Sketch of the History of Mathematics in Scotland to the end of the 18th Century: Part I.

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LEADING DATES.

Foundation of the Universities.

1411. ST ANDREWS.
(1450, St Salvator's; 1512, St Leonard's; 1537, St Mary's).

1450. GLASGOW.

1495. ABERDEEN.
(1593, Marischal College).

1582. EDINBURGH

1550–1617. JOHN NAPIER.
1638–1675. JAMES GREGORY.
1661–1708. DAVID GREGORY.
1666–1742. JAMES GREGORY, *Secundus.*
1687–1768. ROBERT SIMSON.
1692–1720. JAMES STIRLING.
1698–1746. COLIN MACLAURIN.
1717–1785. MATTHEW STEWART.
–1766. JOHN STEWART ("Triangles").
1746–1831. WILLIAM TRAIL.

1748–1819. JOHN PLAYFAIR.
1766–1832. Sir JOHN LESLIE.

1783. ROYAL SOCIETY OF EDINBURGH CONSTITUTED.

In the short sketch which I propose to give of the History of Mathematics in Scotland up to the end of the 18th century I must limit myself mainly to the work of the Universities. An adequate treatment of the subject would involve considerations of a general educational character that would range over the relations of the school to the University, the distribution of the various subjects of study and the place of mathematics in the educational system; but it is, of course, impossible to undertake such an extensive investigation at present, though it seems to me that an investigation, with special reference to mathematics, is greatly needed and might form the subject of a research that would be of real

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value as a contribution to the development of educational ideas. It would be improper, however, to omit all reference to school mathematics, since the school conditions determine, to a considerable extent, those of the University, as current discussions in Scotland clearly show, even though a sound appreciation of the relations between school and University may at times be lacking.

St Andrews, the oldest of our Universities, was founded in 1411 and it seems not inappropriate to quote the opinion of a distinguished historian on the general conditions of Scotland about that time. " The period of the first Stewarts," says Hume Brown (*History of Scotland,* I., 185), " has usually been regarded as one of chronic misery and arrested national development; and if we look only to the record of events it is difficult to avoid this conclusion. . . . Yet compared with the history of England and France throughout the same period that of Scotland has no special pre-eminence in misfortune. . . . Apart from the sensational record of events there are many indications that the nation did not spend its life in misery and that after the danger from England had ceased there was a steady expansion of the people along every line of social progress. In the fifteenth century three of the four Scottish Universities were founded; a succession of poets testify to the existence of an educated opinion; numerous Acts of Parliament, as well as other records, prove that there was a prosperous burgher class, both north and south of the Forth; and we have the testimony of foreigners to the fact that the country was largely under cultivation and that the peasant class of Scotland lived on better terms than their fellows elsewhere. It is only with such facts before us that we can attach their due significance to the wild deeds of king and noble which are apt to determine our judgement regarding every past age." It is well, I think, to bear this opinion in mind when we see the Universities suffering from lack of funds; the difficulty of carrying out any thoroughgoing educational policy is not necessarily due to straightened economic conditions.

Trustworthy information regarding early schools is not abundant; naturally, educational statistics, such as we are accustomed to, do not exist, but evidence is not awanting that long before the 14th century some provision was made for the education of youth according to the ideas of the age. Schools of various kinds were to be found over the greater part of Scotland and, in the more important towns, some provision was made for advanced education. Schools were, at first, under the control of the Church and the all important subject of instruction was " grammar "—a term, which in those days had a much wider connotation than now and was practically equivalent to Latin literature. In fact, down to the 18th century Latin dominated the schools to a degree that is hard for us to realise. The grammar school, whether attached to Cathedrals, Abbeys or Collegiate Churches, was controlled by the ecclesiastical authorities, while the expense of maintenance was met out of municipal funds. In these schools, so far as I can learn, little if any geometry was taught and arithmetic was not highly esteemed as a " culture " subject. I can find no definite statement of the nature or range of arithmetical teaching, but it seems probable that writing and the elements of arithmetic were taught in schools of a lower rank than the grammar schools and that children frequently attended such schools before being enrolled in a grammar school. (*Cf.* Grant, *Burgh Schools,* pp. 29-34, p. 398). In the schools more directly associated with the training of the clergy the *computus* would almost certainly have a place while, if we may judge by the contemporary practice on the Continent, the grammar school arithmetic would be based on that of Nicomachus. The algorisms, the arithmetics of practical life, were apparently considered by the ruling authorities unworthy of a place in higher school education. However that may be, writers on Scottish education are agreed that arithmetic did not in any proper sense form an element of school education till the latter half of the 17th century and that the same may be said regarding algebra and geomery. The low value thus placed on mathematics is not, however, peculiar to Scotland; John Wallis's *Account of some Passages in his Own Life* (quoted in Adamson's *Short History of Education,* p. 185) shows conclusively that matters were no better in England.

