

## Very viscous flows driven by gravity with particular application to slumping of molten glass

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This thesis examines the flow of very viscous Newtonian fluids driven by gravity. It is written with concern for specific applications in the optics industry, with emphasis on the slumping of molten glass into a mould, as in the manufacture of optical components which are in turn used to manufacture ophthalmic lenses. The mould shape is transferred to the upper free surface of the glass that does not contact the mould and which is, therefore, not affected by the roughness of the mould. This free surface is the critical optical surface and the manufacturing process is known as *thermal replication* [1]. However, the work has more general applicability, and disc viscometry, used to determine the viscosity of very viscous fluids, is also considered. In addition, one chapter of the thesis is devoted to the flow of dripping honey as another example of a very viscous flow to which the model can be applied.

The Stokes creeping-flow equations are used to model the very viscous flows of interest. The main solution method is finite elements, and a purpose-written computer program has been developed to solve the creeping-flow equations by this method. The present program is restricted to solving for either two-dimensional or axisymmetric flows but is extendible to three dimensions. In addition, semi-analytic series and asymptotic methods are used for some small portions of the work. These yield information not readily obtainable from the finite-element computations and also help in validating the finite-element code.

The optical applications of this work demand consideration of the topic of computing surface curvature, and therefore second derivatives, from inexact and discrete numerical and experimental data. For this purpose, fitting of B-splines by a least-squares method to coordinate data defining the surface has been used.

Much of the work assumes isothermal conditions, but in the context of the accuracy required in optical component manufacture it is also possible that non-isothermal effects will be important. Consequently, this restriction is eventually relaxed and some consideration given to non-isothermal conditions.

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In order to validate the creeping-flow model and finite-element program, comparisons of numerical simulations with experimental results are performed. A preliminary assessment of the importance of non-isothermal conditions to the thermal-replication process is also made by comparing isothermal and non-isothermal simulations with experimental results. The isothermal model is found to best match the experimental data and to give a good qualitative understanding of thermal replication.

#### REFERENCES

- [1] L. Smith, R.J. Tillen and J. Winthrop, 'New directions in aspherics: glass and plastic', in *Replication and molding of optical components* **896**, (M.J. Riedl, Editor), Proceedings of the Society of Photo-Optical Instrumentation Engineers (Washington, Bellingham, 1988), pp. 160–166.

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