Detached Binary Systems

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RESONANCE BETWEEN PULSATION MODES DUE TO TIDAL PERTURBATION

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ABSTRACT. The effect of tidal perturbation to stellar pulsation is a relatively underdeveloped problem in the theory of variable stars. We derive amplitude equations describing the resonances between pulsational modes and orbital motion taking into consideration the rotation of stars as well. In the case of δ *Scuti* stars the two-mode-tidal resonance was found to be the most powerful effect. If the difference between frequencies of excited and damped mode is close to the orbital frequency, parametric excitation of the damped mode may occur, while the other mode loses energy. We discuss this effect for a wide range of parameters.

1. Introduction

Some preliminary discussion of the general appearance of oscillations generated by the gravitational tide have already appeared at the end of the last century. After these preliminary studies Cowling (1941), Zahn (1977) and others discussed the oscillations of a star by tidal interaction.

If we consider an intrinsically pulsating variable star which is a component of a close binary system we can expect a modification in its oscillation behaviour comparing with a single counterpart. (There are e.g. observational allusions that multiperiodic variables can be found with higher probability among the components of binaries.)

We have learned in the last decade that in realistic models the oscillation modes will not be independent and there will appear an interaction between them. The behaviour of these nonlinear systems can be treated as if the modes of linear approximation would be excited but with variable amplitudes in time. Physically this means that energy transfer would be possible between the excited and damped modes. Dziembowski,Królikowska and Kosovitchev (1988) have found on this way that the amplitude of δ *Scuti* stars will be limited through this mode coupling by the rotation.

Papaloizou and Pringle (1981) extended the description of a rotating star incorporating an external time dependent force into the equations representing the tidal interaction. From the derived equations for the amplitude variations can be concluded that resonance requirements giving rise to mode coupling can be fulfilled by $m\sigma_1 - n\sigma_2 = r\Omega_r$, however, the order of magnitude of coupling coefficients cannot be seen from their formalism.

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2. The equations of tidal perturbation

Dziembowski, Królikowska and Kosovitchev (1988) generalized the formalism of the pulsation of rotating stars developed by Lynden-Bell and Ostriker (1967). The second order adiabatic momentum equation for the displacement vector ξ, including a forcing term is

$$\frac{\partial^2 \xi}{\partial t^2} - \frac{i}{\rho_0}\mathbf{B}(\frac{\partial \xi}{\partial t}) + \frac{1}{\rho_0}\mathbf{C}(\xi) + \frac{1}{\rho_0}\mathbf{N}(\xi,\xi) + \mathbf{T}_1(\xi) + \mathbf{T}_2(\xi,\xi) = 0, \tag{1}$$

where $\mathbf{B}$ and $\mathbf{C}$ are the linear operators described by Lynden-Bell and Ostriker (1967), $\mathbf{N}$ is the nonlinear operator introduced by Dziembowski, Królikowska and Kosovitchev (1988) and $\mathbf{T}_1$ and $\mathbf{T}_2$ are the first and second order (in ξ) operators of the tidal perturbation. The tidal operators are derived from the gravitational potential of interaction $\sum_{m=-\infty}^{\infty} U_m(\mathbf{r})e^{im\Omega t}$, $(U_k = U^*_{-k})$ as follows:

$$\mathbf{T}_1(\xi) = \sum_m \mathbf{T}_1^{(m)}(\xi)e^{im\Omega t} = \sum_m (\xi\nabla)(\nabla U_m)e^{im\Omega t},$$

$$\mathbf{T}_2(\xi,\xi) = \sum_m \mathbf{T}_2^{(m)}(\xi,\xi)e^{im\Omega t} = \frac{1}{2}\sum_m (\xi\xi\nabla\nabla)(\nabla U_m)e^{im\Omega t}. \tag{2}$$

For the standard perturbation method we use the eigenfunctions $\mathbf{h}_k(\mathbf{r})e^{i\omega_k t}$ of the unperturbed, linear-equation ($\mathbf{N} = \mathbf{T}_1 = \mathbf{T}_2 = 0$). Since the operators $\mathbf{B}$ and $\mathbf{C}$ have Hermitian behaviour, the $\mathbf{h}_k$ eigenfunctions are orthogonal. We used the normalization $\int \rho_0 \mathbf{h}_k, \mathbf{h}_l d\mathbf{r} = \delta_{k,l}$.