In the 17th century a great change begins to take place. Definite mention of arithmetic as part of the school curriculum is found at Aberdeen in 1628, at Irvine in 1665, at Dunbar in 1690 and at Stirling before 1697. The earliest notice of mathematics is at Glasgow in 1660 but there seems to be no further reference to it till the next century. About the middle of the 18th century the demand for an education of a more liberal and practical kind—a demand which had been steadily growing for several years and which had been partly met by broadening the curriculum of the grammar schools—gave rise to a new type of school to which the name of " Academy " was given. The oldest of these academies is Perth Academy, founded in 1760,* and it began with a very ambitious programme in mathe-

* Grant, p. 115.

matics, *viz.*, the higher branches of arithmetic; mathematical, physical and political geography; algebra, including the theory of equations, and the differential calculus; geometry, consisting of the first six books of Euclid; plane and spherical trigonometry; mensuration of surfaces and solids; navigation, fortification; analytical geometry and conic sections, natural philosophy, consisting of statics, dynamics, hydro-statics, pneumatics, optics and astronomy. At a later date chemistry was added.

It is, I should think, improbable that this programme was ever carried out with any degree of thoroughness but it marked a decided change for the better and the example of Perth was followed elsewhere so that good school courses of mathematics were established in many towns. These were not due to the demands of the Universities as no University imposed any serious preliminary knowledge of mathematics on its students though it was from the Universities that the teachers were drawn. In many of the parish schools the schoolmasters were competent to impart advanced instruction in Latin, Greek and Mathematics and were proud of such pupils as took advantage of the opportunities offered. I think, however, that while Scotland has good reason to be grateful to the old time parish schoolmaster, it is too often forgotten that many parishes were very badly off in respect of both teachers and buildings. The old *Statistical Account* has many sad details of incompetent schoolmasters and inadequate schools.

It is probable that before the founding of the Universities there was some provision for higher education but such provision was sporadic and unorganised. Scotsmen who desired University instruction must betake themselves to England or to the Continent and when we consider the dangers and the expense involved in such an undertaking in those days it is somewhat remarkable that so many were willing and able to face the difficulties. Scotsmen were to be found both as students and teachers in almost every important University, but Paris was their favourite place of resort. In 1326, the Bishop of Moray founded the Scots College there, to meet the wants of students from his own diocese, and at a later date the College was thrown open to all Scottish students. The pressing need for a University in Scotland itself was, however, strongly felt by the beginning of the 15th century, and before its close three Universities had been founded. It will be sufficient for me to give names and dates. St Andrews University was founded in 1411, the three Colleges of St Salvator's, St Leonards and St Mary's being established in 1450, 1512, and

1537 respectively. Glasgow was the second University, founded in 1451, and was followed by Aberdeen in 1495 (Marischal College was not founded till 1593 and was only united with King's College in 1860). The first three Universities were due to the action of the Church and papal bulls were their charters. Not till 1582 did Edinburgh get its University and in this case " the actual promoters and founders " were " the Town Council and the Ministers of the City " (Grant. *Hist. of Edin. Univ.*, I., 99).

So far as mathematics is concerned the teaching in the Universities must, in view of the position of the subject in the schools, have been of the most elementary character. The educational ideas that dominated the Universities till the middle of the 16th century and that were by no means completely transformed till a much later date were mediaeval; the curricula were drawn up in the interests of the church rather than for the promotion of learning and science. Again, the funds at the disposal of the Universities were quite inadequate to meet the expense of courses such as the charters implied and it must be admitted, I fear, that within the teaching and administrative bodies of the Universities themselves there was frequently a want of harmony that was fatal to educational interests. Many, probably most, of the Regents (or, as we should now call them, Professors) were learned men as judged by the standard of the time, but their training seems to have developed a tendency to concentrate on words rather than facts, on language rather than ideas. The disciplinary code was extremely detailed and the limits prescribed for teachers and students alike were totally inconsistent with that atmosphere of freedom which is essential to all higher education. While it was certainly better for Scotland that it should have institutions devoted to the higher learning I confess that I have derived from my study of the conditions of our Universities for the first two centuries of their existence the conviction that they failed to realise the aspirations of their founders and that Scottish students still required to go abroad—as many did—if they were bent on getting access to the best scholarship of the day.

