wright (1978) have determined the rate of mass loss from the radio flux independently (nearly) of the velocity law and temperature. The infrared observations by Barlow and Cohen, which Mike Barlow has described earlier, are the most complete for ζ Pup of all the O stars. David Van Blerkom will discuss the analysis he has been doing on the $H\alpha$ line; I would like to show the results of some model fitting I have done using the infrared data obtained by Barlow and Cohen.

All the models are computed by solving the spherical transfer equation allowing free-free absorption and electron scattering as opacity sources. The region interior to the sonic point was assumed to radiate a Planck spectrum, and I used a constant infrared Gaunt factor for simplicity. The first attempt at fitting the spectrum was made with the standard velocity law $v(r) = v_\infty(1-R_*/r)^{1/2}$. One temperature was assumed for the stellar photosphere, and another constant temperature was taken for the wind. The rate of mass loss was fixed at the radio determination. The result is shown in Figure 4. The dots are the observations, and it is clear that in the wavelength range they span, the wind is simply too transparent to affect the emergent flux, and what is seen is the photospheric Planck spectrum. However, the observed fluxes define a spectrum slope that is definitely redder than a Planck function at the stellar temperature. (Lowering the assumed stellar temperature would not improve the fit very much.) We also notice in Figure 4 that varying the wind temperature does not make any difference to the emergent flux in the region that is observed.

For a model close to hydrostatic equilibrium, the 11.67 μm radiation comes from a layer with electron density 2×10^{13} cm^{-3}, which implies an outflow velocity of about 10 km s^{-1}. The flow velocity for the 2 μm emitting layer would be corresponding less, about 2 km s^{-1}. Thus one explanation of the redness of the spectrum is that there is a rise in temperature by about a factor two between these two layers.



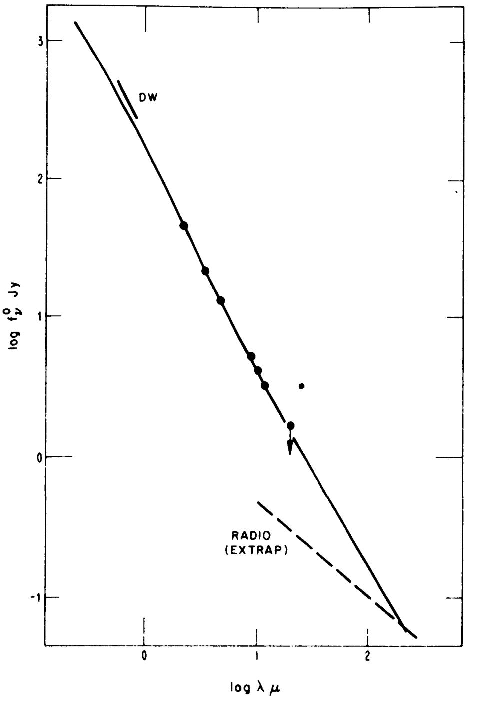


Figure 4. The solid lines are computed infrared fluxes (in Janskys) for a model with $v = v_\infty(1-R/r)^{1/2}$ and the indicated ratios of wind temperature to photosphere temperature. The dots and the dashed line show the Barlow and Cohen observations of ζ Pup.

Figure 5. Same as Figure 4 except the model has $v = v_1(r-R)$ and $T_w \approx T_\star$. The short line marked DW is the visual photometry of Davis and Webb (1974). The dashed line is the radio flux extended to shorter wavelengths with a $\nu^{2/3}$ law, corrected for the Gaunt factor.

A relation between temperature and velocity of the form $T \propto v^{0.5}$ would give the observed spectral index, if the density distribution were close to hydrostatic equilibrium. A model of this kind could be tested by investigating the consequences for the stellar absorption lines, which are formed in about the same region as we know from the Balmer progression of radial velocities. I have not pursued this model, but have instead investigated a model in which the infrared excess is due entirely to geometrical extension.

In order to have the wind affect the 2-10 micron infrared, its emission measure ($\int n_e^2 dr$) must be increased over that for the standard velocity law; this means that the velocity should rise more slowly with radius than for the standard law. I have tried the law $v = v_1(r/R_\star - 1)$. This linear law guarantees a much larger emission measure than that for the square-root law. (The law was used only in the region where $v \geq 25$ km s^{-1}; the Planck boundary condition was applied at $v = 25$.) Of course, the velocity does not increase indefinitely with radius, but turns over and approaches the terminal velocity. This turn-over will have very little effect on the region shortward of 20 μm, as we see in Figure 4, so we need not try to model it. The parameters that can be adjusted in the model are the temperature T_* and the angular size of the photosphere, the velocity coefficient v_1, and the temperature T_w of the wind. Of these, T_* is assumed to be fixed at 50,000 K, and the visual flux then implies an angular diameter equivalent to $R_* = 15.9\ R_\odot$ at a distance of 450 pc (Lamers and Morton 1976). The two remaining parameters can then be used to fit the slope and the magnitude of the emergent flux in the near infrared.