In order to obtain the equations for the amplitudes in the case of tidal resonance we introduce the trial function:

$$\xi(\mathbf{r},t) = \sum_{k=-2,k\neq 0}^{2} Q_k(t)\mathbf{h}_k e^{i\omega_k t}, \tag{3}$$

where $Q_k = Q^*_{-k}$, $\mathbf{h}_k = \mathbf{h}^*_{-k}$, $\omega_k = -\omega_{-k}$. Inserting this function to Eq. (1), multiplying by $\rho_0\mathbf{h}_j e^{i\omega_j t}$ and integrating over the volume we get

$$\sum_k \ddot{Q}_k e^{i(\omega_k+\omega_j)t}\langle \mathbf{h}_j, \mathbf{h}_k\rangle + i\sum_k \dot{Q}_k e^{i(\omega_k+\omega_j)t}\langle \mathbf{h}_j, 2\omega_k\mathbf{h}_k + \frac{1}{\rho_0}\mathbf{B}(\mathbf{h}_k)\rangle +$$

$$\sum_{k,l} Q_k Q_l e^{i(\omega_k+\omega_l+\omega_j)t}\langle \mathbf{h}_j, \frac{1}{\rho_0}\mathbf{N}(\mathbf{h}_k,\mathbf{h}_l)\rangle + \sum_{k,m} Q_k e^{i(\omega_k+\omega_j+m\Omega)t}\langle \mathbf{h}_j, \mathbf{T}_1^{(m)}(\mathbf{h}_k)\rangle +$$

$$\sum_{k,l,m} Q_k Q_l e^{i(\omega_k+\omega_l+\omega_j+m\Omega)t}\langle \mathbf{h}_j, \mathbf{T}_2^{(m)}(\mathbf{h}_k,\mathbf{h}_l)\rangle = 0 \tag{4}$$

where $\langle \mathbf{f}, \mathbf{g} \rangle = \int_V \rho_0 \mathbf{f}\mathbf{g} d\mathbf{r}$.

Resonance coupling occurs in the above equations, if one of the exponential functions varies slowly compared to the others. Averaging out the rapidly varying time (integrating over the fast time) we get the amplitude equations for resonance. The resonance $\omega_1 = \omega_2 + \omega_3 + \Delta\omega$, $|\Delta\omega| \ll |\omega_k|$ leads to the three-mode-resonance of single stars described by Dziembowski, Królikowska & Kosovitchev (1988).

In the following we take into consideration the resonances with the orbital frequency Ω arising through tidal interaction.

2.1. TWO-MODE-TIDAL RESONANCE

Two modes interact with the tidal perturbation, when the frequencies satisfy one of the criteria

$$\omega_1 - \omega_2 = \Omega + \Delta\omega, \text{ or } \omega_1 + \omega_2 = \Omega + \Delta\omega, \tag{5}$$

where $|\Delta\omega| \ll |\omega_k|$ for $k = 1, 2$. For the first case, the Eq. (4) integrating over the fast time leads to the following forms for $j = -1$ and $j = -2$

$$i\dot{Q}_1 \langle \mathbf{h}_1^*, 2\omega_1 \mathbf{h}_1 + \frac{1}{\rho_0} \mathbf{B}(\mathbf{h}_1) \rangle + Q_2 e^{-i\Delta\omega t} \langle \mathbf{h}_1^*, \mathbf{T}_1^{(+1)}(\mathbf{h}_2) \rangle = 0 \tag{6}$$

$$i\dot{Q}_2 \langle \mathbf{h}_2^*, 2\omega_2 \mathbf{h}_2 + \frac{1}{\rho_0} \mathbf{B}(\mathbf{h}_2) \rangle + Q_1 e^{+i\Delta\omega t} \langle \mathbf{h}_2^*, \mathbf{T}_1^{(-1)}(\mathbf{h}_1) \rangle = 0 \tag{7}$$