It is perhaps worth noting that for many years (till the beginning of the 18th century, Grant, I., 147) the system of Regents was in force; the individual subjects were not assigned to different teachers, but to each teacher, or Regent as he was called, was assigned a class. The students of that class—which often meant all the students of a particular year—received their whole instruction during their University course from the same Regent who was thus kept in close touch with them and came to know their individual aptitudes. In the infancy of the Universities the system, I think, had some advantages but it lasted far too long; at King's College in Aberdeen it was not superseded till the end of the 18th century. But one can readily understand that the predilections of the Regents would often lead to difference of emphasis on different subjects and there is some evidence that mathematics was one of the subjects that at times received less than the normal attention.

Up to the Reformation the curriculum in Arts seems to have been practically identical in the three Universities. The Statutes of the Faculty of Arts in Glasgow University outline the course for graduation and I think we may assume that it represents the courses of the other two for all practical purposes. The preponderating subjects are drawn from Aristotelian philosophy; the mathematical subjects are :—The Sphere (of Sacrobosco, I suppose), perspective, algorism and the elements of geometry. The physics, the *de caelo* and the meteorology of Aristotle were also included in the course. Of the mathematical textbooks, other than the Sphere of Sacrobosco which is rather astronomical than mathematical, I can find no mention. The mathematical programme was thus very meagre and probably did not go beyond arithmetic of the scholastic type and the very elements of geometry; so far as I am aware there is no name of any special note in the history of mathematics that is associated with the Universities in these years.

The Reformation had a profound influence on the general life of Scotland but the educational proposals put forward were, as is well-known, very imperfectly realised in practice. Still a change for the better had set in as James Melville's account of his studies at St Andrews shows. Under the date 1572 he writes that in his "the second yeir of my course . . . the Primarius [Mr James Wilkie] a guid, peaceable, sweit auld man, wha luiffed me weill, teached the four speaces of the Arithmetik and sum thing of the Sphere." . . . "In the thrid yeir of our course we hard the fyve buikis of the Ethiks with the aught buiks of the Physiks [and de Ortu et Interitu]." . . ." The fourt and last yeir of our course . . . we lerned the buikis de Caelo and Mateors, also the Spher, more exactlie teachit be our awin Regent" (Diary, pp. 27, 28). When Andrew Melville was made Principal of Glasgow University James accompanied him and was appointed a Regent. In the Diary, p. 49, he gives a sketch of the courses laid down by his uncle and I quote the passage that deals with mathematics and physics :—" He teatched the Elements of

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Euclid, the Arithmetic and Geometrie of Ramus, the Geographie of Dyonisius, the Tables of Hunter, the Astrology of Aratus. . . . From that to the Naturall Philosophie; he teatched the buiks of the Physics, De Ortu, De Caelo, etc., also of Plato and Fernelius. With this he joined the Historie, with the twa lights thairof, Chronologie, Chorography, out of Sleidan, Menarthes and Melancthon." He concludes his sketch with the words, " Finalie, I dar say thar was na place in Europe comparable to Glasgow for guid letters, during these yeires, for a plentifull and guid chepe mercat of all kynd of langages, artes and sciences." It is, perhaps, worth adding that James says (p. 54), " the second yeir of my regenting I teatchit the elements of Arithmetic and Geometrie out of Psellus for shortness."

Again one of the Old Laws of King's College, Aberdeen, promulgated anew in 1641, gives a syllabus of the subjects taught in each of the four classes or years. [Fasti Aberd. p. 231]. No mathematics appear in the first year, but in the second Alsted's *Compendium of Arithmetic and Geometry* is prescribed, while in the fourth year the *De Caelo, de Ortu et Interitu* and, as occasion permits, the Elements of Astronomy, Geography, Optics and Music from Alsted's *Admiranda Mathematica* are among the subjects of study. The textbooks of Alsted are of interest as indicating the nature of the courses, but as none of Alsted's books were published before 1610 it is obvious that they could have formed no element in the instruction till long after the Reformation. The extracts given by Mr Anderson [*The Arts Curriculum.* Aberdeen 1892] from the Report of the Commissioners of 1647-48 do not add anything material to what has been already stated.

There can be no question, I think, that the latter half of the sixteenth century witnessed a decided improvement in University teaching but, after all, the improvement was rather an indication of a more liberal outlook than a solid advance in higher learning; some years had still to elapse before the Universities produced men of genuine mathematical distinction.

So far nothing has been said of the curricula in Edinburgh University. A detailed account, such as we do not possess for any of the other Universities, is given by Sir Alexander Grant in his *Story of the University of Edinburgh;* it will be sufficient for my purpose to state the demands in mathematics at the foundation of the College. In the Bajan or first year, no mathematics; in the Semi-Bajan or second year, towards the close of the session a compendium of Arithmetic was given to the students; in the Bachelor or third year there is no mention of mathematics while in the Magistrand or fourth year " the De Caelo of Aristotle and the Sphere of Johannes de Sacrobosco were read and demonstrations of Practical Astronomy were given. Then the students read the De Ortu, the Meteorologica and the De Anima and also Hunteri Cosmographia (a work on Geography)."

