Linear Difference Equations. By Kenneth S. Miller. Benjamin, New York (1968). x+105 pp.

This slender volume gives a stimulating introduction into the theory and applications of linear difference equations, or more specifically linear recurrence relations. With the presentation aimed at the undergraduate student, the style is concise and the exposition is clear and free of the gaps which makes many similarly condensed texts forbidding to read.

The book commences with an outline of the basic theory of solutions of linear homogeneous and inhomogeneous difference equations which precisely parallels the analogous theory for linear differential equations. The theory is presented in terms of the first order vector-matrix system of the form

$$y_t = A(t)y_{t-1}+w_t \quad (t = a+1, a+2, \ldots) \tag{1}$$

where y_t is a p-dimensional vector defined for $t=a, a+1, \ldots$, w_t is a p-dimensional vector and $A(t)$ a $p\times p$ matrix defined for $t=a+1, a+2, \ldots$

In the second section the theory for higher order difference equations is derived in terms of the results obtained for first-order systems in the first section. In particular we have a discussion of the one-sided Green's function and of the formal adjoint difference operator and corresponding analogues of the Lagrange identity and Green's formula for differential equations.

In the third section some special techniques are discussed, beginning with the composition of linear difference operators. Then there is a presentation of the powerful and elegant method of Laplace integrals for the case of difference equations with coefficients $\alpha_i(t)$ polynomials in t of the form

$$R(t)y_t = \alpha_0(t)y_t+\alpha_1(t)y_{t-1}+\cdots+\alpha_q(t)y_{t-q} = 0 \tag{2}$$

As examples the author obtains by this method integral formulae such as Heine's formula for Bessel functions of the first kind, Mehler's formula for Legendre polynomials and the integral representation of Hermite polynomials. Finally the method of generating functions is presented, and as an amusing exercise the binomial probability function for an experiment with independent trials is derived.

In a final section the application of the theory of linear difference equations to time series is discussed. The problem treated is that of the stochastic linear vector difference equation

$$y_t = A(t)y_{t-1}+u_t \quad t\in I, I = \{\ldots, -1, 0, 1, \ldots\} \tag{3}$$

where u_t is a member of a stochastic process, a p-dimensional vector with mean zero $\mathscr{E}u_t=0$, and second moment covariance matrix $\mathscr{E}u_tu_s=\Sigma_{ts}$. Conditions are given for the auto-regressive equation (3) to determine a valid stochastic process with convergence in mean square to a mean zero vector satisfying (3). For the case of uncorrelated forcing functions u_t and $A(t)$ a constant matrix, conditions are given for (3) to determine a wide-sense stationary stochastic process. A formula is derived for the spectral density function of the y_t process in terms of the spectra density function of the wide-sense stationary stochastic process u_t.

This short monograph is an informative and attractively written elementary account of a useful subject, though it might have been improved by a wider range of coverage. However, references to a wide selection of texts and papers are given which provide the interested reader with the opportunity to read further at a more advanced level in special topics in linear difference equations such as Sturm-Liouville theory or boundary problems and expansion theorems in the regular or singular case which have not been dealt with in this introductory volume.

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Opera Matematica. BY ALEXANDRU GHIKA. Editura Academici R.S.R., Bucarest (1968). 955 pp.

L'œuvre mathématique d'Alexandre Ghika, ex professeur à l'Université de Bucarest est assez vaste; les 1,000 pages de ce volume en sont témoin.

Des résultats profonds dans la théorie des fonctions analytiques marquent le début de sa carrière et sa thèse à Paris (1929). Il s'occupe aussi d'autres problèmes dans l'analyse, souvent comme application des méthodes de l'Analyse Complexe. Notamment, des équations différentielles d'ordre infini, équations intégrales, équations aux dérivées partielles et en différences finies.

Dans la dernière partie de sa vie, Ghika s'intéresse et travaille dans différentes directions de l'Analyse Fonctionnelle. Son influence et son esprit ont fait école, et beaucoup de mathématiciens roumains plus jeunes travaillent aujourd'hui dans cette direction.

En publiant ce volume, l'Académie Roumaine a fait un hommage reconnaissant à la mémoire d'un de ses plus brillants membres.

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Matrices and Linear Algebra. BY H. SCHNEIDER and G. P. BARKER. Holt, New York (1968). ix+385 pp.

The authors have written this text for sophomore, and perhaps freshman, students in physics, engineering, economics, and other fields outside mathematics.