6

Construction of the model

The next task is to find a gauge theory that contains the weak and electromagnetic currents described in the previous chapters. We consider a gauge model of electrons and their neutrinos. At the very beginning we must answer two questions:

(i) which group should we select for the theory; and
(ii) to which representation of the group should we assign the fermion fields?

The currents of the theory must include at least the charged weak current

$$J_\mu^+(x) = \bar{\nu}_e(x)\gamma_\mu(1-\gamma_5)e(x), \tag{6.1}$$

its Hermitian adjoint, and the electromagnetic current

$$J_\mu^{em}(x) = \bar{e}(x)\gamma_\mu e(x). \tag{6.2}$$

We need three vector fields with which to couple them. They correspond to the intermediate gauge bosons, $W^\pm$, and the photon. The smallest group is SU(2). This group, however, is unacceptable because the currents (6.1) and (6.2) do not form an SU(2) algebra. This becomes evident on considering the charges and studying their commutation relations. Consider the charges

$$T^+ = \frac{1}{2}\int d^3x\, \nu_e^+(x)(1-\gamma_5)e(x), \qquad T^- = (T^+)^+. \tag{6.3}$$

The commutator is

$$\begin{aligned}
\left[T^+, T^-\right] &= \frac{1}{4}\int d^3x\, d^3y \left[\nu^+(x)(1-\gamma_5)e(x), e^+(y)(1-\gamma_5)\nu(x)\right] \\
&= \frac{1}{2}\int d^3x\, d^3y \left[\nu^+(x)(1-\gamma_5)\nu(x) - e^+(x)(1-\gamma_5)e(x)\right]\delta^3(x-y), \\
iT^3 &= \left[T^+, T^-\right],
\end{aligned} \tag{6.4}$$

which is not the charge operator corresponding to the electromagnetic current.

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There are now two alternatives:

(i) introduce new leptons and modify the weak current $J_\mu^\pm$ so that we get the right SU(2) algebra, or
(ii) introduce another gauge boson W_3 and its corresponding current. In this alternative there are four gauge bosons, $W^\pm$, Z, and γ, and the group must be enlarged to SU(2) $\times$ U(1).

Both alternatives were actively studied and it became evident only after the discovery of neutral currents that Nature prefers the second solution.

We consider a theory based on the group SU(2) $\times$ U(1). We must decide how the electron and its neutrino transform under SU(2) and U(1), separately. From (6.3) and (6.4) we see that the charges

$$T^+ = \frac{1}{2}\int d^3x\left[\nu^+(1-\gamma_5)e\right], \tag{6.5}$$

$$T^- = \left[T^+\right]^+, \tag{6.6}$$

$$iT^3 = \frac{1}{2}\int d^3x\left[\nu_e^+(1-\gamma_5)\nu_e - e^+(1-\gamma_5)e\right] \tag{6.7}$$

generate an SU(2) algebra. This means that the left-handed fields

$$e_L = \frac{1}{2}(1-\gamma_5)e \quad \text{and} \quad \nu_L = \frac{1}{2}(1-\gamma_5)\nu$$

form an SU(2) doublet,

$$\Psi_L = \begin{pmatrix}\nu_L \\ e_L\end{pmatrix}. \tag{6.8}$$

The charges are defined as

$$T_i = \int d^3x\, \Psi_L^+\left(\frac{\tau_i}{2}\right)\Psi_L \tag{6.9}$$

and

$$Q = -\int d^3x\, e^+e = -\int d^3x\left(e_L^+e_L + e_R^+e_R\right). \tag{6.10}$$

Since we should include Q in the group transformations, Q must be a combination of T_3 and the generator of U(1), denoted by Y:

$$\frac{Y}{2} \equiv Q - T_3 = -\frac{1}{2}\int d^3x\left(e_L^+e_L + 2e_R^+e_R + \nu_L^+\nu_L\right). \tag{6.11}$$