From this syllabus it is plain that mathematics was not placed in any better position than in the older Universities; in fact, the course is not so good as that which Melville set up in Glasgow—so hard is it to break away from long established tradition.

I shall not do more in describing general conditions in the Universities, though their detailed study is of great interest and full of lessons that modern educationalists might study with profit. Throughout the 17th century the political and ecclesiastical conditions in Scotland were very unfavourable to higher learning but, though harassed by many troubles, the Universities carried on, and towards the end of the century the place of mathematics became more assured and worthy exponents of it began to appear.

In the Napier Tercentenary Memorial Volume Napier's achievements are so fully and interestingly set forth that I may confine myself to a short statement on one or two of the more salient topics. The conditions prevailing in Napier's day are graphically presented by Professor Hume Brown in his sketch and the following extract is instructive :—

" Even to-day a certain mystery surrounds the figure of the Laird of Merchiston. Appearing at the time he did, and in an environment seemingly so strangely in contrast with his special pursuits, he strikes us as the most singular of apparitions among his contemporaries. He had one fellow-countryman, indeed, as a predecessor in the study of physical science. In the thirteenth century Michael Scott, like his contemporary Roger Bacon, had given his attention to that study; had gained a continental reputation as wide as Napier's, and an equally evil name among his countrymen of being in league with the infernal powers. But between Michael and Napier we can name no Scot whose interest lay specially in the domain of science, and the explanation is simple. In Scotland, as in other countries, the universities were the exclusive centres of intellectual activity, and the studies at the Universities were under the sole dominion of the Church, which naturally laid its ban on investigations that might imperil its own teaching. By his isolation Napier is thus wrapped in a certain mystery, and the mystery is deepened by the fact that we know so little of him, and that what we do know is at times strangely incongruous with the main preoccupations of his life" (Mem. Vol., p. 34).

John Napier was born in 1550 at Merchiston Castle, near Edinburgh. Of his education we know little. In 1563, he matriculated at St Salvator's College, St Andrews, where he was under the special charge of the principal, Dr John Rutherford, a man of strong character. His stay at St Andrews, however, was short, and he did not graduate. It is not certain, though it is, I think, highly probable that he spent some years on the Continent; in any case he was back in Scotland in 1571, when he is found to be domiciled at Gartness, in the parish of Drymen, Stirlingshire, where his father possessed lands. In the religious conflicts of the time he took a keen interest but, while fervent in spirit, he was also diligent in business and he handed down a very fine inheritance to his son Archibald, the first Lord Napier. He died on the 4th of April 1617, and was buried, it is fairly certain, in the old church of the parish of St Cuthbert's, Edinburgh—the church in which he was an elder.

It is very unfortunate that we know so little of the training Napier had in mathematics and we are reduced to such indications as can be gathered from his published works. In the *Descriptio* (p. 34) he mentions "Regiomontanus, Copernicus, Lansbergius, Pitiscus and others"; in the *Rabdologia* (page 21, Lib. I., Cap. iv), he quotes the *Decimal Arithmetic* of "that most learned mathematician Simon Stevin," and I think it is reasonable to assume that a man of his wealth and keen interest in mathematics would possess and study carefully all the more important of the accessible publications of the time. But while his recorded work is, of course, not independent of that of his predecessors his genius enabled him to strike out along new lines that opened up a new era in mathematics.

Of the *Rabdologia* (Edinburgh: Andrew Hart, 1617) and the *De Arte Logistica,* which only appeared in 1839, I shall say nothing except that in the former the construction and use of the "numbering rods," usually called "Napier's Bones," are fully described. The "Bones" are no doubt ingenious and were extensively used for some years after his death but they are rather an ingenious toy than an arithmetical machine. The books on which Napier's fame depends are the two that expound his logarithms, namely:—

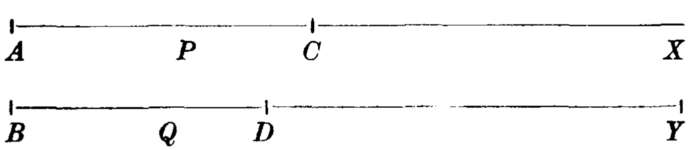
I. *Mirifici Logarithmorum Canonis Descriptio.* (Edinburgh: Andrew Hart, 1614). An English translation by Edward

Wright was published in 1616, after the translator's death, by his son Samuel Wright.