The results of a not-quite-perfect fit are shown in Figure 5. The long solid line is the model, and it fits the IR data very well. It does miss the visual data by 0.1 in $\log f_\nu{}^o$, which is the not-quite-perfect aspect. In fact, the fitting is done in non-dimensional variables, and the model shown turned out to correspond to $T_* = 46{,}000$ K rather than the desired 50,000 K. In addition, my treatment of the photosphere as a Planck boundary condition is not very accurate, so it is not surprising there is some discrepancy. The computational aspects are better for the infrared itself, so we can have some confidence in the values of v_1 and T_w. These are $v_1 = 700$ km s^{-1} and $T_w = 42{,}000$ K.

These are interesting results, but what do they actually imply? Apparently a cool model with a slowly-rising velocity can fit the infrared to within the accuracy of the observations. But by no means is this the only model that will fit. I have described above a model with a temperature rise in the subsonic region that will also work. No doubt there are others. Additional data must be analyzed in conjunction with the infrared. For example, we should analyze the Hα profile, the Balmer velocity progression, and the intensities of He I lines. These are all sensitive to the velocity and temperature distributions in different ways.

Some other aspects of the model derived above may be of interest. The velocity range for the layers in which the observed infrared originates is from the sonic point out to 250 km s^{-1}. These are more than ten times higher than the velocities mentioned earlier in connection with the nearly hydrostatic atmosphere, and the reason is that in an extended atmosphere less particle density is needed to produce a given emission measure. The optical depth in electron scattering at the sonic point is about 0.4. This is large enough to explain the large angular diameter found by Hanbury Brown et al., but not so large that the treatment of the photosphere is nonsense or so that extensive wings would be expected on every line profile.

https://doi.org/10.1017/S0074180900013462 Published online by Cambridge University Press

IV. CONCLUSIONS

The radiatively-driven wind models have been fairly successful in explaining the rates of mass loss and terminal velocities of the winds in O and B stars. I feel we have correctly identified the mechanism of the wind. The effects of line overlap and shifting ionization balance, as well as rotation, will improve the agreement between the predicted and observed shapes of the velocity law, as I have sketched above. I am hopeful that the models including stellar rotation will give a satisfactory account of the Be stars.

The present uncertainty about the temperature indicates that our understanding of the stellar wind is by no means comprehensive. It is likely that the models that have been made do correspond fairly well with the real star in some spherically-averaged way, but there may be significant inhomogeneities, hot and cold regions and so forth, that we have no idea of at present. This is a frustrating situation for the model maker -- imagine trying to study prominences and other features of the solar corona from a distance of 500 parsecs! We will have to study the inhomogeneities by measuring as many different integrals of the velocity and temperature structure as we can, but at some point we will simply accept our incomplete knowledge and go on to study other things.

This work was supported by National Science Foundation Grant AST77-23183 to the University of Colorado.

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DISCUSSION FOLLOWING CASTOR

Hearn: How well does this theory predict the mass loss rates?

Castor: I think it does quite well.

Lamers: The mass loss rate of τ Sco, as derived from the UV lines by Lamers and Rogerson is about 25 times smaller than the mass loss rate predicted if it is proportional to the luminosity. This indicates that the mass loss rates drop very drastically along the main sequence near types B0, i.e., close to the limit $M_{bol} \simeq -6$ found by Snow and Morton.

Castor: The radiatively-driven stellar wind theory was used by Rogerson and Lamers to derive the rate of mass loss in τ Sco. So presumably the theory should give the same answer if it manages to give the observed degree of ionization, and therefore the observed line strengths. The rate of mass loss is proportional to luminosity to a power larger than one, since the number of absorbing lines goes down as the wind becomes more tenuous.

Hearn: What about variability in the wind?

Castor: You get variations out if you put variations in. Presently, I have assumed it steady. However, I do test for instabilities which often appear to be present, so I assume they are there. The over-all wind is some average, with fluctuations superimposed.

Hearn: Well, there are instabilities and instabilities ... Do yours get large enough to destroy the over-all equilibrium?

Castor: I don't know for sure.

Thomas: What is the physical cause of these instabilities?

Castor: The instability analysis has not yet been done. I suspect instabilities could come from the random force. This could occur in a symmetric or asymmetric fashion. I believe it safe to say that these will not grow and disrupt the envelope.

Cassinelli: Does the concept of overlapping lines driving the wind lead to an increase often quoted "maximum mass loss rate" $L/v_\infty c$?

Castor: Yes, if you put in many, many lines, suppose one per Angstrom, and the expansion velocity is, say, six Angstroms, then you could end up with three times $L/v_\infty c$ for the mass loss rate. $L/v_\infty c$ is not an absolute upper limit on the rate, if you have overlapping lines.

Morton: Is there enough flux shortward of the Lyman limit in a star as cool as ε Ori (B0Ia) or ζ Ori (O9.7Ib) to drive the observed wind by radiation pressure?

Castor: I have not checked that. I think so.

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