could certainly recommend this book to a student and particularly Chapters 2 and 4 if he wanted to see nice proofs of any of the following: the theorems of Glivenko, Lévy and Bochner relating characteristic functions and convergence in distribution, the characterisation of Borel measures on Polish spaces by their values on compact sets, and generally for theorems concerning the pointwise and fluctuation behaviour of sums of independent (but not identically distributed) random variables.

However, one is left with the feeling that the features (i) and (ii) mentioned above make this book unsuitable as a general textbook—at least for British undergraduates. The approach is very interesting but over-formal for a first introduction. There is little looking ahead or verbal explanation—and little appeal to the intuition—rather it is an attempt to do intuitive mathematics formally. Undoubtedly this would be good for those who already have the intuition but would cause difficulties for a genuine beginner.

Chapter 1 is concerned with finite trials (finite sample spaces) and develops the idea of probability, random variable, law or distribution and real-valued random variable. It also introduces the idea of direct composition of two finite trials and tree composition (the former is just the product sample space and measure). Because these are introduced before the somewhat simpler ideas of conditional probability and independence (for finite event spaces) one can't help suspecting that students would become confused, particularly on pages 18–21 where one "deduces" appropriate trials to model multiple coin tossing games and multiple selection without replacement before one has even introduced the notion of independence as it applies to the various coin tosses or selections.

The second chapter is a nice description of basic measure theory. It starts with Carathéodory's construction of Lebesgue integral, and proceeds to consider standard measure spaces and measures on Polish spaces (but in common with most others ignores Doob's truly simple proof of Lusin's theorem). The chapter finishes with a discussion of probability measures on $\mathbb{R}$, proving that they are a Polish space with respect to convergence in distribution and giving the usual characterisations of this convergence in terms of characteristic functions. Again, to bolster the motivation of the novice reader some mention of sums of independent random variables would have been a good idea when convolutions of probability measures were treated.

The third chapter is on random variables (rather than measurable functions). Here a certain problem arises. In this book a probability measure μ uniquely determines a σ-algebra $\mathscr{D}(\mu)$ on which it is typically complete. If X is a map which is measurable from $\mathscr{D}(\mu)$ to say the Borel sets on $\mathbb{R}$ then the law μ^X is defined not as a Borel measure but on every set for which $X^{-1}(A)$ is μ-measurable. Thus μ^X might not be completely determined by its values on Borel sets. To overcome this the author follows Kolmogorov by considering random variables as being defined on a restricted class of measurable spaces (perfect, separable). This is fine; however, when combined with the enthusiasm for completions it makes the exposition slightly complicated. Conditional expectation is also treated in a non-standard way.

The final chapter discusses sums of random variables giving Lindeberg's central limit theorem and a law of the iterated logarithm. In this "harder" analysis the book is at its best and that is very good.

This book can easily be recommended to anyone preparing a course on (analytic) probability and to the able student looking for additional insights. It also has some good do-able problems.

It is a great pity for us that Professor Itô has so far only translated the first four chapters of his book; it would have been very interesting to see what the master had to say about stochastic processes.

T. J. LYONS

ApSimon, H. *Mathematical byways in Ayling, Beeling and Ceiling* (Recreations in Mathematics No. 1, Oxford University Press, 1984), 97 pp. £5.95.

This little collection of eleven recreational problems has been accumulated by the author over a period of about thirty years. They involve only elementary algebra, geometry and trigonometry and could be enjoyed by anyone with a mathematical bent over the age of sixteen.

About half the problems are variants of problems I have seen elsewhere. At this level it is not easy to be both entertaining and original. Nevertheless even the old favourites appear in acceptable new dress and cannot be dismissed out of hand even by the puzzle-addict. The other half are new to me and involve mathematical ideas that do not feature too often in puzzles. I particularly enjoyed "Bowling Averages" with its use of inequalities and "Wrapping" which makes geometrical sense of what shopkeepers have been doing for years.

The presentation perhaps encourages cheating in that each problem is immediately followed by a very full solution and exposition, but this in itself is far more than half the interest of the book as each problem is placed by the author in its mathematical context, sometimes with ideas for further hard generalizations.

The style is easy to read and in the solutions makes one wonder what the difficulty was—a rare gift. The printing and diagrams are clear with few misprints.

It is an entertaining and well-presented book which would be a good addition to the library of puzzle-collectors and an acceptable present for anyone interested in recreational mathematics.

MAGNUS PETERSON

BAKER, ALAN, *A concise introduction to the theory of numbers* (Cambridge University Press, 1984), xii+95 pp. £15 cloth, £4.95 paper.

The adjective "concise" is an accurate description of this excellent book. A very bright student could read it without help, but the average honours student will require considerable assistance in following several of the arguments given. As a text for a lecturer to expand on it is admirable. After the usual preliminaries, there are chapters on quadratic residues, forms and fields, and on Diophantine approximation and equations. In the 91 pages of text a surprisingly large amount of material is covered. At the end of each chapter the reader is brought up to date by accounts of recent developments, in many of which the author has played a leading part.

I found the last chapter particularly interesting. In it the equations of Pell, Mordell ($y^2=x^3+k$), Fermat, and Catalan ($x^p-y^q=1$) are discussed. In 15 pages a complete account of these problems is, of course, impossible, but the author has been remarkably skilful in conveying an understanding of the difficulties involved by his comments and choice of examples.

There are exercises at the end of each chapter, some of considerable difficulty. The book is beautifully printed, as one would expect. The only misprint I found was in the index, where, as one knows from experience, printers find it hard to believe that Lebesgue does not have a *q* in place of its *g*.

R. A. RANKIN

CONWAY, J. B. *Subnormal operators* (Research Notes in Mathematics 51, Pitman, 1981), xvii+476 pp. £15.75.

Of the various special classes of Hilbert space operators which have been studied, normal operators are probably the best understood. Up to unitary equivalence, they are just the operators on L^2 spaces given by multiplication by bounded measurable functions and they can be classified in measure theoretic terms. In 1950, P. R. Halmos generalised the notion of normality by introducing the class of subnormal operators. These are the operators which have normal extensions; that is, an operator S on a Hilbert space H is subnormal if there is a normal operator N on a Hilbert space K containing H such that H is N-invariant and S is the restriction of N to H. The motivating example was the unilateral shift, which is defined as multiplication by z on the Hardy space H^2 and has an obvious normal extension acting on L^2 of the circle.

The early developments in the theory of subnormal operators used mainly the tools of abstract operator theory, whereas more recent work has relied heavily on the techniques of function theory and uniform algebras. The reason for this is that each cyclic subnormal operator S can be represented as multiplication by z on $P^2(\mu)$, where μ is a compactly supported measure on the plane and $P^2(\mu)$ is the closure in $L^2(\mu)$ of the (analytic) polynomials. Furthermore, the ultraweakly

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