2. *Mirifici Logarithmorum Canonis Constructio.* (Edinburgh; Andrew Hart, 1619). This work, though written several years before the *Descriptio,* only appeared after Napier's death and contained Notes by Briggs.

The *Descriptio* defines a logarithm, lays down the rules for working with logarithms, illustrating their use particularly by applying them to the solution of triangles, and contains a Table of the logarithms. The book also has an excellent discussion of theorems in Spherical Trigonometry, usually known as Napier's Rules of Circular Parts—though this is a very inadequate statement of the contribution to Trigonometry that the relative pages of the Descriptio make.

The definition of a logarithm is very different from that now so familiar. In the first place Napier speaks always of the logarithm of a *sine,* not of a number; it was the simplification of Trigonometric calculations he had primarily in view and therefore it was the sine that bulked most largely. It is to be remembered that in his day, and for long after, the sine was a line, not a ratio, and the *whole sine* meant the radius of the circle whose half-chords were the sines. In the second place he makes use of two moving points to define a logarithm.



Suppose one point P to set out from the point A and to move along the line AX (of unlimited length) with a uniform velocity V; then suppose a second point Q to set out from B on the line BY, *of fixed length* r, at the same time as P sets out from A, starting with the velocity V and moving *not uniformly* but so that its velocity at any point, as D, is proportional to the distance DY from D to the end Y of the line BY. If C is the point P has reached, moving with velocity V, when Q, moving in the way described, has reached D then the number which measures AC is the logarithm of the sine (or number) which measures DY.[1]

[1] In the *Constructio* the name "Artificial Number" is used instead of "logarithm."

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In Napier's terminology r, the length of BY, is the whole sine; when Q is at B, P is at A so that the logarithm of the whole sine is 0. The logarithms of numbers less than BY are positive ("abundant"); if Q were to the left of B then P would be to the left of A so that the logarithms of numbers greater than the whole sine are negative ("defective"). In the *Constructio* $r=10^7$ so that $\log 10^7 = 0$ and $\log x \gtrless 0$ as $x \lessgtr 10^7$.

The fundamental rule on which all the others depend is that

$$\text{if } a : b = c : d \text{ then } \log a - \log b = \log c - \log d.$$

In the use of logarithms as they were first defined the greatest defect is that log 1 is not zero, so that we cannot say that

$$\log (ab) = \log a + \log b.$$

We first write the proportion, say $ab : a = b : 1$ then

$$\log (ab) - \log a = \log b - \log 1 \text{ or } \log ab = \log a + \log b - \log 1.$$

Of course we might use r instead of 1; thus let $rx = ab$ so that

$$x : a = b : r, \ \log x = \log a + \log b - \log r = \log a + \log b.$$

Then, when x has been found, multiply by r, which is easy if $r=10^7$.

It is quite obvious that the logarithm as thus defined is not so simple in actual work as the logarithm we now use. I may be allowed to refer to my article in the Memorial Volume for a fuller illustration of the various methods of operation.

Cumbrous as these logarithms are in many ways the *Descriptio* was hailed as a great discovery. Among those who recognised its merits, Henry Briggs, Professor at Gresham College, London, must be specially named. He contributed a Preface to Wright's translation, suggested that the logarithm of the tenth part of the whole sine should be a power of 10 (the logarithm of the whole sine being kept 0 as before) and began the calculation of new tables to embody his suggestion. In the summer of 1615, he journeyed to Edinburgh and stayed with Napier for a month when the proposed change was discussed. Napier, as Briggs relates, stated he was himself convinced that a change was needed and would have set about it long before had his health and leisure permitted, but he proposed a more far-reaching change than Briggs had done. He suggested namely, that zero should be the logarithm of *unity*, not of the whole sine, while the logarithm of the whole sine should be a power of ten. Briggs at once recognised the merit of Napier's proposal and set himself to the calculation of the new logarithms. In the following summer he

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again visited Napier and submitted the calculations he had made; further visits were made impossible by Napier's death and thus the calculation of logarithms in the form we now have them fell to Briggs in the first place, his *Arithmetica Logarithmica* appearing in 1624.

The proposal to take unity as the number whose logarithm is to be zero is vital to the ready working of logarithms and it came, as Briggs himself explicitly states, from Napier, but the credit due to Briggs for his labours in the calculation of logarithms can hardly be over-estimated; his name should always be associated with that of Napier in any fair account of the origin of logarithms.

It should be noted that what we now call Napierean logarithms are not the logarithms which Napier calculated and published in the *Descriptio*. The relaton between the two is given by the equation.

Napier's logarithm of $y = r(\log_e r - \log_e y)$ where $r = 10^7$ and $\log y$ is our Napierian logarithm of y.

Napier's method of calculating his logarithms is given in the *Constructio,* but into that subject I cannot enter.