2.2. MONO-MODE-TIDAL RESONANCE

Similarly, a pulsation mode may be in direct resonance with the tidal force, if twice the mode frequency is close to the orbital period or its harmonics: $2\omega_1 = \Omega + \Delta\omega$ with similar calculations as in the case of two-mode-tidal resonance we get the amplitude-equation for the tidal excitation:

$$i\dot{Q}_1 \langle \mathbf{h}_1^*, 2\omega_1 \mathbf{h}_1 + \frac{1}{\rho_0} \mathbf{B}(\mathbf{h}_1) \rangle + Q_1^* e^{-i\Delta\omega t} \langle \mathbf{h}_1^*, \mathbf{T}_1^{(+1)}(\mathbf{h}_1^*) \rangle = 0 \tag{8}$$

By similar calculations we also can derive equations for second order two-mode-tidal resonance with the following criteria: $2\omega_1 + \omega_2 = \Omega + \Delta\omega$ or $2\omega_1 + \omega_2 = \Omega + \Delta\omega$.

3. The two-mode-tidal resonance

Let us see the two-mode-tidal resonance in more detail. To calculate the coupling coefficients we use the perturbation potential introduced by Zahn (1977).

To take into consideration nonadiabatic effects we introduce the linear damping and driving rates γ_k (for more information see Dziembowski 1982). We also introduce a cubic damping (see e.g. Buchler and Goupil 1984) for the excited mode to balance the fixed amplitude solution for the unperturbed oscillation. We get the final form of the amplitude equations of the two-mode-tidal resonance by introducing the new variables $Q_k = q_k e^{i\Phi_k}$, $X = \Phi_2 - \Phi_1 - \Delta\omega t$:

$$\dot{q}_1 = \gamma_1 q_1 - \alpha q_1^3 - C_1 \sin(X) q_2 \tag{9}$$

$$\dot{q}_2 = \gamma_2 q_2 + C_2 \sin(X) q_1 \tag{10}$$

$$\dot{X} = (\frac{C_1 q_1}{q_2} - \frac{C_2 q_2}{q_1}) \cos(X) - \Delta\omega \tag{11}$$

where C_1 and C_2 are the coupling coefficients of tidal interaction. One can easily derive the following identities for the fixed point solution (i.e. when the temporal derivatives are equal to zero):

$$q_1 = (\frac{\gamma_1}{\alpha_1})^{1/2}(1 + \frac{C_1 C_2}{\gamma_1 \gamma_2} \sin^2(X))^{1/2}, \tag{12}$$

$$q_2 = -C_2/\gamma_2 \sin(X) q_1, \tag{13}$$

where X satisfies the following equation:

$$(\frac{-\gamma_2}{\sin(X)} + \frac{C_1 C_2 \sin(X)}{\gamma_2}) \cos(X) = \Delta\omega. \tag{14}$$

We can see from (12) that the ratio of the perturbed and unperturbed value of q_1 is independent from the cubic damping i.e. the amplitude of the pulsation.

Eq. 14 leads to a cubic equation for $\sin(X)$. We solved the above equations numerically and tested the stability of the fixed point solutions.

The amplitudes derived from the equations 12 – 14 for typical δ Scuti parameters (see e.g. Dziembowski and Królikowska 1985) were calculated. In all cases the amplitude of the damped mode remained in a low level (thousands of the excited mode). The value of the excited mode, however, may be changed significantly.

We found, that the tidal perturbation can kill the oscillation for a wide range of parameters. As the amplitude decreases, it losts its stability and in some cases the non-zero fixed point disappears.

From the above consideration we can conclude, that the tidal perturbation may change significantly the observed amplitude of δ Scuti stars.

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