This relation involves the difference between the charge Q and the weak isospin T_3 and it defines a new quantum number: the weak hypercharge. It is analogous to the Gell-Mann–Nishijima formula of the strong interactions which was established by empirical data. With this definition the charge coincides with what we were always

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using as charge, but the weak isospin and hypercharge, when extended to quarks, do not always coincide with the corresponding hadronic quantum numbers. In (6.11) there appear the left-handed and right-handed leptonic number operators

$$N_{\mathrm{R}} = \int \mathrm{d}^3x \, e_{\mathrm{R}}^{+} e_{\mathrm{R}}, \tag{6.12}$$

$$N_{\mathrm{L}} = \int \mathrm{d}^3x \left(e_{\mathrm{L}}^{+} e_{\mathrm{L}} + \nu_{\mathrm{L}}^{+} \nu_{\mathrm{L}} \right). \tag{6.13}$$

Thus the hypercharge operator in SU(2) is the unit matrix which commutes with the other generators of the group,

$$[Y,\ T_i] = 0 \quad \text{for} \quad i = 1, 2, 3. \tag{6.14}$$

From the relation $Y = -(N_{\mathrm{L}} + 2N_{\mathrm{R}})$ we deduce that

$$Y = \begin{cases} 1 & \text{for left-handed states,} \\ 2 & \text{for right-handed states.} \end{cases}$$

This satisfies the original requirements of selecting a group and the representations for the fields with

$$\Psi_{\mathrm{L}} = \begin{pmatrix} \nu_{\mathrm{L}} \\ \mathrm{e}_{\mathrm{L}} \end{pmatrix} \qquad \text{an SU(2) doublet,}$$

$$\Psi_{\mathrm{R}} = \mathrm{e}_{\mathrm{R}} \qquad \text{an SU(2) singlet.}$$

Finally, we give the parts of the Lagrangian describing the fermion and gauge fields. We denote by W_μ^i the gauge fields of SU(2) and by B_μ the field of U(1). The field tensors are written as

$$F_{\mu\nu}^i = \partial_\mu W_\nu^i - \partial_\nu W_\mu^i + g\varepsilon^{ijk} W_\mu^j W_\nu^k, \tag{6.15}$$

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \tag{6.16}$$

and the Lagrangian for the gauge fields is

$$\mathcal{L}_{\mathrm{B}} = -\frac{1}{4} F_{\mu\nu}^i F^{i,\mu\nu} - \frac{1}{4} G_{\mu\nu} G^{\mu\nu}, \tag{6.17}$$

with a summation understood over repeated indices. The gauge fields at this stage are massless. The Lagrangian for the leptons is

$$\mathcal{L}_{\mathrm{F}} = \mathrm{i}\bar{\Psi}_{\mathrm{R}} \gamma_\mu \left(\partial_\mu + \mathrm{i}g' \frac{Y}{2} B_\mu \right) \Psi_{\mathrm{R}} + \mathrm{i}\bar{\Psi}_{\mathrm{L}} \gamma_\mu \left(\partial_\mu + \mathrm{i}g' \frac{Y}{2} B_\mu + \frac{\mathrm{i}}{2} g \tau^k W_\mu^k \right) \Psi_{\mathrm{L}}. \tag{6.18}$$

We notice that the leptons are also massless because there are no $\bar{\Psi}_{\mathrm{R}} \Psi_{\mathrm{L}}$ and $\bar{\Psi}_{\mathrm{L}} \Psi_{\mathrm{R}}$ terms, which indeed are not SU(2) × U(1)-invariant.

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Select bibliography

Glashow, S. L. (1961), *Nucl. Phys*. **22**, 579
Salam, A., and Strathdee, J. (1972), *Nuovo Cimento* **11A**, 397
Weinberg, S. (1967), *Phys. Rev. Lett*. **19**, 1264

An early review of the electroweak theory is

Abers, E. S., and Lee, B. W. (1973), *Phys. Rep*. **9C**, 1

https://doi.org/10.1017/9781009402378.007 Published online by Cambridge University Press