The *Descriptio* (and also the *Constructio*) marks an important advance in theoretical *Trigonometry* as well as in practical calculations. For a good analysis and discussion of Napier's merits in this field I may refer to the second volume of Braunmühl's *Geschichte der Trigonometrie,* pp. 11-18; Professor Sommerville's article in the Memorial Volume is also well worth careful study.

In the 16th century great progress was being made in the development of algebra and in the 17th the progress was still more rapid. Though Scotsmen on the Continent, such as Alexander Anderson, who had settled in Paris, were active workers, Napier had no distinguished successor in Scotland until the second half of the 17th century, when we find the names of two men who made substantial contributions to pure and applied mathematics, James Gregory and his nephew David.

James Gregory (born 1638) was the third son of the Rev. John Gregory, minister of Drumoak, a small parish near Aberdeen. His mother was the daughter of David Anderson of Finzeach in Aberdeenshire, and related to the Alexander Anderson, just mentioned as a teacher in Paris. Gregory is said to have received his first lessons in mathematics from his mother, but in due course he passed on, first to the Grammar School and then to Marischal College, Aberdeen, where he graduated. In 1663 his *Optica Promota* was published in London and he spent some time in that city after the publication of his book in the hope of securing facilities for constructing a telescope on the principles he had laid down in the *Optica*. His efforts were, however, unsuccessful and he went to Italy where he continued his mathematical studies. After a residence of about three years in Padua he returned, in 1668, to Scotland. In 1669 he was appointed to the Chair of Mathematics at St Andrews; in that position he had a busy and, as the years passed, a rather troubled life so that he was glad to accept a call in 1674 to be Professor of Mathematics at Edinburgh where, as he says in a letter to a friend in Paris, " my salary is double and my encouragements much greater." (*Acad. Greg.* p. 49). His Edinburgh professorship was, however, very brief as he died in October 1675.

The mathematical writings of Gregory that were published under his own supervision, in addition to the *Optica Promota,* are :—

1. *Vera Circuli et Hyperbolae Quadratura in propria sua proportionis specie inventa et demonstrata* : Patavii, 1667.
2. A reprint of the *Quadratura* with an important addition, *Geometriae Pars Universalis, inserviens quantitatum curvarum transmutationi et mensurae* : Patavii, 1668.
3. *Exercitationes Geometricae* : London, 1668.

Much of Gregory's most original work is to be found in Rigaud's *Correspondence of Scientific Men of the Seventeenth Century;* had his letters been published as they were received he would have taken a much larger place in mathematical history than has usually been assigned him. (For an account of these letters see *Proc. Edin. Math. Soc.*, vol 41.)

The *Quadratura* and the *Geometriae Pars Universalis* are not altogether easy to read but the difficulty is due not so much to any obscurity in the reasoning as to the cumbrous phraseology of the geometrical form in which the demonstrations are carried out. If the books were re-written and the symbolism of modern mathematics employed throughout very much of the difficulty would disappear.

Quadratura. If OAB is a sector of a circle (or ellipse or hyperbola), O being the centre and AC, BC the tangents at A and B let the triangle OAB be called an inscribed triangle and the quadrilateral $OACB$ a circumscribed quadrilateral.

Let EDF be the tangent to the arc at D where OC cuts it;

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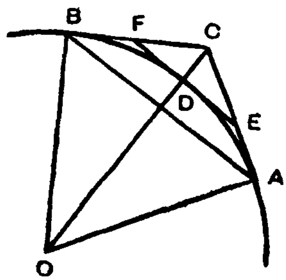
then the quadrilateral $OADB$ is a second inscribed quadrilateral and the polygon $OAEFB$ a second circumscribed figure.



Proceeding in this way Gregory constructs pairs of inscribed and circumscribed figures, thus forming two sets

$$(S)\begin{cases} u_1,\ u_2,\ u_3\ \ldots\ u_n & \text{inscribed polygons} \\ v_1,\ v_2,\ v_3\ \ldots\ v_n & \text{circumscribed polygons.} \end{cases}$$

For the circle $u_1, u_2, \ldots$ form an increasing sequence, $v_1, v_2, \ldots$ a decreasing sequence and these are connected by the relations

$$u_n = \sqrt{(u_{n-1}\, v_{n-1})}, \quad v_n = \frac{2u_n\, v_{n-1}}{u_n + v_{n-1}}, \quad \underset{n\to\infty}{L} (v_n - u_n) = 0.$$

The two sequences (S) are called a "converging series," the corresponding pairs u_n, v_n are called "converging terms," and the common limit is called "the termination of the series." It is from this beginning that the term "convergence" comes into use in connection with series.

In the course of the work Gregory shows great skill in carrying through complicated calculations in spite of very inadequate symbolism but the main interest now-a-days does not lie in the calculations. The important point is that he seeks to prove that t, the termination of the converging series, can not be an algebraic function of any pair of the terms u_n, v_n. To find the termination he says we must find a function such that $f(u_n, v_n) = f(u_{n+1}, v_{n+1})$ and then if t is the termination $t = f(u_n, v_n) = f(u_1, v_1)$. Here u_n, v_n are *any pair, i.e.* are variables and if such a function can be found it will give t. Now Gregory tries to prove, and believes he does prove, that f is not an *algebraic function* and that therefore circular (and also logarithmic) functions are not algebraic. His method of defining the *degree* of his algebraic function is not at all clear or satisfactory and would need fuller development, but, whatever decision we come to on that matter, it is a very remarkable attempt and is a striking proof of Gregory's philosophic acumen. His work applies to the hyperbola as well as to the circle so that logarithms come within its scope.

Geometriae Pars Universalis.—The aim of the *Geometriae Pars Universalis* is wider. It may be briefly described as an attempt to reduce to a coherent system the various methods that had been applied in the investigation of the rectification and quadrature of curves, the cubature of solids, the determination of centres of gravity and problems of maxima and minima. He says that in the more obvious propositions he uses the methods of Cavalieri, but that his chief object is to establish his propositions with geometric rigour. He shows wide reading but the methods adopted in the principal propositions seem to me to indicate a specially close affinity with Fermat's work of which he might have obtained a knowledge through Herigone's *Supplementum Cursus Mathematici* (vol. 6 of the *Cursus*) which was published in 1644.

The subtangent is an essential element in his work and in the 7th Proposition he shows how to find it or (what is the same thing) how to find $\frac{dy}{dx}$ for "those curves which Descartes calls geometrical." He works out the gradient for the curve

$$a^3 y^3 = c^3 (x^3 + ax^2)$$

but lays down no general rule; each particular case is simply stated when required and a reference is given to this Proposition for the method of determining it. So far as algebraic curves are concerned he evidently was master of $\frac{dy}{dx}$ but his lack of algebraic symbolism makes itself painfully felt in all these investigations.

His fundamental theorem in quadrature is, curiously enough, that which determines the surface of that part of a cylinder of height h with generators perpendicular to the xy-plane and the arc of the curve $y = f(x)$ between the points for which x has the values a and b as guiding curve. In modern symbols, if s represents the arc, the surface is

$$\int_a^b h \frac{ds}{dx} dx = \int_a^b h \sqrt{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}} dx.$$

His proof is extremely careful; the curve is expressly stated to be such that $f(x)$ is either monotonic or made up of a finite number of monotonic parts. If h is taken as unity the integral gives the length of the arc; but he extends the proposition to various cases in which h is a function of x and also to cases in which y is taken as the variable of integration. In this way he is able to effect many integrations that do not seem to us to come naturally within his scope. For example he finds (in the *Exercitationes*) the integrals of sec x and tan x; he also uses, as if it were quite familiar the integral of sin x, which is very easily established by his methods.

Again if (a, a') and (b, b') are the end-points of the graph of y he establishes the theorem

$$\int_a^b y\,dx = \int_{a'}^{b'} y\frac{dx}{dy}dy$$

and applies it to give a beautiful proof of the quadrature of $ax^{p/q}$.

In a series of propositions he discusses the mensuration of the surface of paraboloids and hyperboloids of revolution and of spheroids and rectifies parabolic arcs. He works out in detail the rectification of the curve $ay^{2n} = x^{2n+1}$ for the case $n = 2$ and states that his method applies for every positive integral value of n; the proof was, I think, well within his competence.

Gregory's developments in the *Geometria* and the *Exercitationes* show plainly that what we now call the Differential and Integral Calculus was near at hand, but while the great range of results and the ingenuity of the demonstrations are worthy of recognition the vital connection between differentiation and integration is not yet stated as is necessary for the advance that followed from Barrow's developments (*Lect. Geom.*, X, x.)

Gregory's name is usually attached to a series for $\tan^{-1}x$ and except in this connection and in relation to the phrase " convergent series " it is rarely mentioned. But the source from which the series issued sent forth many more theorems of great importance which seem to have been unnoticed. I may refer to my paper in the *Proc. Edinb. Math. Soc.* for an account of these phases of Gregory's work. I would like, however, to emphasise the fact—for I think it is a fact—that the assertion, so often repeated in the *Commercium Epistolicum,* of Gregory's indebtedness to Newton is simply not true; it is very little to the credit of the compilers of that volume that they misrepresented and deliberately concealed the great number of quite independent results that Gregory had attained. From the paper referred to it will be seen that Gregory had worked out for himself (i) *The Binomial Theorem*[1] in its most general form; (ii) *the general expression for $f(x)$* in terms of $f(b)$ and of the finite differences $\Delta f(b)$, $\Delta^2 f(b)$ ... and (iii) *general series* for the sines and cosines of multiple angles, with a large variety of series for the mensuration of the circle. Later, *after* seeing one of Newton's series, he developed many series and for the inspiration, though not for the methods, he was in these cases indebted, I think, to the simple *statement* (without explanations of any kind) of the Newtonian series.

After James Gregory's death the Chair of Mathematics in Edinburgh remained vacant for some years, the work being carried on meantime by a lecturer called John Young, but on 29th October 1683, his nephew, David Gregory, was appointed Professor. David Gregory was born in 1661, was educated at the Grammar School of Aberdeen and studied for a time at Marischal College, but came to Edinburgh to complete his College training. The fact that he was appointed before graduation, though he had passed the necessary examinations, is a striking testimony to his reputation. A MS Volume of notes of his course of lectures has been preserved and the range of subjects indicated by these notes, says Professor Chrystal, " will bear comparison with our curriculum as it is now [*i.e.* in the eighties of last century]." But the great point of interest attaching to David Gregory is that he has, in Chrystal's words, " the honour of having been the first to give public lectures on the Newtonian philosophy. This he did five and thirty years before these doctrines were accepted as part of the public instruction in the University of their inventor " (Grant's *Story of Edin. Univ.*, II., 296). In 1692 he left Edinburgh to take up the position of Savilian Professor of Astronomy at Oxford.

In this sketch I can deal only with Gregory's work when at Edinburgh. He seems to have been a teacher of very remarkable powers. A manuscript in Latin of a course on Practical Geometry was left in Edinburgh and was used by his successor in class teaching. An English translation was published by Maclaurin in 1745; the translation contained considerable additions by Maclaurin himself and was long in use. I have read the book and it seems to me to be an excellent piece of work.

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[1] The general Binomial Theorem

$$(P + PQ)^{\frac{m}{n}} = P^{\frac{m}{n}} + \frac{m}{n} AQ + \frac{m-n}{2n} BQ + \frac{m-2n}{3n} CQ + -$$

was first *published* in Wallis's Algebra (1685).

In the Simson Collection in Glasgow University Library there is a book with the title *Arithmeticae et Algebrae Compendium. In usum juventutis Academiae* (Edin. 1736), catalogued under David Gregory's name; but there is no name of the author in the book itself nor any explanation of its origin. It is a well-written textbook and compares not at all unfavourably with Maclaurin's. Some information as to its origin would be welcome.[1]

In 1684 Gregory published a tractate of 50 pages with the title *Exercitatio Geometrica de Dimensione Figurarum sive Specimen Methodi Generalis dimetiendi quasvis Figuras.* In the Introduction he refers to his uncle's work and states that he has found in such Notebooks of his uncle's as had come into his hands only examples but no systematic treatment of his methods. He had therefore to depend on himself for the discussion of many of the problems dealt with. The tractate shows a thorough mastery of the special processes developed in Gregory's communications to Collins. He applies the method of indivisibles exactly in the manner of Newton and is particularly clear in his explanation of what we should call the element of an integral, *ydx, yds, sdx,* etc. The problems include practically every type of those that had been handled up to that time. But he is specially good in the treatment of series. He uses freely the binomial theorem for the indices $\frac{1}{2}$, $-\frac{1}{2}$, $\frac{1}{3}$. $\frac{3}{2}$ and even expands $\sqrt{x}\,(2a-x)^{3/2}/(a-x)$. These examples are earlier than the similar cases given by John Craig, in his *Methodus figurarum . . . quadraturas determinandi* (1685). He states but does not prove that the expansion of $(x+a)^n$ for all values of n may be readily found.

As judged by the Tractate, David Gregory was a worthy successor of his uncle and added to the reputation of the Academic Gregorys.

David Gregory was succeeded in the Chair by his brother, James Gregory (*Secundus*), born in 1666. He graduated at Edinburgh in 1685, was Professor of Philosophy at St Andrews from 1685 to 1692 and was in that year appointed by the Town Council of Edinburgh to be Professor of Mathematics. Professor Chrystal remarks of him that "he seems to have been an able teacher but did not otherwise add to the reputation of the Gregory family." He held the Chair till 1725 when he retired on a pension that was drawn in part from his successor's salary; he died in 1742.

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[1] Further investigation has convinced me that the attribution to Gregory is incorrect (the suggestion, it should be said, was *not* Simson's) and that Maclaurin is the author or source.